Solutions for practice in 4.1 Linear and Nonlinear Systems of Equations

1. Use substitution

$x - 2y = 0$ $\Rightarrow$ $x = 2y$

$3x - y^2 = 0$

Then

$3x - y^2 = 0$

$3(2y) - y^2 = 0$

$6y - y^2 = 0$

$y(6 - y) = 0 \Rightarrow y = 0$ OR $y = 6$

- $y = 0$ $\quad x = 2y = 2 \cdot 0 = 0$ $\quad x = 0, y = 0$ $\quad (0, 0)$
- $y = 6$ $\quad x = 2y = 2 \cdot 6 = 12$ $\quad x = 12, y = 6$ $\quad (12, 6)$

2. Solve graphically

$x^2 - 6x - 27 + y^2 = 0$

$y - x = 3$

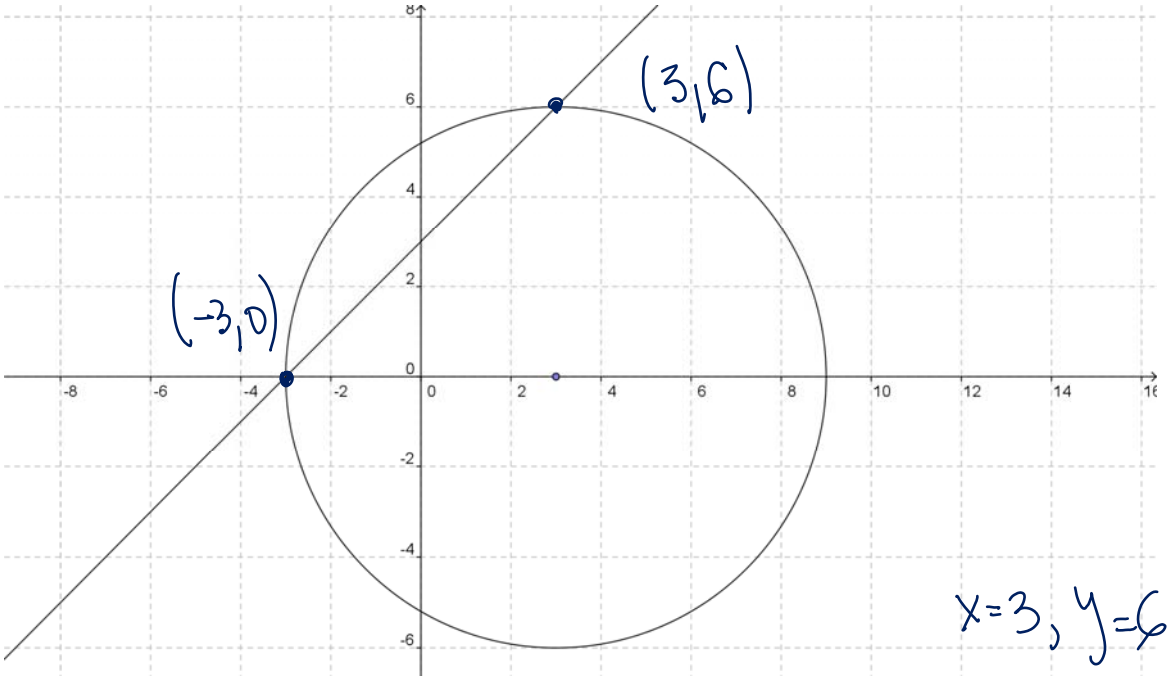
① $x^2 - 6x - 27 + y^2 = 0$ $\quad \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$x^2 - 6x + 9 - 9 - 27 + y^2 = 0$

$(x-3)^2 + y^2 = 36 \Rightarrow$ circle with center at $(3,0)$ and radius 6

$y - x = 3$ line with slope 1 and

$y = x + 3$ y-intercept $(0,3)$



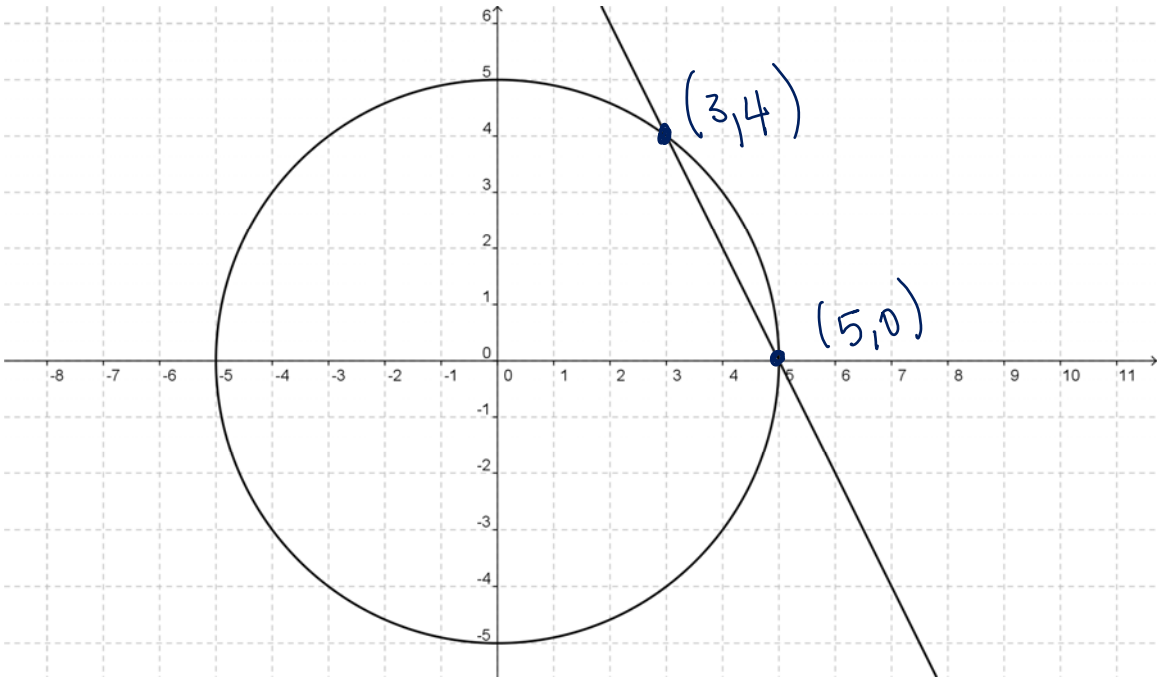
$x = 3, y = 6$

or $x = -3, y = 0$

3. Solve:

$x^2 + y^2 = 25$ — circle with center @ the origin and radius 5

$2x + y = 10$ — line with slope -2 and y-intercept $(0, 10)$



$x = 3, y = 4$

or

$x = 5, y = 0$

4. You are offered two jobs selling cleaning supplies. One company offers a straight commission of 6% of your sales. The other company offers a salary of $300 per week plus 3% of sales. How much would you have to sell in a week in order to make the straight commission a better job?

x = your sales

y = your salary

straight commission: $y = 0.06x$

salaried job: $y = 0.03x + 300$

Note that straight commission has higher slope, but the y-intercept @ the origin, while salaried job has y-intercept @ $(0, 300)$. This means that for x smaller than the x-coordinate of the intersection point you'd be making less on the straight commission job. Likewise you'll make more money on the straight commission if your sales are larger than the x-coordinate of the intersection.

$$\begin{cases} y = 0.06x \\ y = 0.03x + 300 \end{cases}$$

$$0.06x = 0.03x + 300$$

$$0.03x = 300 \Rightarrow x = \frac{300}{0.03} = \frac{30000}{3} = 10000$$

You'd have to sell more than $1000 worth of cleaning supplies.