

Solutions for practice problems in 3.2 Logarithmic Functions

1. Determine x and put these in logarithmic form.

$$2^{-5} = \frac{1}{x} \quad \Leftrightarrow \quad \log_2 \left(\frac{1}{x} \right) = -5$$

$$\text{Also: } 2^{-5} = \frac{1}{x} \quad \Leftrightarrow \quad \frac{1}{2^5} = \frac{1}{x} \Rightarrow x = 2^5$$

$$\left(\frac{3}{4} \right)^{-2} = x \quad \Leftrightarrow \quad \log_{\frac{3}{4}} x = -2$$

$$\text{Also: } \left(\frac{3}{4} \right)^{-2} = x \quad \Leftrightarrow \quad \left(\frac{4}{3} \right)^2 = x \Rightarrow \frac{16}{9} = x$$

$$\left(\frac{4}{25} \right)^{-\frac{1}{2}} = x \quad \Leftrightarrow \quad \log_{\frac{4}{25}} x = -\frac{1}{2}$$

$$\text{Also: } \left(\frac{4}{25} \right)^{-\frac{1}{2}} = x \quad \Leftrightarrow \quad \left(\frac{25}{4} \right)^{\frac{1}{2}} = x \Rightarrow \frac{5}{2} = x$$

$$10^x = 10,000 \quad \Leftrightarrow \quad \log_{10} 10,000 = x$$

$$\text{Also: } 10^x = 10,000 \quad \Leftrightarrow \quad 10^x = 10^4 \Rightarrow x = 4$$

2. Determine x and put these in exponential form.

$$\log_3 \frac{1}{27} = x \Leftrightarrow 3^x = \frac{1}{27} \Leftrightarrow 3^x = \frac{1}{3^3} \Leftrightarrow 3^x = 3^{-3} \\ \Rightarrow x = -3$$

$$\log_{10} x = -3 \Leftrightarrow 10^{-3} = x \Rightarrow x = \frac{1}{10^3} \Leftrightarrow x = 0.001$$

$$\log_x \frac{27}{64} = -\frac{1}{3} \Leftrightarrow x^{-\frac{1}{3}} = \frac{27}{64} \Leftrightarrow x^{\frac{1}{3}} = \frac{64}{27} \Rightarrow \\ x = \left(\frac{64}{27}\right)^3$$

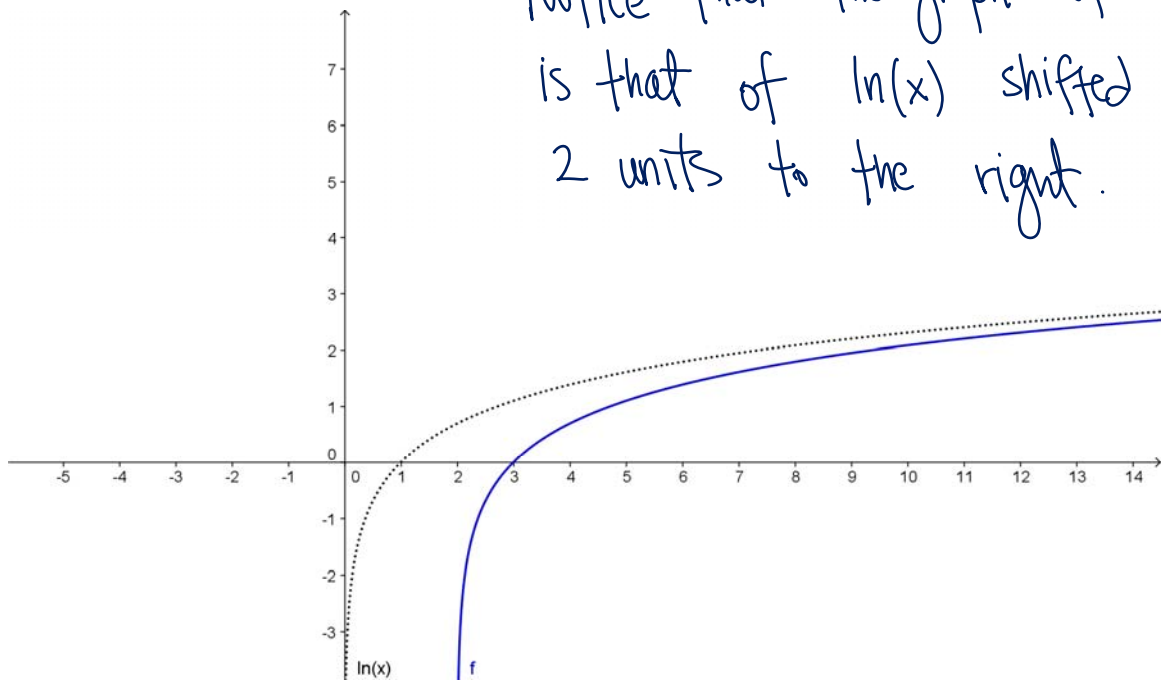
$$\log_x 1 = 0 \Leftrightarrow x^0 = 1$$

x any positive real number $\neq 1$

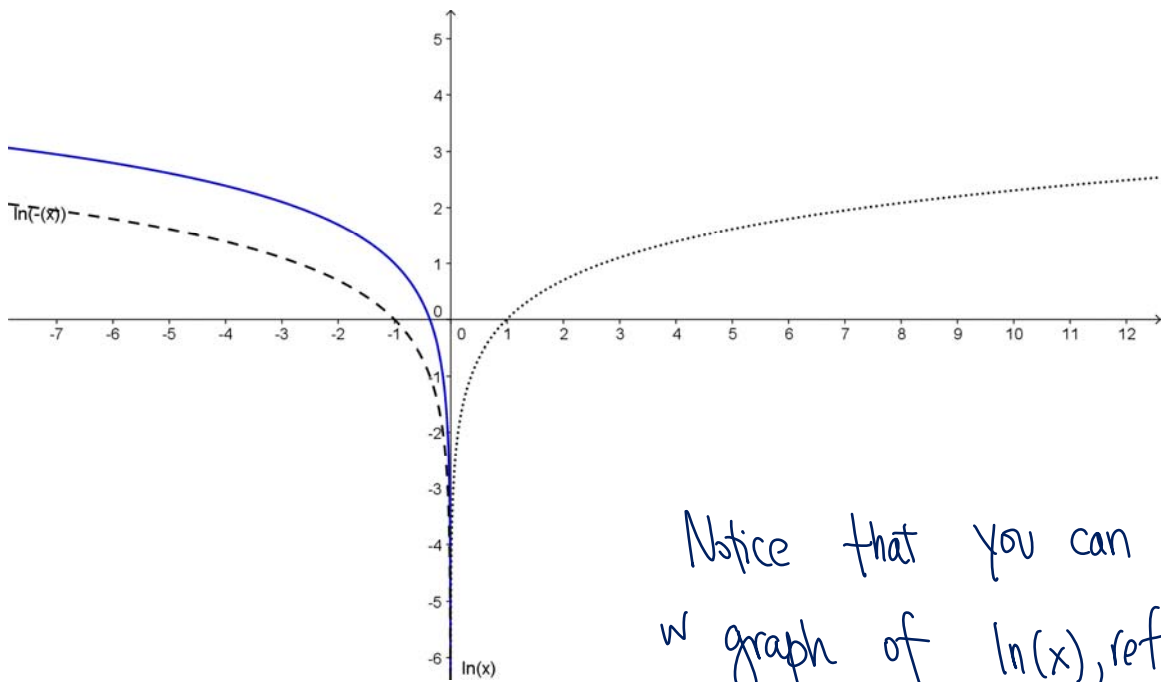
3. Sketch the graph:

$$f(x) = \ln(x-2)$$

Notice that the graph of f is that of $\ln(x)$ shifted by 2 units to the right.



$$f(x) = \ln(-x) + 1$$



Notice that you can start w graph of $\ln(x)$, reflect it about y-axis to get the graph of $\ln(-x)$ and finally shift that graph 1 unit up.

4. Determine these values without a calculator:

$$\ln e^{-2} = -2$$

$$\ln(1) = 0$$

$$\ln\left(\frac{1}{e^3}\right) = -3$$

$$\ln e = 1$$

$$\ln(5e) = 1 + \ln 5$$

$$\ln\left(\frac{3}{e^2}\right) = \ln 3 - \ln e^2 = \ln 3 - 2$$