Solutions for practice in 5. 3 The inverse of a square matrix

1. Determine the inverse of these matrices if they exist:

a. $\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$ b. $\begin{vmatrix} 0 & -2 & 1 \\ 3 & 0 & 4 \end{vmatrix}$ c. $\begin{bmatrix} 1 & 4 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$

a) $\left[\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ -1 & 5 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 0 & 17 & 1 & 3 \end{array}\right] \sim$

$\sim \left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 0 & 1 & \frac{1}{17} & \frac{3}{17} \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{17} & -\frac{2}{17} \\ 0 & 1 & \frac{1}{17} & \frac{3}{17} \end{array}\right]$

b) Not possible since it's not a square matrix

c) $\left[\begin{array}{ccc|ccc} 1 & 4 & 7 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -4 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \sim$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -4 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{3} \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{3} \end{array}\right]$

d) $\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right] \sim$

$$\sim \begin{bmatrix} 1 & 0 & -4 & | & 7 & -2 & 0 \\ 0 & 1 & 3 & | & -3 & 1 & 0 \\ 0 & 0 & 1 & | & -5 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & | & -13 & 6 & 4 \\ 0 & 1 & 0 & | & 12 & -5 & -3 \\ 0 & 0 & 1 & | & -5 & 2 & 1 \end{bmatrix}$$

2. Use your work on problem 1 to solve these systems:

a. $3x + 2y = -5$ b. $x + 2y + 2z = 8$

   $-x + 5y = 6$ $3x + 7y + 9z = 1$

                            $-x - 4y - 7z = 0$

a)
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{17} & -\frac{2}{17} \\ \frac{1}{17} & \frac{3}{17} \end{bmatrix} \begin{bmatrix} -5 \\ 6 \end{bmatrix} \supset \begin{bmatrix} -\frac{25}{17} - \frac{12}{17} \\ -\frac{5}{17} + \frac{18}{17} \end{bmatrix} \supset \begin{bmatrix} -\frac{37}{17} \\ \frac{13}{17} \end{bmatrix}$$

$$x = -\frac{37}{17} \qquad y = \frac{13}{17}$$

b)
$$\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -13 \cdot 8 + 6 \cdot 1 + 4 \cdot 0 \\ 12 \cdot 8 - 5 \cdot 1 - 3 \cdot 0 \\ -5 \cdot 8 + 2 \cdot 1 + 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} -98 \\ 91 \\ -38 \end{bmatrix}$$

$$X = -98 \quad , \quad y = 91 \quad , \quad z = -38$$