3.2–3.3 Practice (Monotonicity / Concavity / Local Extrema)

Ex 1 For $f(x)=\frac{x^2}{x^2+1}$, find all min/max pts, inflection pts & sketch the graph.

EX2 Find all max/min pts, inflection pts, where $f(x)$ is increasing/decreasing, where $f(x)$ is concave up/down & sketch graph.

$$y = f(x) = x\sqrt{x-2}$$

Ex 3 Sketch graph of a continuous fn that satisfies these conditions:

- $f(0) = f(3) = 3$
- $f(2) = 4$
- $f(4) = 2$
- $f(6) = 0$
- $f'(x) > 0$ on $(0,2)$
- $f'(x) < 0$ on $(2,4) \cup (4,5)$
- $f'(2) = f'(4) = 0$
- $f'(x) = -1$ on $(5,6)$
- $f''(x) < 0$ on $(0,3) \cup (4,5)$
- $f''(x) > 0$ on $(3,4)$

Ex 4 Find all min/max pts, inflection pts, increasing/decreasing intervals, concave up/down intervals, for (a) $f(x) = x^2 - \frac{2}{x}$ and sketch the graph.

Ex 4 (cont)

(b) $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta}$ $\quad \theta \in [0, 2\pi]$

3.4 Lecture (Story Problems)

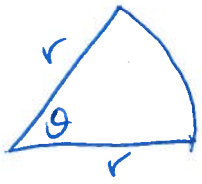
Ex 1 For what number does the principal square root exceed 8 times the number by the largest amt?

Steps

1. Draw a picture and/or list info given.
2. Write down what needs to be optimized.
3. If have more than one input variable, find an eqn to eliminate one of the input vars.
4. Differentiate fn.
5. Set derivative = 0 or find where derivative is undefined, i.e. look for stationary/singular pts.
6. check to ensure it's min or max (whichever you want).

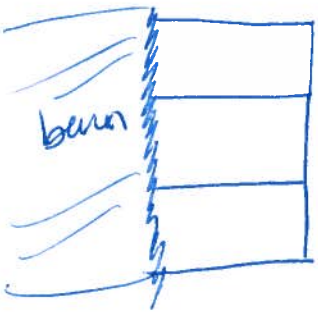
Ex 2 Show that the rectangle w/ max perimeter that can be inscribed in a circle is a square.

Ex 3 A flower bed will be in the shape of a sector of a circle of radius r and vertex angle θ. Find r and θ if its area is a constant A and perimeter is a minimum.



Ex 4 Find the volume of the largest open box that can be made from a piece of cardboard that is 24" by 9". Find the dimensions of the box that yields the max volume. (You'd form the box by cutting identical squares from each corner & fold up.)

EX 5 A farmer has 80 ft of fencing. He needs to enclose 3 identical pens along one side of the barn. What dimensions for the total enclosure make the area of the pens as large as possible?



3.5 Practice (Graphing Functions)

Ex 1 Analyze & graph.

(a) $f(x) = \tan^2 x$

(b) $f(x) = \dfrac{x^2+x-6}{x-1}$

(c) $f(x) = |x|^3$

(Note: $D_x(|x|) = \frac{x}{|x|}$)

3.6 Practice (MVT for Derivatives)

Ex 1 Find all possible c given by MVT, if any exist.

(a) $f(x) = x + \frac{1}{x}$ on $[-1, 2]$

(b) $f(x) = x + \frac{1}{x}$ on $[2, 4]$

Ex 2 Suppose $F'(x) = 5$ and $F(0) = 4$. Find $F(x)$.

Ex 3 Show that if f is the quadratic fn $f(x) = ax^2 + bx + d$, $a \neq 0$, then the c given by MVT is always the midpt on any given interval $[\alpha, \beta]$.

3.7 Practice (Bisection and Newton's Method)

Ex 1 Use Bisection method to approximate real root of $x - 2 + 2\cos x = 0$ on $[1, 2]$

EX 2 Use Newton's method to approximate the smallest root of $2\cos x - \sin x = 0$.

3.8 Practice (Antiderivatives)

Ex 1 Evaluate.

(a) $\int \left(\frac{\sqrt{2x}}{x} + \frac{3}{x^5} \right) dx$

(b) $\int \frac{x\,(x+1)^2}{\sqrt{x}}\, dx$

Ex 2 Evaluate.

(a) $\int x^2 \sqrt[3]{x^3+5}\, dx$

(b) $\int \sin x \cos x \sqrt{1+\sin^2 x}\, dx$

3.9 Practice (Intro D.E.s)

Ex 1 Solve

(a) $\frac{dy}{dx} = y^3(x^3 - x)$ $y(x)$ goes thru $(0, 4)$

(b) $\frac{dy}{dx} = -y^2 x (x^2+2)^3$ thru $(0, 1)$.