

Math5700
Induction Proof Homework

A. For the Fibonacci sequence, defined recursively as

$a_1=1$, $a_2=1$, $a_n=a_{n-1}+a_{n-2}$, $n \geq 2$, I claim the direct formula is

$$a_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{\sqrt{5} \cdot 2^n} \text{ for all } n=1,2,3,\dots$$

Prove this.

B. Prove that for all natural numbers n , $n^2 - n$ is even.

C. Make a conjecture about the sum $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}$ and prove your claim.

D. For f given recursively by $f(0)=0$, $f(n)=f(n-1)+3n+2$ for all $n = 1, 2, \dots$ find an explicit formula for $f(n)$ and prove your formula is valid.