Chapter 1 Supplementary Exercises: 1, 3, 5, 7, 11, 12, 17, 19, 20, 21

Supplemental Practice Problems:

1) Linearly (In)Dependent Sets

Construction

(a) Give an example of two vectors in $\mathbb{R}^2$ that are linearly dependent. $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix}$

(b) Give an example of two vectors in $\mathbb{R}^2$ that are linearly independent. $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}$

(c) Give an example of three vectors in $\mathbb{R}^2$ that are linearly dependent. $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}$

(d) Give an example of three vectors in $\mathbb{R}^2$ that are linearly independent. not possible

(e) Give an example of three vectors in $\mathbb{R}^3$ that are linearly dependent. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}2\\3\\2\end{bmatrix}$

(f) Give an example of three vectors in $\mathbb{R}^3$ that are linearly independent. $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

2) Find all solutions, if any, to the following systems of equations.

(a) (see below)

$$\begin{cases} x_1 - 3x_2 = -3 \\ -x_1 + x_2 = -1 \\ 2x_1 - 5x_2 = -4 \end{cases}$$

three lines in $\mathbb{R}^2$

(b) (see below)

$$\begin{cases} -2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + 2x_2 - 5x_3 = -2 \end{cases}$$

3) Find all solutions, if any, to the following matrix equations.

(a) $\begin{bmatrix} 2 & -4 & 10 \\ 3 & 1 & 1 \\ -2 & 3 & -8 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$

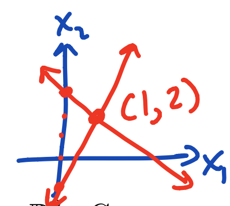
(b) $\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & 2 \\ -1 & 0 & -1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

> see below

4) Consider the matrix equation $\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$.

① $3x_1 - x_2 = 1 \iff x_2 = 3x_1 - 1$

② $2x_1 + 2x_2 = 6 \iff x_2 = -x_1 + 3$



(a) Show that the equation has a unique solution and find that solution. $\begin{bmatrix}1\\2\end{bmatrix}$

(b) Write the corresponding system of equations and graph the two corresponding lines in $\mathbb{R}^2$. Geometrically, how do you interpret your solution from (*a*)? pt of intersection

(c) Write the corresponding linear combination problem. Verify that your solution from (a) gives the correct linear combination.

$x_1\begin{bmatrix}3\\2\end{bmatrix} + x_2\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}1\\6\end{bmatrix}$

5) Consider the matrix equation $\begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

columns of A are lin. dep

$\begin{bmatrix} -2 & 1 & 2 \\ 6 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} -2 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ N.S.

(a) Show that the system has no solution. parallel lines

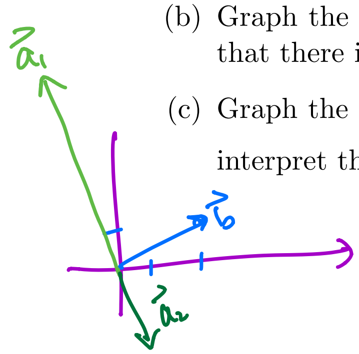
(b) Graph the lines of the corresponding system of equations. How does this graph relate to the fact that there is no solution?

$x_2 = 2x_1 + 2$

$x_2 = 2x_1 - \frac{1}{3}$

$6x_1 - 3x_2 = 1$

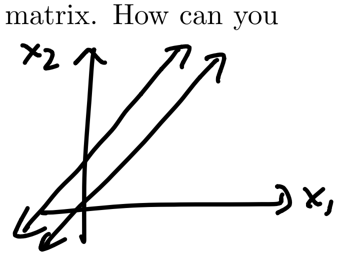
(c) Graph the vector $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ along with the column vectors, $\mathbf{a}_1$ and $\mathbf{a}_2$, of the matrix. How can you interpret the fact that there is no solution in terms of linear combinations?



$\vec{a}_1 = \begin{bmatrix}-2\\6\end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix}1\\-3\end{bmatrix}$

$\vec{b}$ is not a linear combo of $\vec{a}_1$ and $\vec{a}_2$

$\iff \vec{b} \notin \text{span}\{\vec{a}_1, \vec{a}_2\}$



2a)

$$x_1 - 3x_2 = -3$$
$$-x_1 + x_2 = -1$$
$$2x_1 - 5x_2 = -4$$

$$\begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ -4 \end{bmatrix}$$

$$(2)\begin{bmatrix} 1 & -3 & -3 \\ -1 & 1 & -1 \\ 2 & -5 & -4 \end{bmatrix} \quad \begin{array}{c} \\ (-\frac{1}{2}) \\ (-\frac{1}{3}) \end{array}\begin{bmatrix} 1 & -3 & -3 \\ 0 & -2 & -4 \\ 0 & -3 & -6 \end{bmatrix} \quad (-1)\begin{bmatrix} 1 & -3 & -3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}(3) \quad \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{cases} x_1 = 3 \\ x_2 = 2 \end{cases}$$

soln: $\boxed{\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}}$

#2b) $-2x_1 - x_2 + 3x_3 = 5$
$3x_1 + 2x_2 - 5x_3 = -2$

$$\begin{bmatrix} -2 & -1 & 3 & 5 \\ 3 & 2 & -5 & -2 \end{bmatrix} \quad \begin{array}{c}(-3)\\ \end{array}\begin{bmatrix} 1 & 1 & -2 & 3 \\ 3 & 2 & -5 & -2 \end{bmatrix} \quad (-1)\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -1 & 1 & -11 \end{bmatrix}$$

RREF
$$\left[\begin{array}{ccc:c} 1 & 0 & -1 & -8 \\ 0 & 1 & -1 & 11 \end{array}\right]$$

$x_1 - x_3 = -8 \iff x_1 = x_3 - 8$
$x_2 - x_3 = 11 \iff x_2 = x_3 + 11$
x_3 free $\iff x_3 = x_3$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 - 8 \\ x_3 + 11 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} + \begin{bmatrix} -8 \\ 11 \\ 0 \end{bmatrix}$$

$$\boxed{\vec{x} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -8 \\ 11 \\ 0 \end{bmatrix}}$$

$\Rightarrow$ soln to $A\vec{x} = \vec{0}$ (homogeneous eqn)

$$\vec{x} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3a)

$$\begin{bmatrix} 2 & -4 & 10 \\ 3 & 1 & 1 \\ -2 & 3 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 10 & 6 \\ 3 & 1 & 1 & 5 \\ -2 & 3 & -8 & 4 \end{bmatrix}\left(\tfrac{1}{2}\right) \quad \begin{array}{c} (-3) \\ (+1) \end{array}\begin{bmatrix} 1 & -2 & 5 & 3 \\ 3 & 1 & 1 & 5 \\ 0 & -1 & 2 & 10 \end{bmatrix} \quad (-7)\begin{bmatrix} 1 & -2 & 5 & 3 \\ 0 & 7 & -14 & -4 \\ 0 & 1 & -2 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 & 3 \\ 0 & 0 & 0 & 66 \\ 0 & 1 & -2 & -10 \end{bmatrix} \Rightarrow 0 = 66$$ which is not true

$\Rightarrow$ N.S.

3b)

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & 2 \\ -1 & 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 1 \\ 0 & 1 & -1 & 2 & 2 \\ -1 & 0 & -1 & -1 & 3 \end{bmatrix} \quad (-2)\begin{bmatrix} 1 & 2 & 0 & 4 & 1 \\ 0 & 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 1 \\ 0 & 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 0 & 4 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}(-2)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -3 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$x_1 = -2x_4 + -3$
$x_2 = -x_4 + 2$
$x_3 = x_4$
$x_4 = x_4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_4\begin{bmatrix} -2 \\ -1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{x} = x_4\begin{bmatrix} -2 \\ -1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

6) Consider the following vectors in $\mathbb{R}^3$.

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$$

For each of the sets below, determine whether the set is linearly dependent or independent. If the set is linearly dependent, give a dependency relation between the vectors.

(a) $\{\mathbf{u}, \mathbf{v}\}$ yes lin indep $\vec{u} \neq c\vec{v}$

(b) $\{\mathbf{u}, \mathbf{x}\}$ $\vec{u} = -1\vec{x} \Rightarrow \{\vec{u}, \vec{x}\}$ not l.i.

(c) $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$

(d) $\{\mathbf{u}, \mathbf{v}, \mathbf{y}\}$ > see below

7) Find all solutions, if any, to the following linear combination (or vector equation) problems.

(a) Determine if $\mathbf{w} = \begin{bmatrix} 5 \\ 6 \\ -12 \end{bmatrix}$ is a linear combination of $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$. (see below)

(b) Determine if $\mathbf{w} = \begin{bmatrix} -1 \\ 13 \end{bmatrix}$ is a linear combination of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

it has to be since $\vec{v}_1, \vec{v}_2$ & $\vec{v}_3$ are lin. dep.

8) Homogeneneous, $A\mathbf{x} = \mathbf{0}$ and Nonhomogeneous Systems, $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \neq \mathbf{0}$

(a) What condition(s) on the row echelon form of the matrix A guarantee(s) that the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions? row of zeros

(b) What condition(s) on the row echelon form of the matrix A guarantee(s) that the nonhomogeneous equation $A\mathbf{x} = \mathbf{b}$ always has at least one solution no matter the entries of $\mathbf{b}$? (see below)

(c) What condition(s) of the numbers of rows and columns of A always give infinitely many solutions to the homogeneous problem? (see below)

(d) What condition(s) on the numbers of rows and columns of A guarantee that there will be lots of vectors $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ is inconsistent? (see below)

9) Consider the homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$ with the matrix A and its reduced row echelon form given below: 3×5

$A: \mathbb{R}^5 \to \mathbb{R}^3$

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 4 & 1 \\ -2 & -4 & 0 & -2 & -2 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find and express the solution, if any, to this system in linear combination form. (see below)

(b) Are the columns of A linearly independent or dependent? (more columns than rows)

(c) For what $\mathbf{b} \neq \mathbf{0} \in \mathbb{R}^3$, does a solution exist? Find a solution to such a nonhomogeneous matrix equation.

$\vec{b} \in \mathbb{R}^3$

10) Suppose $\{\mathbf{v}_1, \mathbf{v_2}, \mathbf{v_3}\}$ is a linearly dependent set in $\mathbb{R}^n$. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation. Explain why $\{T(\mathbf{v}_1), T(\mathbf{v_2}), T(\mathbf{v_3})\}$ must be linearly dependent in $\mathbb{R}^m$.

A is $m\times n$

#10

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ lin. dep

$\Leftrightarrow \exists\ c_1, c_2, c_3$ s.t. they're not all zero (i.e $c_1^2+c_2^2+c_3^2 \neq 0$) and

$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$ has nontrivial soln.

$T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = T(\vec{0})$

$c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + c_3T(\vec{v}_3) = T(\vec{0}) = \vec{0}$

we know that not all c_1, c_2, c_3 are zero

which means $T(\vec{v}_1), T(\vec{v}_2)$ & $T(\vec{v}_3)$ are lin. dep.

6c) $\begin{array}{ccc} \vec{u} & \vec{v} & \vec{w} \end{array}$
$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & -3 \\ -1 & -2 & 1 \end{bmatrix}$ find RREF $\quad (-2)$

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -7 \\ 0 & -1 & 3 \end{bmatrix}$ $\quad \left(\frac{-1}{4}\right) \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -7 \\ 0 & 0 & -4 \end{bmatrix} (-1) \quad \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} (-9) \; (7)$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \vec{u}, \vec{v}, \vec{w}$ are $\boxed{\text{lin. indep.}}$

6d) $\begin{array}{c} \begin{array}{ccc} \vec{u} & \vec{v} & \vec{y} \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ -1 & -2 & -3 \end{bmatrix} \end{array}$

notice! $\vec{y} = \vec{u} + \vec{v}$

$\Rightarrow \vec{u}, \vec{v}, \vec{y}$ are $\boxed{\text{lin. dep.}}$

7)

$$\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2$$

$$\begin{bmatrix} 5 \\ 6 \\ -12 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 1 & -2 & 6 \\ -2 & 4 & -12 \end{array}\right]$$

$$(-2)\begin{bmatrix} 1 & -2 & 6 \\ 2 & 3 & 5 \\ 0 & 7 & -7 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 6 \\ 0 & 7 & -7 \\ 0 & 7 & -7 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 6 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{bmatrix}\left(\tfrac{1}{7}\right) \quad \begin{bmatrix} 1 & -2 & 6 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}(2)$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad c_1 = 4, \quad c_2 = -1$$

$\Rightarrow$ $\boxed{\text{yes } \vec{w} \text{ is linear combo of } \vec{v}_1 \text{ \& } \vec{v}_2 \text{ w/ } c_1 = 4,\ c_2 = -1}$

8b) $A\vec{x} = \vec{b}$ has at least one soln $\forall \vec{b} \in \mathbb{R}^m$

No

every column has pivot position

ex $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\qquad \vec{b} = \begin{bmatrix} 5 \\ 2 \\ 10 \end{bmatrix}$ $\Rightarrow$ N.S. for $A\vec{x} = \vec{b}$

$\vec{x} \in \mathbb{R}^2, \ b \in \mathbb{R}^3$

$\boxed{\text{every row has pivot positn}}$

ex $A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

8c) ex $A = \begin{bmatrix} 1 & 0 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

$A\vec{x} = \vec{0}$

$x_2 = -3x_3 - 2x_4$

$x_1 = -5x_3 - x_4$

x_3, x_4 free

$\boxed{\# \text{rows} < \# \text{cols}}$

solns $\vec{x} = \begin{bmatrix} -3 \\ -5 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} x_4$

ex $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

8d) $m>n$ because if $A: \mathbb{R}^n \to \mathbb{R}^m$ and $m>n$, then the output space is bigger than the input space & it can't all get mapped to which means there will be lots of $\vec{b}$ outputs for which $A\vec{x}=\vec{b}$ has no solution.

9a)

$x_1 = -2x_2 - x_4 - x_5$
$x_3 = -2x_4 + x_5$
x_2, x_4, x_5 free

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 - x_4 - x_5 \\ x_2 \\ -2x_4 + x_5 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\boxed{\vec{x} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}}$$

9c) augmented matrix

$$\begin{array}{c} (-2) \\ \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 1 & b_1 \\ 2 & 4 & 1 & 4 & 1 & b_2 \\ -2 & -4 & 0 & -2 & -2 & b_3 \end{array}\right]
\quad
\begin{array}{c} \\ (-1) \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 1 & b_1 \\ 0 & 0 & 1 & 2 & -1 & b_2 - 2b_1 \\ 0 & 0 & 1 & 2 & -1 & b_2 + b_3 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 1 & b_1 \\ 0 & 0 & 1 & 2 & -1 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & 0 & b_2 + b_3 - b_2 + 2b_1 \end{array}\right]
\quad
\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 1 & b_1 \\ 0 & 0 & 1 & 2 & -1 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & 0 & 2b_1 + b_3 \end{array}\right]$$

$\Rightarrow$ for a solution to exist, we need

$$2b_1 + b_3 = 0 \iff \boxed{b_3 = -2b_1}$$

i.e. for $A\vec{x} = \vec{b}$ to have a solution, we need $\vec{b}$ to be of form

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ -2b_1 \end{bmatrix} = b_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + b_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

i.e. $\vec{b}$ needs to be a linear combo of $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.