Problems

1. $\int \dfrac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx$

2. $\int \cos^3 x\ \sin^4 x\ dx$

3. $\int \dfrac{2x^4-2x^3+6x^2-5x+1}{x^3-x^2+x-1}\,dx$

4. $\int x\sec^2 x\,dx$

5. $\int \dfrac{x^2}{(1-9x^2)^{\frac{3}{2}}}\,dx$

6. $\int \cos(\sqrt{x})\,dx$

7. $\int \dfrac{3x-5}{\sqrt{1-x^2}}\,dx$

8. $\int \dfrac{3x^3-18x^2+29x-4}{(x+1)(x-2)^3}\,dx$

9. $\int \dfrac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx$

10. $\int \dfrac{\sec^2 x}{(1+\tan x)^2}\,dx$

11. $\int \dfrac{x^2-x-21}{2x^3-x^2+8x-4}\,dx$

12. $\int x^2\ e^{3x}\,dx$

13. $\int \dfrac{2x-1}{x^2-6x+13}\,dx$

14. $\int \dfrac{e^{2x}}{4+e^{4x}}\,dx$

15. $\int \arcsin x\,dx$

16. $\int x^5\ \ln x\,dx$

17. $\int \sqrt{x}\ e^{\sqrt{x}}\,dx$

18. $\int \tan^3 x\ \sec^5 x\,dx$

19. $\int \dfrac{\sqrt{4x^2-9}}{x}\,dx$

20. $\int \arcsin(2x)\,dx$

21. $\int y\ \arctan(y)\,dy$

22. $\int \dfrac{4x}{(x^2+1)^3}\,dx$

23. $\int \dfrac{\sqrt[3]{x+8}}{x}\,dx$ (This is a very challenging one.)

24. $\int x\sqrt{2-3x}\ dx$

25. $\int \sqrt{x(6-x)}\ dx$

26. $\int \cos(4x)\cos(3x)dx$

27. $\int 3x(4^x)dx$

28. $\lim_{x\to\infty} (\sqrt{x^2+4} - \arctan x)$

29. $\int_4^\infty \dfrac{1}{x\sqrt{x}}dx$

30. $\lim_{x\to\frac{\pi}{2}^-} (\cos x\ \ln(\cos x))$

31. $\lim_{x\to 0} (1+3x)^{\frac{1}{2x}}$

32. $\lim_{x\to 0^+} (x^2\ \ln x)$

33. $\int_3^\infty \dfrac{1}{x-1}dx$

34. $\lim_{x\to\infty} \left(\dfrac{\ln(\ln x)}{\ln x}\right)$

35. $\lim_{x\to 0} \left(\dfrac{\arcsin(2x)}{\arcsin(x)}\right)$

36. $\int_{-\infty}^2 \dfrac{1}{5-2x}dx$

37. $\lim_{x\to\infty} (x^2-1)e^{-x^2}$

38. $\lim_{x\to 0^+} (2x+1)^{\cot x}$

39. $\lim_{x\to 0^+} \left(\dfrac{1+x-e^x}{x^3}\right)$

40. $\int_{-\infty}^\infty \dfrac{1}{1+x^2}dx$

41. $\lim_{x\to -3} \left(\dfrac{x}{x^2+2x-3}-\dfrac{4}{x+3}\right)$

42. $\int_{-\infty}^\infty \dfrac{1}{e^x+e^{-x}}dx$

43. $\int_0^\infty xe^{-x}dx$

44. $\lim_{x\to 5} \left(\dfrac{\sqrt{x-1}-2}{x^2-25}\right)$

45. $\lim_{x\to 0} \left(\dfrac{e^x+e^{-x}}{x^2}\right)$

46. $\int_{-2}^0 \dfrac{1}{\sqrt{4-x^2}}dx$

47. $\displaystyle\int_{-1}^{1} x^{\frac{-4}{3}}\, dx$

48. $\displaystyle\int_{0}^{\frac{\pi}{2}} \tan^2 x\, dx$

49. $\displaystyle\int_{0}^{9} \frac{x}{\sqrt[3]{x-1}}\, dx$

50. $\displaystyle\int_{-1}^{2} \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$

Answers

1. $2\left(\sqrt[6]{x}+1\right)^3 - 9\left(\sqrt[6]{x}+1\right)^2 + 18\left(\sqrt[6]{x}+1\right) - 6\ln\left|\sqrt[6]{x}+1\right| + C$

2. $\dfrac{1}{5}\sin^5 x - \dfrac{1}{7}\sin^7 x + C$

3. $x^2 + \ln|x-1| + \dfrac{3}{2}\ln(x^2+1) + C$

4. $x\tan x + \ln|\cos x| + C$

5. $\dfrac{1}{27}\left(\dfrac{3x}{\sqrt{1-9x^2}} - \arcsin(3x)\right) + C$

6. $2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C$

7. $-3\sqrt{1-x^2} - 5\arcsin x + C$

8. $2\ln|x+1| + \ln|x-2| + \dfrac{3}{x-2} - \dfrac{1}{(x-2)^2} + C$

9. $\dfrac{3}{5}\left(1+e^x\right)^{\frac{5}{3}} - \dfrac{3}{2}\left(1+e^x\right)^{\frac{2}{3}} + C$

10. $\dfrac{-1}{1+\tan x} + C$

11. $\dfrac{-5}{2}\ln|2x-1| + \dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + \dfrac{3}{2}\ln(x^2+4) + C$

12. $\dfrac{1}{3}x^2 e^{3x} - \dfrac{2}{9}x e^{3x} + \dfrac{2}{27}e^{3x} + C$

13. $\ln\left((x-3)^2+4\right) + \dfrac{5}{2}\arctan\left(\dfrac{x-3}{2}\right) + C$

14. $\dfrac{1}{4}\arctan\left(\dfrac{e^{2x}}{2}\right) + C$

15. $x\arcsin x + \sqrt{1-x^2} + C$

16. $\dfrac{1}{6}x^6 \ln x - \dfrac{1}{36}x^6 + C$

17. $2x\,e^{\sqrt{x}} - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

18. $\dfrac{1}{7}\sec^7 x - \dfrac{1}{5}\sec^5 x + C$

19. $\sqrt{4x^2-9}-3\arccos\left(\dfrac{3}{2x}\right)+C$

20. $x\arcsin(2x)+\dfrac{1}{2}\sqrt{1-4x^2}+C$

21. $\dfrac{1}{2}\left(y^2\arctan(y)-y+\arctan(y)\right)+C$

22. $\dfrac{-1}{(x^2+1)^2}+C$

23. $3\sqrt[3]{x+8}+2\ln\left|\sqrt[3]{x+8}-2\right|-22\ln\left|\dfrac{\sqrt{3}}{\left(\sqrt[3]{x+8}+1\right)^2+3}\right|-10\sqrt{3}\arctan\left(\dfrac{\sqrt[3]{x+8}+1}{\sqrt{3}}\right)+C$

24. $\dfrac{-4}{27}(2-3x)^{\frac{3}{2}}+\dfrac{2}{45}(2-3x)^{\frac{5}{2}}+C$

25. $\dfrac{9}{2}\arcsin\left(\dfrac{x-3}{3}\right)+\dfrac{1}{2}(x-3)\sqrt{9-(x-3)^2}+C$

26. $\dfrac{1}{14}\sin(7x)+\dfrac{1}{2}\sin x+C$

27. $\dfrac{3}{\ln 4}\left(x(4^x)-\dfrac{4^x}{\ln 4}\right)+C$

28. ∞

29. 1

30. 0

31. $e^{\frac{3}{2}}$

32. 0

33. ∞

34. 0

35. 2

36. ∞

37. 0

38. e^2

39. $-\infty$

40. π

41. DNE

42. $\dfrac{\pi}{2}$

43. 1

44. $\dfrac{1}{40}$

45. ∞

46. $\dfrac{\pi}{2}$

47. diverges

48. diverges

49. $\dfrac{243}{10}$

50. diverges