

INTRODUCTION TO POLYNOMIAL CALCULUS

PROBLEMS

1. Straight Lines

In problems (1) through (6) find the slope of the line containing the indicated two points.

- (1) (0, 1) and (1, 2) $m = 1$
 (2) (2, 3) and (4, 7) $m = 2$
 (3) (1, 1) and (3, 2) $m = \frac{1}{2}$
 (4) (1, 4) and (3, 2) $m = -1$
 (5) (-2, 3) and (3, 1) $m = -\frac{2}{5}$
 (6) (-2, 0) and (0, 2) $m = 1$

In problems (7) through (12) find the equation of the line with the indicated slope and passing through the indicated point.

- (7) slope 2 and point (0, 0) $y = 2x$
 (8) slope 5 and point (1, 2) $y = 5x - 3$
 (9) slope -3 and point (2, -1) $y = -3x + 5$
 (10) slope $\frac{1}{2}$ and point (1, 1) $y = \frac{1}{2}x + \frac{1}{2}$
 (11) slope $-\frac{2}{3}$ and point (0, 5) $y = -\frac{2}{3}x + 5$
 (12) slope 7 and point (-2, 0) $y = 7x + 14$

- (13) Find the equation of the line with slope 3 and y -intercept 1 $y = 3x + 1$
 (14) Find the equation of the line with slope $\frac{4}{3}$ and y -intercept 2 $y = \frac{4}{3}x + 2$
 (15) Find the slope and y -intercept for the line with equation $6x - 2y = 4$ $m = 3; (0, -2)$
 (16) Find the slope and y -intercept for the line with equation $2x + 5y = 3$ $m = -\frac{2}{5}; (0, \frac{3}{5})$
 (17) Find the equation of the line which passes through (1, 1) and is parallel to the line $y = 3x + 2$.
 (18) Find the equation of the line which passes through (2, -1) and is parallel to the line which passes through (2, 0) and (3, 2). $y = 2x - 5$
 (19) Find the equation of the line which passes through (1, 0) and is perpendicular to the line $y = 3x + 2$ $y = -\frac{1}{3}x + \frac{1}{3}$
 (20) Find the equation of the line which bisects the line segment from (0, 0) to (2, 4) at a right angle. $y = -\frac{1}{2}x + \frac{5}{2}$
 (21) Find the equation of the line which passes through (0, 1) and is perpendicular to the line $x = 3$. $y = 1$
 (22) Find the equation of the line which passes through (2, 0) and is perpendicular to the line $y = 1$. $x = 2$
 (23) If a perpendicular line is drawn from the point (1, 1) to the line $2y - x = 4$, at what point does it meet this line? What is the distance from the point (1, 1) to the line $2y - x = 4$. $(\frac{2}{5}, \frac{11}{5}); \frac{3\sqrt{5}}{5}$
 (24) What is the distance from the point (0, 1) to the line $y = 2x - 3$? $\frac{4\sqrt{5}}{5}$
 (25) What is the distance from the line $y = 2x$ to the parallel line $y = 2x + 3$? $\frac{3\sqrt{5}}{5}$

2. Slope of a Curve

In problems 1 - 8, you are to find the slope of the curve $y = f(x)$ at the point where x has the indicated value by calculating $\frac{f(x+h)-f(x)}{h}$ and determining what number it approaches as h approaches 0.

- (1) $f(x) = 3x + 2, x = 1$ (1) 3
 (2) $f(x) = x^2, x = 0$ (2) 0
 (3) $f(x) = x^2, x = 2$ (3) 4
 (4) $f(x) = x^2 - 3, x = 1$ (4) 2
 (5) $f(x) = x^2 + 2x - 1, x = 0$ (5) 2
 (6) $f(x) = 3x^2 - 2, x = 1$ (6) 6
 (7) $f(x) = x^3, x = 1$ (7) 3
 (8) $f(x) = x^3, x = 0$ (8) 0

In problems 9 - 14 you are to find $f'(x)$ by calculating $\frac{f(x+h)-f(x)}{h}$ and determining what it approaches as h approaches 0.

- (9) $f(x) = x$ (9) $f'(x) = 1$
 (10) $f(x) = 2x + 5$ (10) $f'(x) = 2$
 (11) $f(x) = 3x^2$ (11) $f'(x) = 6x$
 (12) $f(x) = x^2 - 2x + 3$ (12) $f'(x) = 2x - 2$
 (13) $f(x) = x^3$ (13) $f'(x) = 3x^2$
 (14) $f(x) = x^3 + x^2$ (14) $f'(x) = 3x^2 + 2x$

3. Derivative of a Polynomial

- (1) Find the derivative of x^9 (1) $9x^8$
 (2) Find the derivative of $2x^{50}$ (2) $100x^{49}$
 (3) Find the derivative of $3x - 6$ (3) 3
 (4) Find the derivative of $x^3 - 2x + 4$ (4) $3x^2 - 2$
 (5) Find the derivative of $2x^4 + x^3 - 5x^2 + x + 2$ (5) $8x^3 + 3x^2 - 10x + 1$
 (6) Find the derivative of $x^{11} - 2x^9 + 15x$ (6) $11x^{10} - 18x^8 + 15$
 (7) Find the slope of the curve $y = x^3$ at the point (1, 1) (7) $m = 3$
 (8) Find the slope of the curve $y = x^2$ at the point (0, 0) (8) $m = 0$
 (9) Find the slope of the curve $y = x^3 - x^2$ at the point (1, 0) (9) $m = 1$
 (10) Find the slope of the curve $y = x^4 - 2x^3 + 5x + 3$ at the point where $x = -1$ (10) $m = -5$
 (11) Find the slope of the curve $y = 2x^{50} - 50x^2$ at the point where $x = 1$ (11) $m = 0$
 (12) For what values of x does the curve $y = x^2 - 2x + 3$ have positive slope? Negative slope? Zero slope?
 (12) pos: $x > 1$; neg: $x < 1$; zero: $x = 1$
 (13) If a ball is thrown straight up in such a way that its height t seconds later is

$$s(t) = -16t^2 + 32t + 6$$

- find the velocity of the ball at t seconds after it is thrown. At what time t does the ball reach its maximum height (hint: the velocity will be positive before this time and negative after it). How high does the ball get? (13) $v(t) = -32t + 32$, $t = 1 \text{ sec}$;
 (14) In the previous problem, what is the acceleration of the ball at any time t ? $s = 22 \text{ ft}$.

$$(14) a(t) = -32 \text{ ft/sec}^2$$

4. Antiderivatives of Polynomials

- (1) Find $\int (2x - 3) dx = x^2 - 3x + C$
- (2) Find $\int (3x^2 - 4x + 5) dx = x^3 - 2x^2 + 5x + C$
- (3) Find $\int (x^5 + 2x^3 + 1) dx = \frac{1}{6}x^6 + \frac{1}{2}x^4 + x + C$
- (4) Find $\int (10x^9 - 8x) dx = x^{10} - 4x^2 + C$
- (5) Find the antiderivative of $x^2 - 5$ that has value 2 when $x = 0$. $\frac{x^3}{3} - 5x + 2$
- (6) Find the antiderivative of $8x^3 - 2x$ that has the value 4 when $x = 1$. $2x^4 - x^2 + 3$
- (7) Find the antiderivative of $2x^3$ that has the value 1 when $x = 1$. $\frac{1}{2}x^4 + \frac{1}{2}$
- (8) Find the antiderivative of $x^3 - x$ that has the value 1 when $x = 2$. $\frac{1}{4}x^4 - \frac{1}{2}x^2 - 1$
- (9) If a ball is thrown straight up with initial velocity of 64 ft/sec, what will its velocity be after t seconds? At what time t will it achieve its maximum height? $v(t) = -32t + 64$;
- (10) If the ball in the last problem was thrown from an initial height of 6 feet, what will its height be after t seconds? What is the maximum height it achieves? $t = 2 \text{ sec}$
 $s(t) = -16t^2 + 64t + 6$; 70 ft

5. Definite integrals

- Find $\int_1^5 (x^2 - 2x + 1) dx$.
- Find $\int_0^2 (x^3 + 2) dx$.
- Find $\int_0^1 (x^4 - x^5) dx$.
- Find $\int_0^1 (x^n - x^{n+1}) dx$, for any $n \geq 0$.
- Find the area under the curve $y = x^2 + 5x$ from $x = 3$ to $x = 4$.
- Find the definite integral of $y = x^{10} - x^9$ from $x = 1$ to $x = 3$.
- A particle travels along a horizontal line so that its velocity at time t is $v(t) = 2t + 3t^2 + 1$ feet per second. Suppose that at time $t = 1$ the particle is at the origin. What is the location of the particle at time $t = 3$?