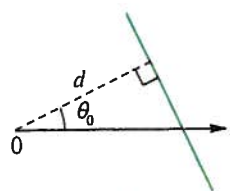
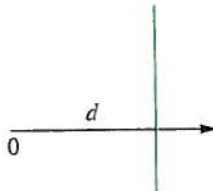
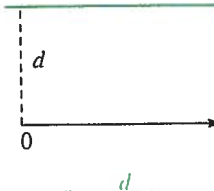
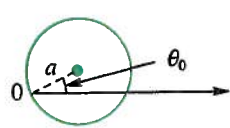
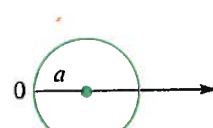
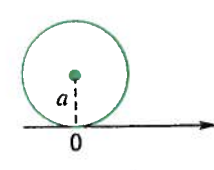
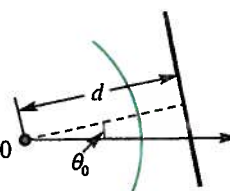
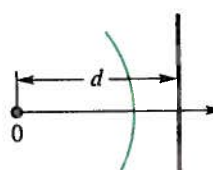
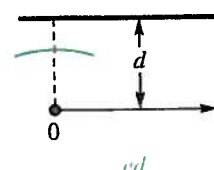


10.5 Practice (Polar Coordinates)

Summary of Polar Equations

Type of Figure	General Case	$\theta_0 = 0$	$\theta_0 = \pi/2$
Line	 $r = \frac{d}{\cos(\theta - \theta_0)}$	 $r = \frac{d}{\cos \theta}$	 $r = \frac{d}{\sin \theta}$
Circle	 $r = 2a \cos(\theta - \theta_0)$	 $r = 2a \cos \theta$	 $r = 2a \sin \theta$
Ellipse ($0 < e < 1$) Parabola ($e = 1$) Hyperbola ($e > 1$)	 $r = \frac{ed}{1 + e \cos(\theta - \theta_0)}$	 $r = \frac{ed}{1 + e \cos \theta}$	 $r = \frac{ed}{1 + e \sin \theta}$

Ex1 Plot and give 3 other ways to write point in polar coords.

(a) $(-1, \frac{15}{4}\pi)$ (b) $(-2\sqrt{2}, \frac{19}{2}\pi)$

Polar to Cartesian

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Cartesian to Polar

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

Ex 2 Convert to rectangular coords.

(a) $(1, \frac{15\pi}{4})$

(b) $(-2\sqrt{2}, \frac{14}{2}\pi)$

Ex 3 Convert to polar coords.

(a) $(0, -2)$

(b) $(5\sqrt{2}, -5\sqrt{2})$

Ex 4 Name curve and sketch.

(a) $r = \frac{-9}{\cos \theta}$

(b) $r = -4 \cos \theta$

Ex4 (cont)

$$(c) \quad r = \frac{4}{2 + \cos(\theta - \pi/6)}$$

$$(d) \quad r = \frac{4}{2 + 2\cos(\theta - \pi/6)}$$

$$(e) \quad r = \frac{4}{2 + 4\cos(\theta - \pi/6)}$$

10.6 Practice (Graphs of Polar Equs)

Ex 1 Name and Sketch graph / look for symmetry.

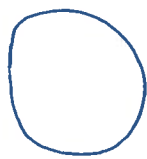
(a) $r = 4 - 3\sin\theta$

(b) $r = 2 - 2\cos\theta$

(c) $r^2 = -9\cos(2\theta)$

Limacons

$$r = a \pm b\cos\theta \quad \text{or} \quad r = a \pm b\sin\theta$$



$a > b$



(cardioid)

$a = b$



$a < b$

Lemniscate

$$r^2 = \pm a\cos(2\theta) \quad \text{or}$$

$$r^2 = \pm a\sin(2\theta)$$



Rose

$$r = a\cos(n\theta) \quad \text{or} \quad r = a\sin(n\theta)$$

n odd \Rightarrow n "leaves" (or petals)

n even \Rightarrow $2n$ " " "



Spiral

$$r = a\theta$$



Ex1 (cont)

(d) $r = 4 \cos(3\theta)$

(e) $r = 4 \cos(4\theta)$

(f) $r = \frac{1}{2}\theta, \theta \geq 0.$

Ex2 Sketch curves + give pts of intersection.
 $r = 6 \sin \theta$ and $r = \frac{6}{1+2 \sin \theta}$

10.7 Practice (Calculus in Polar Coords)

Ex1 Find area inside small loop of $r = 2 - 4\cos\theta$

Area "between" two polar curves

$$A = \frac{1}{2} \int_a^b \text{radius}^2 d\theta$$

$$\text{radius}^2 = (f(\theta))^2$$

(or possibly

$$(f(\theta))^2 - (g(\theta))^2$$

outer radius squared - inner radius squared)

Tangent line Slope

given $r = f(\theta)$ curve

$$m = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$(\text{i.e. } m = \frac{dy/d\theta}{dx/d\theta})$$

Ex2 Find area in Q2 inside $r = 2 + 2\sin\theta$ and outside $2 + 2\cos\theta = r$

Ex 3 Find slope of tangent line to

(a) $r = 4 - 3\cos\theta$ at $\theta = \pi/3$

(b) $r = \sin(2\theta)$ at $\theta = \pi/3$

Extra Ex A goat is tethered to the edge of a circular pond of radius a by a rope of length ka ($0 < k \leq 2$). Find its grazing area.

