Math1220 Midterm 1 Review Problems
(6.1-6.5, 6.8, 6.9, 7.1)

1. Find the equation of the tangent line to the graph of $y=\cos^{-1}(\ln(x^4))$ when x = 1.

2. Find $f^{-1}(x)$ for $f(x)=\left(\frac{2x-1}{2x+5}\right)^3$.

3. Find $\frac{dy}{dx}$ for each function. **<u>(Don't simplify.)</u>**
   (a) $y=\ln(\cos^2(3x))+\sin^{-1}(3x-2)$
   (b) $y=(5x+3)^{2x^2}$
   (c) $y=(1+x^4)^{\pi}+\pi^{1+x^4}$
   (d) $y=sech(\cos(2x))$
   (e) $y=\ln(3x-2)+2x^{-6}+4x^3-\sin(5x)+9$
   (f) $y=e^{\frac{1}{3x}}+\frac{1}{e^{3x}}$
   (g) $y=(x^3-1)^{\ln x}$
   (h) $y=\cosh^{-1}(\cos x+3)$

4. Evaluate each integral.
   (a) $\int \frac{20x+5}{2x^2+x-7}dx$
   (b) $\int \frac{-5}{x+x(\ln x)^2}dx$
   (c) $\int_1^3 4^{2x-7}dx$
   (d) $\int_0^{\frac{\pi}{6}} 2^{\cos x} \sin x\, dx$
   (e) $\int_0^1 \frac{2t^2+1}{2t^3+3t-4}dt$
   (f) $\int_{-2}^0 6^{2x+4}dx$
   (g) $\int \frac{e^{2x}}{e^{2x}+5}dx$
   (h) $\int \frac{5x^2}{\sqrt{1-x^6}}dx$
   (i) $\int \frac{x}{x^4+4}dx$
   (j) $\int \frac{x^3}{x^4+4}dx$
   (k) $\int_{\frac{\pi}{2}}^{\pi} \frac{2\cos x}{1+\sin^2 x}dx$

(l) $\int_0^{\ln 2} \sinh x \, dx$

5. Find $(f^{-1})'(5)$ given $f(x)=2x^5+4x-1$.

6. Show that $f(x)=\dfrac{\sin x+1}{\cos x}$ is monotonic on the interval $\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)$, i.e. that its inverse exists on that domain.

7. Show that $f(x)=6-\tan^{-1}(2x)-5(x-1)^3$ has an inverse on its domain. (Explain your reasoning.) Then, find $(f^{-1})'(11)$

8. Find the limits.

(a) $\lim_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x}$

(b) $\lim_{x\to\infty}(1)^{5x}$

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Answer Key

1. $y=-4x+4+\dfrac{\pi}{2}$

2. $f^{-1}(x)=\dfrac{1+5\sqrt[3]{x}}{2-2\sqrt[3]{x}}$

3.

(a) $y'=\dfrac{2\cos(3x)(-\sin(3x))(3)}{\cos^2(3x)}+\dfrac{3}{\sqrt{1-(3x-2)^2}}$

(b) $y'=(5x+3)^{2x^2}\left(4x\ln(5x+3)+\dfrac{5(2x^2)}{5x+3}\right)$

(c) $y'=\pi(1+x^4)^{\pi-1}(4x^3)+\pi^{1+x^4}(\ln\pi)(4x^3)$

(d) $y'=-sech(\cos(2x))\tanh(\cos(2x))(-\sin(2x))(2)$

(e) $y'=\dfrac{3}{3x-2}-12x^{-7}+12x^2-5\cos(5x)$

(f) $y'=e^{\frac{1}{3x}}\left(\dfrac{-1}{3x^2}\right)+\dfrac{1}{e^{3x}}(-3)$

(g) $y'=(x^3-1)^{\ln x}\left(\dfrac{1}{x}\ln(x^3-1)+\dfrac{3x^2(\ln x)}{x^3-1}\right)$

(h) $y'=\dfrac{-\sin x}{\sqrt{(\cos x+3)^2-1}}$

4. Evaluate each integral.

(a) $5\ln|2x^2+x-7|+C$

(b) $-5\arctan(\ln x)+C$

(c) $\dfrac{4^{-1}-4^{-5}}{\ln 16}=\dfrac{255}{1024\ln 16}$

(d) $\dfrac{-1}{\ln 2}\left(2^{\sqrt{3}/2}-2\right)$

(e) $\dfrac{-1}{3}\ln 4$

(f) $\dfrac{6^4-1}{\ln 36}$

(g) $\dfrac{1}{2}\ln\left(e^{2x}+5\right)+C$

(h) $\dfrac{5}{3}\left(\arcsin\left(x^3\right)\right)+C$

(i) $\dfrac{1}{4}\arctan\left(\dfrac{x^2}{2}\right)+C$

(j) $\dfrac{1}{4}\ln\left(x^4+4\right)+C$

(k) $-\dfrac{\pi}{2}$

(l) $\dfrac{1}{4}$

5. $\dfrac{1}{14}$

6. $f'(x)=\dfrac{\sin x+1}{\cos^2 x}$ and since $\sin x$ is always between -1 and 1, then $1+\sin x$ must be between 0 and 2 (inclusive) which is always nonnegative. The denominator is also always positive on the interval $\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)$. This means that the derivative is always nonnegative in the given domain of the function. This implies that the function is monotonically increasing.

7. $f'(x)=-\left(\dfrac{2}{1+4x^2}+15(x-1)^2\right)$ which is always positive inside the parentheses since all coefficients are positive and the powers on x are even. Thus, the derivative is always negative which means the inverse function exists.

$(f^{-1})'(11)=\dfrac{1}{f'(0)}=\dfrac{-1}{17}$

8.

(a) $\lim_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{5x}=e^{15}$

(b) $\lim_{x\to\infty}(1)^{5x}=1$