MATH 1050-90
PRACTICE EXAM 1
(Sections 1.1-1.9, 2.1-2.5)

The purpose of the practice exam is to give you an idea of the following:

- *length of exam*
- *difficulty level of problems*
- *your instructor's problem writing style*

Your actual exam will have different problems. You should review your homework, quizzes, lecture notes etc., not just perfect taking the practice exam. However, you can use the practice exam to gauge how well you know the material. To do this, take it under the same conditions as a normal exam (no notes, no calculator, time yourself). Then score your problems against the solution key.

The following instructions are on the exam:

- Use a PENCIL, erase or cross out errors.
- **SHOW ALL WORK.** No points will be given for answers without justification.
- Circle your answer so it is easy to locate.
- NO CALCULATORS, NOTES, PHONES, ETC.
- Answers should be simplified (reduced).
- The value of each question is shown.
- Finish in 80 minutes (one hours + 20 minute grace period). You are responsible for keeping track of the time. The proctor does not say "time is up". For each minute you take beyond 80 min, your score will be reduced by 0.5%.

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Parent Functions

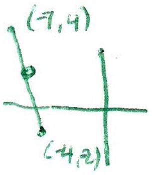
Parabola: polynomial $y = a(x-h)^2 + k$	Square root $y = a\sqrt{x-h} + k$	Cubic $y = a(x-h$
Absolute value $y = a\|x-h\| + k$	Rational $y = \frac{ax \pm b}{cx \pm d}$	

1. *(12pts)* **Points and lines.** Given the points

A (-4,-2) B (-7,4) C (-1,9)

Find the following. Simplify if possible.

a. the midpoint of the segment with endpoints A and B:



$\left(\frac{-7+-4}{2}, \frac{4-2}{2}\right)$

$=\left(\frac{-11}{2}, \frac{2}{2}\right)$

$=\left(\frac{-11}{2}, 1\right)$ +3

b. the distance from A to C:

(oops we did distance from A to B)

$d=\sqrt{(-7-(-4))^2+(4-(-2))^2}$

$=\sqrt{(-3)^2+(6)^2}$

$=\sqrt{9+36}$ +3

$=\sqrt{45} = \sqrt{3^2\cdot 5} = 3\sqrt{5}$

c. the slope of the line containing B and C:

$m=\frac{9-4}{-1-(-7)} = \frac{5}{-1+7}$

$=\frac{5}{6}$ +2

d. The equation of the line passing through ~~B and C and PERPENDICULAR to A.~~

(*in slope-intercept form, y=mx+b*):

A & perpendicular to the line containing B & C.

+4

slope $=-\frac{6}{5}$

$y=-\frac{6}{5}x+b$

$-2=-\frac{6}{5}\cdot -4+b$

$\frac{-10}{5}=\frac{24}{5}+b$

$-\frac{34}{5}=b$

$y=\frac{-6}{5}x-\frac{34}{5}$

2. *(16 pts)* **Functions**

$f(x)=x-7$ and $g(x)=2-x^3$.

Find the following. Simplify if possible.

a. $(f\circ g)(x) = f(2-x^3)$

$=2-x^3-7$

$=-x^3-5$ +4

b. $(g\circ f)(5)$ $g(f(5))$

$=g(5-7)$ +4

$=g(-2)$

$=2-(-2)^3$

$=2-(-8)$

$=2+8$ $=10$

c. $g^{-1}(x)$ +4

$y=2-x^3$

$x=2-y^3$

$x-2=-y^3$

$2-x=y^3$

$\sqrt[3]{2-x}=y$

$g^{-1}(x)=\sqrt[3]{2-x}$

d. $\left(\frac{g}{f}\right)(x) = \frac{2-x^3}{x-7}$ +2

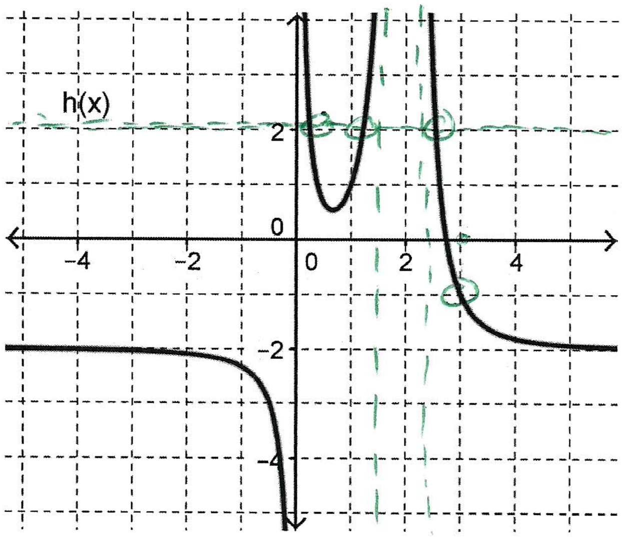
e. What is the domain of the function in d?

$\mathbb{R}, x\neq 7$ +2

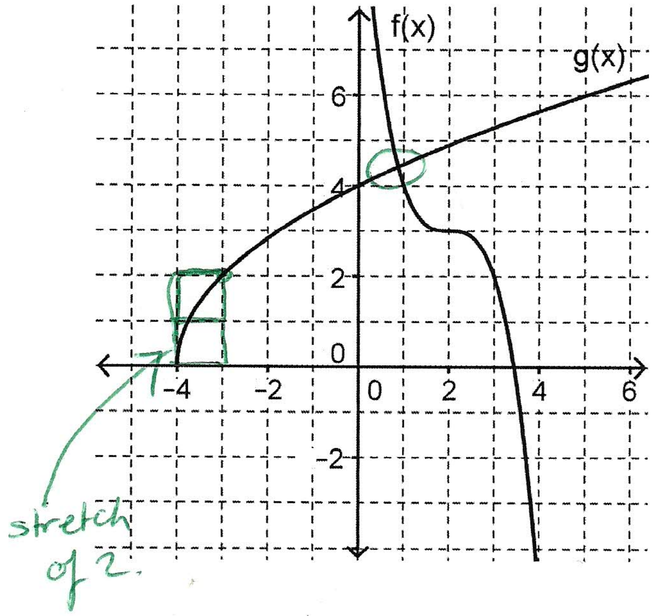
or

$(-\infty, 7)\cup(7,\infty)$

Graph A



Graph B



3. *(8 pts)* **Analyzing GRAPH A.**

   a. h(3) = -1 +1

   b. For which value(s) of x is h(x) = 2? $\approx 0.3, 1.2, 2.5$ +3

   c. Domain *(write as interval)* $(-\infty, 0) \cup (0, 1.5) \cup (2.3, \infty)$ +2

   d. Range *(write as interval)* ~~[illegible]~~ $(-\infty, -2) \cup (-2, \infty)$ +2

4. *(7 pts)* **Analyzing GRAPH B.**

   a. For which x does f(x) = g(x)? $(0.9, 4.4)$ +1

   no stretch.
   cubic. → 2 ↑ 3
   reflected about x-axis.

   b. f(x) = $-(x-2)^3 + 3$ +1 +1 +1 +1 *(Hint, parent functions are shown on the cover page)*

   c. g(x) = $2\sqrt{x+4}$ sq root +1 +1 +1 *(Hint, parent functions are shown on the cover page)*

5. *(6 pts)* **Simplify the expression.** $(7i+3)^2+\sqrt{-100}+i^9$

Write the answer in the form a+bi

$(7i)^2+2(7i)(3)+3^2+\sqrt{100}\cdot i+i^4\cdot i^4\cdot i$

$=-49+42i+9+10i+i$

$=-40+53i$

+6
- subtract 1 for each error

6. *(14 points)* **Quadratic functions.** Given this quadratic function $f(x) = -4x^2+24x-31$

a. Complete the square to rewrite it in the form $f(x) = a(x-h)^2+k$.

$f(x) = -4x^2+24x-31$

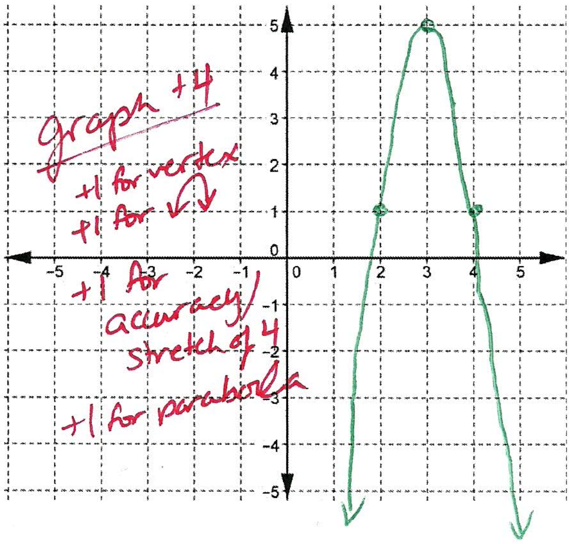
$= -4(x^2-6x+\_\_\_) \quad -31$

$= -4(x^2-6x+9) \quad +36-31$

$= -4(x-3)^2+5$

+5 Subtract 1 for each error

c. Graph f(x) showing Be accurate.



b. Use the quadratic formula to find the ~~roots~~ zeros of f(x). Simplify.

$f(x) = -4x^2+24x-31$

$x = \dfrac{-24 \pm \sqrt{(24)^2-4(4)(31)}}{2(-4)}$

$= \dfrac{-24 \pm \sqrt{576-496}}{-8}$

$= \dfrac{-24 \pm \sqrt{80}}{-8}$

$= \dfrac{-24 \pm 4\sqrt{5}}{-8}$

$= \cancel{3 \pm 4\sqrt{5}}$

answer should be (3 +- sqrt(5)/2)

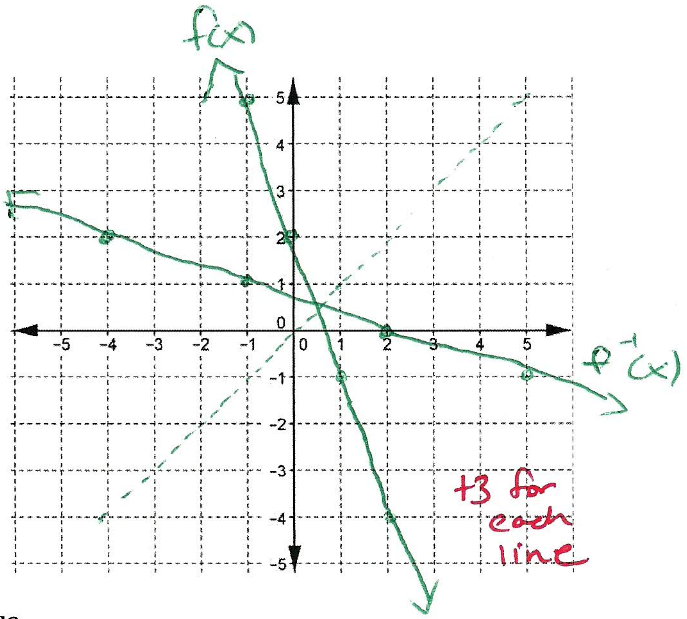
+5 Subtract 1 for each error

24	16	576
24	31	−496
96	16	80
480	480	
576	496	

$80 = 2\cdot 40 = 4\cdot 20 = 4\cdot 4\cdot 5$

13 pts

7. (~~5pts~~) **Functions, inverses and graphs**



Given the function $f(x) = 2 - 3x = -3x + 2$

a. Find $f^{-1}(x)$

$y = 2 - 3x$

$x = 2 - 3y$

$x - 2 = -3y$

+4 $\frac{x}{-3} - \frac{2}{-3} = y$

$f^{-1}(x) = -\frac{1}{3}x + \frac{2}{3}$

b. Graph $f(x)$ and $f^{-1}(x)$ on the same graph.

c. How are the graphs of $f(x)$ and $f^{-1}(x)$ related?

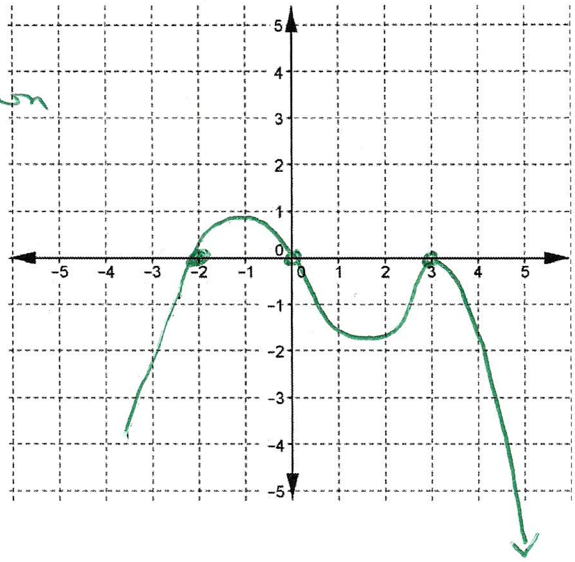
+3
- they are reflections of each other in the line $y = x$

or
- their x- & y- coordinates are swapped. for example (0,2) is on f(x) & (2,0) on $f^{-1}(x)$.

8. (~~4~~6pts) **Polynomials.** Draw a (polynomial) function that looks like it meets these requirements:

- +1 Degree 4 – or
- +1 Has negative leading coefficient → faces down
- Has zeros at x=-2, x=0 and a DOUBLE zero at x=3

+2 double zero +2



Numbers on this page too challenging for exam.

Try same questions with

$2x^3-9x^2+4x+18$

However, in case you worked with the original numbers, their solution is below.

9. *(9 pts)* **Polynomials:** Given: $p(x) = 4x^3 - 12x^2 - x + 15$

a. Calculate: p(x)÷(x-5) *(Use synthetic division or long division)*

$$\begin{array}{r|rrrr} 5 & 4 & -12 & -1 & 15 \\ & & 20 & 40 & 195 \\ \hline & 4 & 8 & 39 & 210 \end{array}$$

$\rightarrow \frac{p(x)}{(x-5)} = 4x^2-8x+39+\frac{210}{x-5}$

b. p(5) =

By remainder theorem, $p(5)=210$

or $p(5)=4(5)^3-12(5)^2-5+15=500-300-5+15=210$

c. What is the relationship between a. and b.? (Hint, think of a theorem.)

The remainder of $\frac{p(x)}{x-5}$ is the value $p(5)$

d. What is the y-intercept of p(x)?

$p(0)=15$ → $(0,15)$

e. Factor p(x).

Hint: either x-1 or x+1 is a factor of the polynomial. Decide which one. Factor it out and then factor the remaining polynomial more.

$$\begin{array}{r|rrrr} 1 & 4 & -12 & -1 & 15 \\ & & 4 & -8 & -9 \\ \hline & 4 & -8 & -9 & 6 \end{array}$$

not a factor

$$\begin{array}{r|rrrr} -1 & 4 & -12 & -1 & 15 \\ & & -4 & 16 & -15 \\ \hline & 4 & -16 & \cancel{-}15 & 0 \end{array}$$

$(x+1)$ is a factor!

$p(x)=(x+1)(4x^2-16x+15)$

=(x+1)(2x-5)(2x-3)

~~$4x^2-16x-15=0$~~

~~$(2x- \quad)(\quad ?$~~

Not sure how to factor.

~~use q.f.~~

~~$x=\frac{16\pm\sqrt{16^2-4\cdot4(-15)}}{2(4)} = \frac{16\pm\sqrt{16^2+16\cdot15}}{8} = \frac{16\pm\sqrt{16\cdot31}}{8} = \frac{16\pm4\sqrt{31}}{8} = \frac{1}{2}\pm\frac{1}{4}\sqrt{31}$~~

$$\begin{array}{rr} 16 & 16 \\ \times 16 & 15 \\ \hline 96 & \\ 160 & \\ \hline 256 & \end{array}$$

Factorization: $(x+1)\left(x-\frac{1}{2}-\frac{1}{4}\sqrt{31}\right)\left(x-\frac{1}{2}+\frac{1}{4}\sqrt{31}\right)$

9 alternatiee (Replaced original 9)

$$p(x) = 2x^3 - 9x^2 + 4x + 15$$

a.) $\frac{p(x)}{x-5} = ?$

$$\begin{array}{r|rrrr} 5 & 2 & -9 & 4 & 15 \\ & & 10 & 5 & 45 \\ \hline & 2 & 1 & 9 & 60 \end{array}$$

$$\frac{p(x)}{x-5} = 2x^2 + x + 9 + \frac{60}{x-5}$$

b.) $p(5) = 2 \cdot 5^3 - 9 \cdot 5^2 + 4 \cdot 5 + 15$

$= 2 \cdot 125 - 9 \cdot 25 + 20 + 15$

$= 250 - 225 + 20 + 15$

$= 25 + 35$

60

c.) The remainder of $\frac{p(x)}{x-5}$ is the same as $p(5)$. This is demonstrating the remainder theorem.

d.) y-int:

$p(0) = 15$

$\rightarrow (0, 15)$

e.)

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 4 & 15 \\ & & 2 & -7 & -3 \\ \hline & 2 & -7 & -3 & 12 \end{array}$$

$(x-1)$ is not a factor

$$\begin{array}{r|rrrr} -1 & 2 & -9 & 4 & 15 \\ & & -2 & 11 & -15 \\ \hline & 2 & -11 & 15 & 0 \end{array}$$

$p(x) = (x+1)(2x^2 - 11x + 15)$

$2x^2 - 11x + 15 = 0$

$(2x-5)(x-3)$

$\rightarrow p(x) = (x+1)(x-3)(2x-5)$