Math 1210 Midterm Review

(Sections 3.4, 3.6, 3.8, 3.9, 4.1, 4.2, 4.3, 4.4)

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<u>Instructions</u>: Please show all of your work. All answers should be completely simplified, unless otherwise stated. Report answers as exact, i.e., no approximations. No calculators or electronics of any kind are allowed.

1. Find the antiderivative of each of the following. Do not forget the constant term.

   (a) $f(x) = \frac{2x^7+3}{x^3}$

   (b) $f(x) = \frac{3x^8+x^2+1}{x^2}$

   (c) $f(x) = (x^2+5)^{10}x$

(d) $f(x) = (x^3 + 6x)^5(x^2 + 2)$

(e) $f(x) = \sin^9 x \cos x$

2. **Solve the following differential equations.**

   (a) $\frac{dy}{dx} = x^2 + 1$, $y(1) = 1$

(b) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}, x > 0, y > 0,\ y(1) = 4$

(c) $\frac{du}{dt} = u^3(t^3 - t),\ u(1) = 4$

(d) $\frac{dz}{dt} = t^2 z^2,\ z(1) = \frac{1}{3}$

(e) $\frac{dy}{dx} = y^2 x(x^2 + 2)$, $y(1) = 1$

3. **An application of the mean value theorem**
Suppose $f(x)$ is an everywhere differentiable function defined on the whole real line and $|f'(x)| \leq 10$ for all x. Show that $|f(7) - f(5)| \leq 20$.

4. **An application of the intermediate value theorem**
Show that $f(x) = 2x^3 - 9x^2 + 1 = 0$ has exactly one solution on each of the intervals $(-1, 0)$,$(0, 1)$, and $(4, 5)$.

5. **Find the values of the following sums, assuming** $\sum_{i=1}^{100} a_i = 30$ **and** $\sum_{i=1}^{100} b_i = 40$

(a) $\sum_{n=1}^{100}(3a_n + 4b_n)$

(b) $\sum_{n=0}^{99}(4a_{n+1} - 3b_{n+1})$

(c) $\sum_{n=1}^{100}(n-1)(4n+2)$

(d) $\sum_{n=1}^{100}((2n-1)^2+a_n)$

(e) $\sum_{n=1}^{100}(3a_n+n^2-n)$

6. **Definite integrals**
   Calculate the following integrals by using (1) the definition and (2) techniques for computing integrals.
   (a) $\int_0^2 (x^2+1)dx$ by using the definition

(b) $\int_0^2 (x^2 + 1)dx$ by direct calculation

(c) $\int_{-10}^{10} (x^2 + x)dx$ by using the definition

(d) $\int_{-10}^{10}(x^2 + x)dx$ by direct calculation

7. **Integrals of odd and even functions**
   Let f be an odd function and g be an even function, and suppose that $\int_0^1 |f(x)|dx = \int_0^1 g(x)dx = 3$. Use geometric reasoning to calculate each of the following:

   (a) $\int_{-1}^{1} f(x)dx$

   (b) $\int_{-1}^{1} g(x)dx$

   (c) $\int_{-1}^{1} |f(x)|dx$

(d) $\int_{-1}^{1}[-g(x)]dx$

(e) $\int_{-1}^{1} xg(x)dx$

(f) $\int_{-1}^{1} f^3(x)g(x)dx$

8. **Use The First Fundamental Theorem Of Calculus to find the derivatives of the following functions.**

   (a) $G(x) = \int_{x}^{1} 2tdt$

(b) $G(x) = \int_0^x (2t^2 + \sqrt{t})dt$

(c) $G(x) = \int_{-x^2}^{x} \frac{1}{1+t^2} dt$

(d) $G(x) = \int_1^{x^2} x^2 t dt$

9. **Newton's method**
Approximate the real root of $f(x) = 4x^3 + x - 5 = 0$ accurate to four decimal places. Choose $x_0 = 2$ as your initial value. Please do use Newton's method to find the real root even if you might or might not see what the real root is. The purpose of this exercise is to give a practice of Newton's method.

10. **Comprehensives**
Show that the rectangle with maximum perimeter that can be inscribed in a circle is a square. It is OK to assume we are working with the unit circle, that is the circle has a radius equal to one.

11. **Comprehensives**
A farmer wishes to fence off four identical adjoining rectangular pens. The farmer has exactly 1600 feet of fencing, which he wishes to use completely. What should the width and length of each pen be so that the pen has maximum area?