Math1210 Final Exam Review Problems
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Name ______________________________ Date _____________

1. Find the area of the region bounded by the curves $y=\sqrt{2x}$, $y=0$ and $x-y=4$.

2. Find the equation of the tangent line to the curve $f(x)=5x^2-3x+4\sqrt{x+1}$ at $x=0$.

3. Find each limit, if it exists.

(a) $\lim_{x\to -3} \frac{x^2-9}{2x^2+x-15}$

(b) $\lim_{x\to -\infty} \frac{3x^3+5x-1}{-4x^2+2}$

(c) $\lim_{x\to 0} \frac{\tan(3x)\cos(x)}{\sin(2x)}$

4. Evaluate these integrals.

(a) $\int \frac{1}{\sqrt[3]{27x^5}}\,dx$

(b) $\int \frac{2t^3}{\sqrt{t^4-6}}\,dt$

(c) $\int_1^2 \frac{y^3+3y-2}{y^3}\,dy$

(d) $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos\theta\,d\theta$

(e) $\int_{-2}^{2} x\sin^2(2x)\cos(2x)\,dx$

5. Find the indicated derivative of the given functions.

(a) $D_x(\sqrt[3]{x^3+\sqrt[3]{x^2-5x}})$ (Do **not** bother to simplify!)

(b) $\frac{d}{dx}\left(\frac{\sin x+2x^5-3}{4x^3+9x}\right)$ (Do **not** bother to simplify!)

(c) $D_x((x^2+2x)^4\tan x)$ (Do **not** bother to simplify!

(d) $\frac{dy}{dx}$ given $y^3-3xy=2x^4-5x^2+1$

6. For $f(x)=\dfrac{3(x+2)^2}{(x-1)^2}$

(given $f'(x)=\dfrac{-18(x+2)}{(x-1)^3}$ and $f''(x)=\dfrac{18(2x+7)}{(x-1)^4}$).

(a) Find the asymptotes.

(b) Fill in the sign line for $f'(x)$.

(c) Find all local minimum and maximum **points**.

(d) Fill in the sign line for $f''(x)$.

(e) Find all inflection point x-value(s).

(f) Sketch the graph of $f(x)$.

7. **Setup** (you do NOT have to evaluate) the integrals for the volume of the solid generated by revolving the region bounded by
$y=4-x^2$, $x=0$, and $y=0$

(a) about the y-axis.

(b) about the x-axis.

(c) about the line y = -3.

(d) about the line x = 4.

8. Find the arc length of the curve given by
$x=\frac{1}{6}t^6$ and $y=\frac{1}{4}t^4$ for $0 \le t \le 2$.

9. A force of 10 pounds is required to keep a spring stretched 3 feet beyond its normal length. Find the work done in stretching the spring 6 feet beyond its natural length.

10. Find the surface area generated by revolving $y=2\sqrt{x+2}$ about the x-axis for $-2 \le x \le 1$.

11. Find the centroid for the region bounded by the curves $y=3x-6$, $y=3\sqrt{x}$, and $x=0$.

12. Sand is pouring from a pipe at a rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose height is always ¼ of the diameter of the base, then how fast is the height increasing when the pile base has a radius of 8 feet? (Note: The volume of a cone is $V=\frac{1}{3}\pi r^2 h$.)

13. Solve the following differential equation

$$x^2\frac{dy}{dx}=\frac{3x^{-3}+2x^2-1}{y^3} \quad \text{such that} \quad x=1 \quad \text{when} \quad y=1 .$$

14. Use differentials to approximate the increase in surface area of a soap bubble when its radius increases from 3 inches to 3.025 inches.

15. A farmer wishes to fence off three identical adjoining rectangular pens, each with 300 square feet of area. What should the width and length of each pen be so that the least amount of fence is required?

16. For all rectangles with a diagonal of 2 inches, find the dimensions of the one with the maximum area.

17. Use the **definition of the derivative** to find $f'(x)$ for $f(x)=\frac{3}{x+2}$.

18. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 ft above the ground?

19. Use the definition of the definite integral, that is $\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$, to find $\int_0^2 (x^2+3)\,dx$.

20. Find the value of c that satisfies the Mean Value Theorem for Integrals for $f(x)=x^3$ on [0, 2].

21. Use the definition of the derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ for $f(x)=x^2+3x-1$.

22. Find the derivative of $y=2x^8-3x^5+x^2-9x$

23. Evaluate the integral $Y=\int (x^2-5)\,dx$ such that $Y=2$ when $x=0$.

24. Find each of these limits or state that the limit does not exist.

(a) $\lim_{x\to 3}\frac{\sqrt{(x+5)(x-3)^4}}{(3x-9)^2}$

(b) $\lim_{x \to -1} \frac{\sqrt{x^2+2}}{5x}$

(c) $\lim_{x \to 2} \frac{x^3-4x+1}{x^2+3x-10}$

25. Find each limit, if it exists.

(a) $\lim_{x \to \infty} \frac{2x^2+3x-7}{x^3+9x}$

(b) $\lim_{x \to \infty} \frac{5x+2\sqrt{x^3}+1}{-\sqrt{3x^3}}$

26. State whether this function is continuous or not. If the function is discontinuous, give the x-values where the discontinuities occur and also state why it's discontinuous.

$f(x)=\frac{x^2-25}{x+5}$

27. For $f(x)=\frac{-4x}{x+3}$, find the vertical and horizontal asymptotes and sketch the graph.

28. Find the derivative of the following functions.
(Do NOT bother to simplify your answers!)

(a) $g(x)=\frac{4x^3-2x^2+x}{x^2-9}$

(b) $y=(5x^{-2}+3x)(2x^4-7x)$

29. Evaluate these limits.

(a) $\lim_{t \to 0} \frac{4\cos^2 t}{1-2\sin t}$

(b) $\lim_{\theta \to 0} \frac{\tan(3\theta)}{\sin(4\theta)}$

30. Find y' for the following functions. **(Do NOT bother simplifying.)**

(a) $y=\frac{\sin x-\cos x}{3\cos x}$

(b) $y=(\tan x)(x^{-3}+5x^2-7)$

31. Find $D_x y$ for $y=\cos[\sin(\cos(3x+1))]$ (Do not bother to simplify.)

32. Find $\frac{dy}{dx}$ for $y=\left(\frac{x^5-2}{\tan x}\right)^3$ (Do not bother to simplify.)

33. Find $f'''(x)$ for $f(x)=\frac{2}{x+4}$

34. Find the equation of the tangent line to $y=(x^3-2)^5(x^4+1)^2$ at x=1.

35. Find $\frac{dy}{dx}$.

(a) $x^2y-3\sqrt{y}=4x^2-7x$ (You need to at least get $\frac{dy}{dx}$ by itself, but don't simplify past that.)

(b) $y=\frac{3}{x^4+\sin x}$ **(Don't bother to simplify.)**

36. For $y=5x^3+3x^2-1$, if x changes from 1 to 1.03, approximately how much does y change?

37. Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose height is always ¼ the diameter of the base, how fast is the height increasing when the pile is 4 feet high? (Note: volume of a right circular cone is $V=\frac{1}{3}\pi r^2 h$.)

38. Identify all the critical points for $f(x)=2x^2+8x+5$ on [-3, 1]

39. For $f(x)=2x^3-9x^2+12x$

(a) Fill in the sign line for $f'(x)$.

(b) Find all local min and max **<u>point(s)</u>**.

(c) Fill in the sign line for $f''(x)$.

(d) Find all x-values of inflection point(s).

(e) Sketch the graph using all this information.

40. Find $\frac{dy}{dx}$ for $y=3\sin x-4x^{-2}+\sqrt{5x-1}$. **(Don't bother to simplify.)**

41. A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides at right angles to the sheet. How many inches should be turned up to give the gutter the greatest volume? (You can assume that the length of the gutter is some fixed value of L.)

42. For $f(x)=\frac{(x-2)^2}{x}$

(a) Fill in the sign line for $f'(x)$.
(b) Find all local min and max points, if there are any.
(c) Fill in the sign line for $f''(x)$.
(d) Find all inflection points, if there are any.
(e) Find the vertical asymptote.
(f) Sketch the graph using all this information.

43. Evaluate.

(a) $\int (x^4-2\cos x+\sqrt[3]{x^2})\,dx$

(b) $\int \frac{-2x}{(2x^2-7)^3}\,dx$

44. For $f(x)=x^2-3x+2$ on [-1, 3], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

45. Solve this equation using the Bisection Method. (Just fill in table for first five rows.)

$f(x)=x^3-x^2+1$ on [-1, 0]

n	a_n	b_n	m_n	$f(a_n)$	$f(b_n)$	$f(m_n)$

46. Solve problem 45 with Newton's method, accurate to four decimal places.

n	x_n

47. Evaluate $\sum_{i=1}^{10} [(i-3)(5i+2)]$

48. Solve this differential equation.

$$\frac{dy}{dx}=(3x^2-5)\,y^2 \quad \text{and} \quad y=1 \quad \text{when} \quad x=1$$

49. Evaluate the definite integral **<u>using the definition</u>** (the tedious way).

$\int_0^4 (2x-3)\,dx$. (Note: Here is the definition. $\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$)

50. Evaluate $\int_0^{\sqrt{\pi}} (x\cos(x^2))\,dx$

51. Find the average value of $f(x)=3x^5-6x^2+5$ on $[0,1]$.

52. Evaluate. $\int_3^5 \frac{x\cos(\sqrt{x^2-9})}{\sqrt{x^2-9}}\,dx$

53. Find $G'(x)$ given $G(x)=\int_\pi^x x\sin(t^2-3)\,dt$

54. Find the area of the region bounded by $y=(x-1)^2+2$, $y=0$, $x=0$ and $x=3$.

55. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y=\frac{x^2}{3}$, $x=1$, and $y=0$.

56. Find the volume of the solid generated by revolving about the y-axis the region bounded by $y=\frac{\sqrt{x}}{2}$, $x=4$, and $y=0$.

57. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y=2x$ and $y=2x^4$.

58. Find the length of the curve given by $x=4\sin t$ and $y=4\cos t-5$ for $0\le t\le\pi$.

59. Find the area of the surface generated by revolving the curve given by $y=2\sqrt{x}$ for $0\le x\le 3$ about the x-axis.

60. A force of 6 pounds is required to keep a spring stretched ½ foot beyond its normal length. Find the work done in stretching the spring 2 feet beyond its natural length.