Practice for section 2.3 Synthetic division

1. Divide each of these using synthetic division if practical:

$(x^4 - 3x^2 + x - 2) \div (x^2 + 1)$ $\qquad$ $(x^5 - 1) \div (x - 1)$

$(3x^3 + x - 1) \div (x + 2)$ $\qquad$ $(3x^3 + x - 2) \div (x - 2)$

a) $(x^4 - 3x^2 + x - 2) \div (x^2 + 1) = x^2 - 4$

$-\ x^4 + x^2$

$-4x^2 + x$

$-\ -4x^2 - 4$

$x + 4 - 2$

$\boxed{x + 2}$

b) $(x^5 - 1) \div (x - 1) = x^4 + x^3 + x^2 + x + 1$

$-\ x^5 - x^4$

x^4

$-\ x^4 - x^3$

x^3

$-\ x^3 - x^2$

x^2

$-\ x^2 - x$

$x - 1$

$-\ x - 1$

0

You can use synthetic division here

c) $3x^3 + x - 1 \div (x + 2) = 3x^2 - 6x + 13$

$-\ 3x^3 + 6x^2$

$-6x^2 + x$

$-\ -6x^2 - 12x$

$13x - 1$

$-\ 13x + 26$

$\boxed{-27}$

You can use synthetic division here

d) $3x^3 + x - 2 \div (x-2) = 3x^2 + 6x + 13$

$$\begin{array}{r} 3x^3 + x - 2 \\ -\ (3x^3 - 6x^2) \\ \hline 6x^2 + x \\ -\ (6x^2 - 12x) \\ \hline 13x - 2 \\ -\ (13x - 26) \\ \hline \boxed{24} \end{array}$$

You can use synthetic division here

2. For this function f(x) = $x^3 - 7x + 6$

Divide by (x-2) find f(2) $= 2^3 - 7 \cdot 2 + 6 = 8 - 14 + 6 = 0$

$x^3 - 7x + 6 \div x - 2 = x^2 + 2x - 3$

$$\begin{array}{r} x^3 - 7x + 6 \\ -\ (x^3 - 2x^2) \\ \hline 2x^2 - 7x \\ -\ (2x^2 - 4x) \\ \hline -3x + 6 \\ -\ (-3x + 6) \\ \hline 0 \end{array}$$

$f(x) = x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3)$

$f(2) = 0$

Divide by (x+1) find f(-1) $= (-1)^3 - 7(-1) + 6 = -1 + 7 + 6 = 12$

$x^3 - 7x + 6 \div x + 1 = x^2 - x - 6$

$$\begin{array}{r} x^3 - 7x + 6 \\ -\ (x^3 + x^2) \\ \hline -x^2 - 7x \\ -\ (-x^2 - x) \\ \hline -6x + 6 \\ -\ (-6x - 6) \\ \hline 12 \end{array}$$

$f(x) = x^3 - 7x + 6 = (x+1)(x^2 - x - 6) + 12$

$f(-1) = 12$

Divide by (x+2)

f(-2) = $(-2)^3 - 7(-2) + 6 = -8 + 14 + 6 = 12$

$x^3 - 7x + 6 \div x + 2 = x^2 - 2x - 3$

$$
\begin{array}{l}
x^3 - 7x + 6 \\
\underline{-x^3 \mp 2x^2} \\
-2x^2 - 7x \\
\underline{\mp 2x^2 \mp 4x} \\
\quad -3x + 6 \\
\quad \underline{\mp 3x \mp 6} \\
\qquad 12
\end{array}
$$

$f(x) = x^3 - 7x + 6 = (x+2)(x^2 - 2x - 3) + 12$

$f(-2) = 12$

Divide by (x+3)

find f(-3) $= (-3)^3 - 7\cdot(-3) + 6 = -27 + 21 + 6 = 0$

$x^3 - 7x + 6 \div x + 3 = x^2 - 3x + 2$

$$
\begin{array}{l}
x^3 - 7x + 6 \\
\underline{-x^3 - 3x^2} \\
-3x^2 - 7x \\
\underline{\mp 3x^2 \mp 9x} \\
\quad 2x + 6 \\
\quad \underline{-2x - 6} \\
\qquad 0
\end{array}
$$

$f(x) = x^3 - 7x + 6 = (x+3)(x^2 - 3x + 2)$

$f(-3) = 0$

What can you conclude?

If $(x-k)$ divides a polynomial $f(x)$ then k is the root of f $(f(k) = 0)$ and we can write $f(x) = (x-k)\,g(x)$, where $g(x)$ is a polynomial whose degree is smaller than degree of f.