1.5   Practice (on Squeeze Theorem / Limits)

__Ex1__   Find the limits.

(a) $\lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

(b) $\lim\limits_{x \to -\infty} \dfrac{3\sqrt[3]{x^5} + x - 7}{\sqrt[3]{5x} + 3x + 1}$

(c) $\lim\limits_{x \to \infty} \sqrt[5]{\dfrac{2x^3 - 5x + 1}{\pi x^3 + 3}}$

(d) $\displaystyle\lim_{x \to \infty} \sin\left(x + \frac{1}{x}\right)$

1.6 Continuity Practice

Ex 2 Redefine this fn so it's continuous.

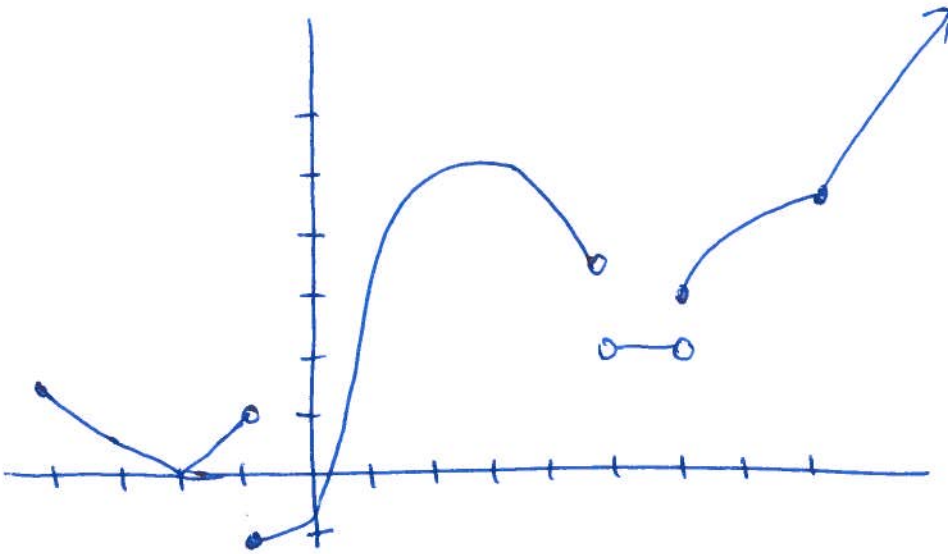(a) $f(x) = \dfrac{\sqrt{x} - 1}{x - 1}$

(b) $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } 0 < x \le 1 \\ 3 - x & \text{if } x > 1 \end{cases}$ Is this continuous? If not, where and why does it fail continuity?

Continuity at $x=c$

① $\lim\limits_{x \to c} f(x)$ exists

② $f(c)$ exists

③ $\lim\limits_{x \to c} f(x) = f(c)$

Ex 3 Is this fn continuous? If not, tell where it's discontinuous & why. On what intervals is it continuous?

2.1 Practice (The Derivative)

Derivative def'n

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(instantaneous slope at any point of a curve $y = f(x)$)

Ex 1 Find derivative

of $f(x) = 4x^2 - 3$.

Ex 2 Find the slope formula of $y = \sqrt{x-1}$ at any point x. Then use that formula to find the slope of the curve at $x = 10$.

Ex 3 Find the equation of the tangent line to $y = \dfrac{3}{x^2}$ at $x = -1$.

Ex 4   what is a secant line?
        what is a tangent line?

Ex 5   If a particle moves along a coordinate line
so that its directed distance from the origin after
t seconds   is $(-t^2 + 4t)$ feet, when did the particle
come to   a momentary stop?

2.2. Practice (More Derivatives)

Use the defn of derivative to find these derivatives.

Ex 1   derivative for $f(x) = \dfrac{x-1}{x+1}$

Ex 2   derivative for $f(x) = \dfrac{x+2}{x}$

Defs of derivative

① $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

② $f'(x) = \lim\limits_{t \to x} \dfrac{f(t) - f(x)}{t-x}$

Ex3   given this derivative, what's the fn?

$\lim\limits_{x \to y} \dfrac{\sin x - \sin y}{x-y}$

⑨

2.3 Practice (Derivative Rules)

Use "shortcuts" to find these derivatives, for the given fns.

EX1 (a) $y = x^{12} + 5x^{-2} - \pi x^{-10}$

(b) $y = (3x^{-2} + 2x)(x^4 - 3x + 7)$

(c) $y = \dfrac{3}{x^5} - x^{-1} + \dfrac{\pi}{x^6}$

(d) $y = \dfrac{5x^2 + 6x - 3}{x^{-2} + 1}$

Ex 2 Find the derivative of these fns.

(a) $y = \dfrac{5}{36 - x^2}$

(b) $y = 3x^2(x^9 + x^8 - 100)$

EX 3 Find eqn of tangent line to $y = \dfrac{1}{x^2 + 4}$

at $x = 1$.

<u>Ex 4</u> Find all points of $y = \frac{1}{3}x^3 + x^2 - x$ graph where tangent line has slope of 1.

1.4 Practice (Trigonometric limits)

EX 1 $\displaystyle\lim_{\theta \to 0} \frac{\sin(5\theta)}{\theta}$

Ex 2 $\displaystyle\lim_{x \to 0} \frac{3x \tan x}{\sin x}$

Ex 3 $\displaystyle\lim_{x\to 0} \frac{\sin(4x) - 2x}{x\cos x}$

Ex 4 $\displaystyle\lim_{x\to 0} \frac{\cos x}{3x}$