Key Definitions: Section 2.1

- The identity matrix I_n is

- A diagonal matrix is

- A zero matrix is

- The transpose of a matrix A is

Major Theorems: Section 2.1

Section 2.1

Theorem 1 Properties of Matrix Addition and Scalar Multiplication *Let A, B, and C be matrices of the same size, $m \times n$, and let r and s be scalars.*

(a) $A + B =$

(b) $(A + B) + C =$

(c) $A + 0 =$

(d) $r(A + B) =$

(e) $(r + s)A =$

(f) $r(sA) =$

Theorem 2 Properties of Matrix Multiplication *Let $A, B,$ and C be matrices and r be a scalar such that the sums and products below are defined. Then,*

(a) $A(BC) =$

(b) $A(B + C) =$

(c) $(B + C)A =$

(d) $r(AB) =$

(e) $I_m A =$

Theorem 3 Transpose Properties *Let A and B be matrices whose sizes are appropriate for the following sums and products.*

(a) $(A^T)^T =$

(b) $(A + B)^T =$

(c) *For any scalar r,* $(rA)^T =$

(d) $(AB)^T =$

Note: *The transpose of a product of matrices equals the product of their transposes in reverse order.*

Supplemental Practice Problems:

1. Consider the following matrices

$$A = \begin{bmatrix} 2 & -1 & 4 \\ -3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ 2 & 5 \\ 3 & 3 \end{bmatrix}, \quad \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

   Calculate (if possible) each of the following matrix products:

   (a) AB

   (b) BA

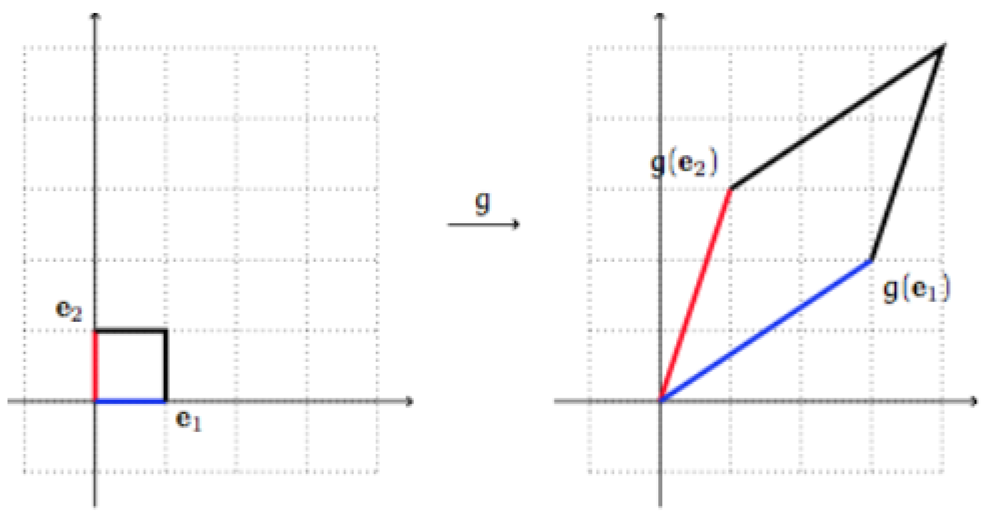
   (c) AC

   (d) CA

2. Let R be the rectangle with vertices (-2, -1), (-2, 2), (3, 2), (3, -1). Consider the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $T(x_1, x_2) = (4x_1 - 3x_2, -x_1 + x_2)$.

   Find the standard matrix A of the linear transformation T, and sketch the image of the rectangle R under T.

3. Let R be the rectangle with vertices (-2, -1), (-2, 2), (3, 2), (3, -1). Consider the linear transformation $S : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ which maps the unit square to the parallelogram pictured below.



   Find the standard matrix B associated to S.

4. Consider the matrices $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & b \\ c & -1 \end{bmatrix}$ where b and c are unknowns. Find values of b and c such that $AB = BA$.