8.1 Practice (Indeterminate Forms of Type 0/0)

Ex1 (a) $\lim_{x \to 0^-} \frac{3\sin x}{\sqrt{-x}}$

L'Hopital's Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

(if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$)

(b) $\lim_{x \to 0} \frac{e^x - 1}{e^{-x} - 1}$

Ex2 (a) $\lim_{x \to 4} \dfrac{x-4}{e^{2(x-4)} - 1}$

(b) $\lim_{x \to 0} \dfrac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

Ex3 (a) $\lim_{x \to 0} \dfrac{\tan x - x}{\arcsin x - x}$

(b) $\lim_{x \to 0} \dfrac{x^2 \sin\left(\frac{1}{x}\right)}{\tan x}$

8.2 Practice (Other Indeterminate Forms)

Ex 1 (a) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

(b) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{x}{\ln x}\right)$

L'Hopital's Rule

$$\lim_{x\to u}\frac{f(x)}{g(x)}=\lim_{x\to u}\frac{f'(x)}{g'(x)}$$

if $\lim_{x\to u} f(x)=\pm\infty$

and $\lim_{x\to u} g(x)=\pm\infty$

Indeterminate Forms

$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $0\cdot\infty$,

$\infty-\infty$, 0^0, ∞^0, 1^∞

Non-indeterminate Forms

$\frac{0}{\infty}\to 0$

$\frac{\infty}{0}\to\infty$

$\infty+\infty\to\infty$

$\infty\cdot\infty\to\infty$

$0^\infty\to 0$

$\infty^\infty\to\infty$

$1^0\to 1$

Ex 2 (a) $\lim_{t\to 0^+}\left(\frac{1}{5}(2^t)+\frac{4}{5}(5^t)\right)^{1/t}$

(b) $\lim_{x\to\pi}\left(\cot x\ln(\cos x)\right)$

Ex 3 (a) $\lim_{x\to\infty}\left[\ln(x+2)-\ln(x-3)\right]$

(b) $\lim_{x\to 0^+}\left((x^x)^x\right)^x$

8.3 Practice (Improper Integrals)

Ex 1 (a) $\int_1^\infty \operatorname{csch} x \, dx$

$\int_{-\infty}^{a} f(x)dx = \lim_{b \to -\infty} \int_b^a f(x)dx$

$\int_a^\infty f(x)dx = \lim_{b \to \infty} \int_a^b f(x)dx$

$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_0^\infty f(x)dx$

converges only if both pieces converge

if limit $\to \pm\infty$, then integral diverges

(b) $\int_1^\infty \frac{1}{x^2+x} dx$

Ex 2 (a) $\displaystyle\int_{10}^{\infty} \frac{x}{1+x^2}\,dx$

(b) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^3}\,dx$

Ex 3 (a) $\displaystyle\int_{-\infty}^{\infty} \frac{2x}{\sqrt{x^2+25}}\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{xe^{-x^2/2}}{\sqrt{2\pi}}\,dx$

8.4 Practice (Improper Integrals: Infinite Integrals)

Ex 1 (a) $\displaystyle\int_{\sqrt{5}}^{\sqrt{8}} \frac{x}{(16-2x^2)^{2/3}}\,dx$

(b) $\displaystyle\int_0^{\pi/4} \frac{\sec^2 x}{(\tan x - 1)^2}\,dx$

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$

if $\displaystyle\lim_{x \to b^-} f(x) = \pm\infty$

(i.e. there is VA at $x = b$)

and $f(x)$ continuous on $[a, b)$

$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$

if $\displaystyle\lim_{x \to a^+} f(x) = \pm\infty$

(i.e. there is VA at $x = a$)

and $f(x)$ cont. on $(a, b]$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$c \in (a, b)$

$$= \lim_{t \to c^-} \int_a^t f(x)\,dx + \lim_{p \to c^+} \int_p^b f(x)\,dx$$

if there is VA at $x = c$

<u>Ex 2</u> (a) $\int_{-1}^{1} (2x+1)^{-5/6}\, dx$

(b) $\int_{0}^{5} \frac{x}{9-x^2}\, dx$

<u>Ex 3</u> (a) $\int_{0}^{\pi/2} \frac{\cos x}{\sqrt[5]{\sin x}}\, dx$

(b) $\int_{0}^{27} \frac{x^{1/3}}{x^{2/3}-9}\, dx$