

Solutions for practice problems in 3.4 Solving equations

1. Solve for x (exact answer, no calculator).

$$2^x = 15 \quad \log 2^x = \log 15$$
$$x \log 2 = \log 15$$
$$x = \frac{\log 15}{\log 2}$$

$$x^4 = 12$$

$$x = 12^{\frac{1}{4}}$$

$$\text{or } x = \sqrt[4]{12}$$

$$\log_5 x = -3$$

$$5^{-3} = x$$

$$\frac{1}{5^3} = x \quad \Leftrightarrow \quad x = \frac{1}{125}$$

$$\ln(2x) = -5$$

$$2x = e^{-5}$$

$$2x = \frac{1}{e^5}$$

$$x = \frac{1}{2e^5}$$

2. Solve for x (exact answer, no calculator.)

$$\ln(x+1) + \ln x = 2$$

$$\ln(x(x+1)) = 2$$

$$x(x+1) = e^2$$

$$x^2 + x - e^2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 4e^2}}{2}$$

$$e^{2x-1} = 5$$

$$\ln e^{2x-1} = \ln 5$$

$$(2x-1) \ln e = \ln 5$$

$$2x-1 = \ln 5$$

$$2x = 1 + \ln 5$$

$$x = \frac{1 + \ln 5}{2}$$

3. If you invest \$5,000 in an account which pays 3.2%, how long will it take to triple your money?

$$A = Pe^{rt}$$

we will assume that interest is compounded continuously

$$P = 5000$$

$$A = 3P = 15000$$

$$r = 0.032$$

$$15000 = 5000 e^{0.032t} \quad / \div 5000$$

$$3 = e^{0.032t}$$

$$\ln 3 = \ln e^{0.032t}$$

$$\ln 3 = 0.032t \text{ fine}$$

$$t = \frac{\ln 3}{0.032} = 34.33 \text{ years}$$

Note: It doesn't matter what the original amount is:

$$A = 3P \text{ so}$$

$$3P = Pe^{rt}$$

$$3 = e^{rt}$$

4. If the half-life of an isotope is 5715, how many years will it take for 10 g to dissipate to 1 g?

$$\frac{1}{2}P = Pe^{r \cdot 5715}$$

$$\frac{1}{2} = e^{r \cdot 5715}$$

$$\ln \frac{1}{2} = \ln e^{r \cdot 5715}$$

$$\ln \frac{1}{2} = 5715r$$

$$r = \frac{\ln \frac{1}{2}}{5715} = \frac{-\ln 2}{5715}$$

$$1g = 10g \cdot e^{-\frac{\ln 2}{5715}t}$$

$$\frac{1}{10} = e^{-\frac{\ln 2}{5715}t}$$

$$\ln \frac{1}{10} = -\frac{\ln 2}{5715}t$$

$$-\ln 10 = -\frac{\ln 2}{5715}t$$

$$t = \ln 10 \cdot \frac{5715}{\ln 2}$$

$$t = 18984.82 \text{ years}$$