

4.1 Practice (Intro to Area)

Ex 1 Write in sigma notation.

(a) $a_1 - a_2 + a_3 - a_4 + a_5 - \dots$

(b) $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \dots$

Ex 2 Find each sum (use special sum formulas)

(a) $\sum_{k=1}^{20} (2k^2 - 3)$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(b) \sum_{j=1}^n (3j+1)^2$$

4.2 Practice (Definite Integral)

Ex 1 Calculate, using defn of definite integral.

$$(a) \int_{-1}^2 (3x^2 + 5) dx$$

Definite Integral

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i \Delta x$$

(right end pt)

$$(b) \int_0^5 2x^3 dx$$

Ex 2 ^{given} f is odd fn; g is even fn; $\int_0^1 |f(x)| dx = \int_0^1 g(x) dx = 3$
Use geometric thinking to evaluate:

$$(a) \int_{-1}^1 f(x) dx$$

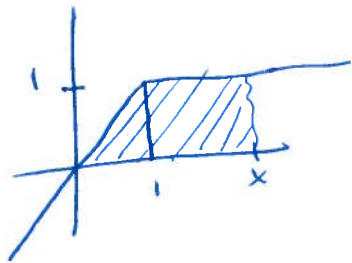
$$(b) \int_{-1}^1 g(x) dx$$

$$(c) \int_{-1}^1 |f(x)| dx$$

$$(d) \int_{-1}^1 f^3(x) g(x) dx$$

4.3 Practice (First Fundamental Thm of Calculus)

Ex1 Find formula for accumulation fn $A(x)$ to represent area.



$$D_x \left[\int_a^x f(t) dt \right] = f(x)$$

$a = \text{constant}$

$f(x)$ continuous on $[c, d]$ $a, x \in (c, d)$

Ex2 Find $G(x)$.

(a) $G(x) = \int_x^1 \sqrt{t^2 + 1} dt$

(b) $G(x) = \int_2^{\tan x} e^{-t^2} dt$

Ex 3 Find $G'(x)$. $G(x) = \int_{x^4}^x \sec(t) dt$

4.4 Practice (Second Fundamental Thm of Calculus)

Ex 1 Evaluate.

(a) $\int_1^3 \frac{x^4 - 5}{x^2} dx$

$$\int_a^b f(x) dx = F(b) - F(a)$$

$f(x)$ continuous on $[a, b]$
 $F(x)$ is any antiderivative

$$(b) \int_0^1 (x^{4/3} - 2x^{1/3}) dx$$

Ex 2 Use u-substitution to evaluate.

$$(a) \int x^3 \cos(x^4 + 1) dx$$

$$(b) \int x^{-3} \sec(x^{-2}-3) \tan(x^{-2}-3) \sqrt[6]{\sec(x^{-2}-3)} dx$$

EX 3 Evaluate.

$$(a) \int_1^2 \frac{x^3 + 2}{\sqrt{x^4 + 8x}} dx$$

$$(b) \int_1^4 \frac{(\sqrt{x}-1)^3}{\sqrt{x}} dx$$

4.5 Practice (Mean Value Theorem (MVT) for Integrals)

Ex1 Find avg. value
on $[0, \pi/2]$ of $f(x) = \sin^2 x \cos x$

MVT

$$f(c) = \frac{1}{b-a} \int_a^b f(t) dt \quad \left. \vphantom{\int_a^b} \right\} \text{called avg. value}$$

$f(x)$ continuous on $[a, b]$
 $c \in (a, b)$

Ex2 Find all values of c guaranteed by MVT, on
 $[0, 2]$ for $f(x) = x^3$.

Ex 3 Use symmetry to help evaluate.

$$(a) \int_{-\pi/4}^{\pi/4} (x \sin^3 x + x^2 \tan x) dx$$

$$(b) \int_{-3}^3 (\sin x - \cos x)^2 dx$$

5.1 Practice (Area of a Plane Region)

Ex 1 Find area between these curves.

(a) $y = \sqrt{x}$, $y = x - 4$, $x = 0$

(b) $x = (3 - y)(y + 1)$, $x = 0$

Ex 2 Find area between curves;

(a) $y = (x-3)(x-1)$, $y = x$.

(b) $x = 4y^4$, $x = 8 - 4y^4$