Math1220 Midterm 1 Review Problems
(6.1-6.5, 6.8, 6.9, 7.1, 7.2)

1. Find the equation of the tangent line to the graph of $y=\cos^{-1}(\ln(x^4))$ when x = 1.

2. Find $f^{-1}(x)$ for $f(x)=\left(\frac{2x-1}{2x+5}\right)^3$.

3. Find $\frac{dy}{dx}$ for each function. **(Don't simplify.)**

   (a) $y=\ln(\cos^2(3x))+\sin^{-1}(3x-2)$
   (b) $y=(5x+3)^{2x^2}$
   (c) $y=(1+x^4)^{\pi}+\pi^{1+x^4}$
   (d) $y=sech(\cos(2x))$
   (e) $y=\ln(3x-2)+2x^{-6}+4x^3-\sin(5x)+9$
   (f) $y=e^{\frac{1}{3x}}+\frac{1}{e^{3x}}$
   (g) $y=(x^3-1)^{\ln x}$
   (h) $y=\cosh^{-1}(\cos x+3)$

4. A certain radioactive substance has half-life of 10 years. How long will it take for 50 grams to decay to 4 grams? (Simplify answer as much as possible without calculator.)

5. Evaluate each integral.

   (a) $\int \arcsin(2x)\,dx$
   (b) $\int \frac{20x+5}{2x^2+x-7}\,dx$
   (c) $\int \frac{-5}{x+x(\ln x)^2}\,dx$
   (d) $\int_1^3 4^{2x-7}\,dx$
   (e) $\int y \arctan(y)\,dy$
   (f) $\int_0^{\frac{\pi}{6}} 2^{\cos x} \sin x\,dx$
   (g) $\int_0^1 \frac{2t^2+1}{2t^3+3t-4}\,dt$
   (h) $\int_{-2}^{0} 6^{2x+4}\,dx$
   (i) $\int \frac{e^{2x}}{e^{2x}+5}\,dx$
   (j) $\int \frac{5x^2}{\sqrt{1-x^6}}\,dx$

(k) $\int \frac{x}{x^4+4}\,dx$

(l) $\int \frac{x^3}{x^4+4}\,dx$

(m) $\int 3x(4^x)\,dx$

(n) $\int_{\frac{\pi}{2}}^{\pi} \frac{2\cos x}{1+\sin^2 x}\,dx$

6. Find $(f^{-1})'(5)$ given $f(x)=2x^5+4x-1$.

7. Show that $f(x)=\frac{\sin x+1}{\cos x}$ is monotonic on the interval $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, i.e. that its inverse exists on that domain.

8. Find the area of the region bounded by $y=\sinh x$, $y=0$ and $x=\ln 2$.

9. Show that $f(x)=6-\tan^{-1}(2x)-5(x-1)^3$ has an inverse. (Explain your reasoning.) Then, find $(f^{-1})'(11)$

10. Find the limits.

(a) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5x}$

(b) $\lim_{x\to\infty}(1)^{5x}$