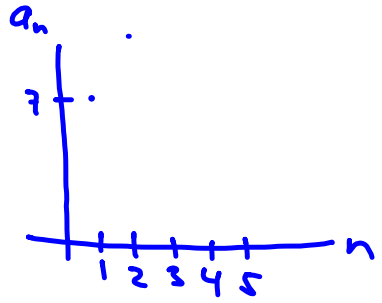
9.2 Sigma Notation

$$\sum_{n=1}^{8} n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$$

arithmetic sequence:

$a_n = 3n + 4$



sum: $S_n = \sum_{j=1}^{n} a_j = \frac{n(a_1 + a_n)}{2}$

$$\sum_{n=1}^{10} (3n+4) = \sum_{j=1}^{10} (3j+4)$$

$$= \frac{10[(3+4)+(30+4)]}{2}$$

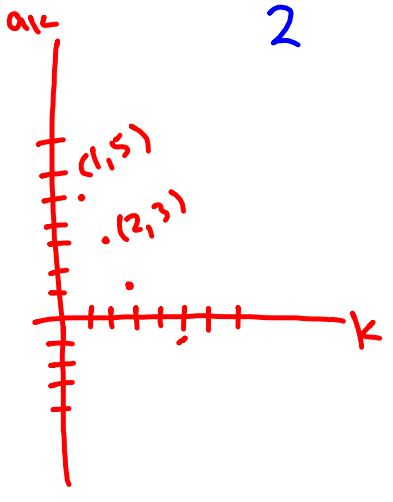
5, 3, 1, −1, . . .

$a_k = -2k + 7$

$$\sum_{k=4}^{12} (-2k+7)$$

$$= \sum_{k=1}^{12} (-2k+7) - \sum_{k=1}^{3} (-2k+7)$$

$$= \frac{12(5+-17)}{2} - \frac{3(5+1)}{2}$$



eqn for circle:

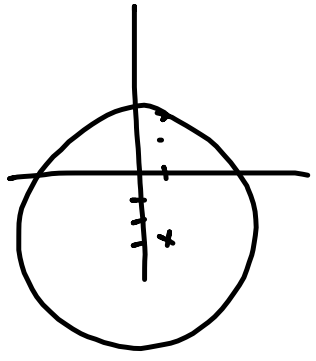
$(x-h)^2+(y-k)^2=a^2$

(h,k) center

a = radius

ex $(x-1)^2+(y+3)^2=25$

center $(1,-3)$ radius $=5$



$f(x)=4x-1 \qquad g(x)=\frac{-2}{x}+1 \qquad h(x)=\sqrt{x-5}$

(a)

$(fg)(1)=f(1)\,g(1)=(4-1)(-2+1)=3(-1)=-3$

(b) $(f\circ g)(x)=f(g(x))=f\left(\frac{-2}{x}+1\right)=4\left(\frac{-2}{x}+1\right)-1$

$=\frac{-8}{x}+4-1=\frac{-8}{x}+3$

(c) $g^{-1}(x)=?$ $\qquad y=\frac{-2}{x}+1$

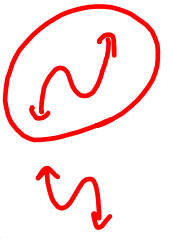
$x=\frac{-2}{y}+1 \iff x-1=\frac{-2}{y}$

$\overset{(-2)}{\iff}\frac{1}{x-1}=\frac{y}{-2}\ (-2)$

$g^{-1}(x)=\frac{-2}{x-1}=y$

Roots / zeros of Polynomials

ex $2x^3 - 3x^2 - 23x + 12 = p(x)$



possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

2	2	-3	-23	12
		4	2	-42
	2	1	-21	-30

4	2	-3	-23	12
		8	20	-12
	2	5	-3	0

$p(x) = (x-4)(2x^2+5x-3)$

$= (x-4)(2x-1)(x+3)$

$x = 4$

$x - 4 = 0$

$2x^2 + 5x - 3$

$= \cancel{(2x+3)(x-1)}$

$= \cancel{(2x+1)(x-3)}$

$= (2x-1)(x+3)$

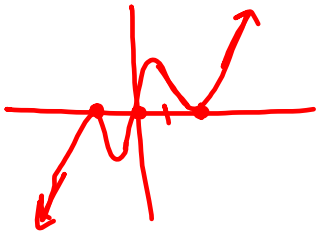
zeros/roots:	4	$\frac{1}{2}$	-3
multiplicity:	1	1	1

ex $m(x) = (x-2)^2 x^3 (x+1)^4$ (9th degree polynomial)

zeros	2	0	-1
multiplicity	2	3	4

odd multiplicity $\Rightarrow$ graph goes thru x axis

even mult $\Rightarrow$ graph touches x axis



System of Linear Eqns.

want $\begin{bmatrix} 1 & x & x & : & x \\ 0 & 1 & x & : & x \\ 0 & 0 & 1 & : & x \end{bmatrix}$

pf
$$x-y+z=4$$
$$x+3y-2z=-3$$
$$3x+7y+2z=6$$

$$\begin{bmatrix} 1 & -1 & 1 & : & 4 \\ 1 & 3 & -2 & : & -3 \\ 3 & 2 & 2 & : & 6 \end{bmatrix} \quad (-1) \quad -R_1+R_2$$

$$\begin{bmatrix} 1 & -1 & 1 & : & 4 \\ 0 & 4 & -3 & : & -7 \\ 3 & 2 & 2 & : & 6 \end{bmatrix} \quad \left(\frac{1}{4}\right)$$

$$\begin{bmatrix} 1 & -1 & 1 & : & 4 \\ 0 & 1 & -3/4 & : & -7/4 \\ 3 & 2 & 2 & : & 6 \end{bmatrix} \quad (-3)$$

$$\begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & 1 & -3/4 & -7/4 \\ 0 & 5 & -1 & -6 \end{bmatrix} \quad (-5)$$

$$\begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & 1 & -3/4 & -7/4 \\ 0 & 0 & 11/4 & 11/4 \end{bmatrix} \quad \left(\frac{4}{11}\right)$$

$$-6=\frac{-24}{4}$$

$$\begin{matrix} ① \\ ② \\ ③ \end{matrix}\begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & 1 & -3/4 & -7/4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

③ $z=1$

② $y-\frac{3}{4}z=-\frac{7}{4}$

$y-\frac{3}{4}=-\frac{7}{4}$

$y=-\frac{7}{4}+\frac{3}{4}=-\frac{4}{4}$

$y=-1$

① $x-y+z=4$

$x-(-1)+1=4$

$x+2=4$

$x=2$

Solutn: $(2,-1,1)$