Notes on Limits

Setup: You have to find $\lim_{x \to c} f(x)$

1. If c is a finite number, first try plugging in c.

   - If you get a finite number back, then you're done.

   - If you get the $\frac{0}{0}$ case, then you need to simplify more to find the limit. Keep going until you can plug in c for x.

   - If you get the $\frac{finite\ number}{0}$ case, then it will either (1) go to ∞, (2) go to $-\infty$, or (3) it will not exist (DNE). You need to check the right and left-hand limits.
     - If the RH limit and the LH limit both go to ∞, then the limit also goes to ∞.
     - If the RH limit and the LH limit both go to $-\infty$, then the limit also goes to $-\infty$.
     - If the RH limit goes to $-\infty$ and the LH limit goes to ∞ (or the other way around), then the limit does not exist (DNE).

2. If c is $\pm\infty$ (and $f(x)$ is a rational function or at least has numerator and denominator that can be written as a collection of terms to powers), then

   - If the highest degree of the numerator > highest degree of the denominator, then the limit goes to $\pm\infty$. (You have to analyze the particular problem to decide if it's positive or negative infinity.)

   - If the highest degree of the numerator = highest degree of the denominator, then the limit is the quotient of the leading coefficients.

   - If the high degree of the numerator < highest degree of the denominator, then the limit is zero.

Asymptotes:

1. To find Vertical Asymptotes (VA)==>
   Look for x-values that will make the function undefined (e.g. x-values that make the denominator zero). Let's say that x = b makes function undefined.

   - If $\lim_{x \to b} f(x)$ equals a finite number, then there is only a "hole" (a.k.a. removable discontinuity) at x = b.

   - If $\lim_{x \to b} f(x)$ does not exist (or goes to $\pm\infty$), then x = b is a VA.

   * (A quick way to determine this is to try plugging in x = b in the function. If it goes to $\frac{0}{0}$, then it's a "hole." If it goes to $\frac{finite\ number}{0}$, then it's a VA.)

   A graph NEVER touches or crosses a vertical asymptote!!

2. To find Horizontal Asymptotes (HA)==>
   The HA can be found by calculating $y = \lim_{x \to \pm\infty} f(x)$. If the limit goes to $\pm\infty$, then there is no HA.

   *(If there is no HA and you have a rational function, you can always find the "slant asymptote" by doing long division.)

   Remember that the HA just describes the behavior of the graph as x gets really huge (either negatively or positively). A graph can cross and touch the horizontal/slant asymptotes as many times as it wants...but as x gets huge, it will only approach the asymptote.