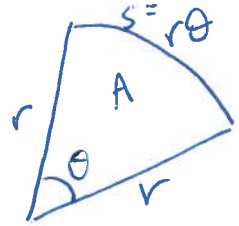
3.4 Ex 3

A flower bed will be in the shape of a sector of a circle of radius r and vertex angle θ. Find r and θ if its area is a constant A and perimeter is a minimum.



perimeter

$P = r + r + r\theta = 2r + r\theta = r(2+\theta)$

$(r > 0)$

$A = \pi r^2\left(\frac{\theta}{2\pi}\right)$

$A = \frac{1}{2}r^2\theta$

$\theta = \frac{2A}{r^2}$

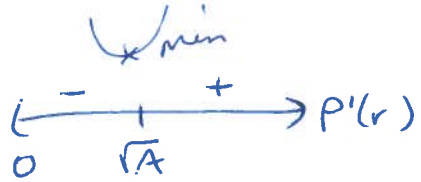
$P(r) = r\left(2 + \frac{2A}{r^2}\right)$

$P(r) = 2r + \frac{2A}{r}$

$P'(r) = 2 - \frac{2A}{r^2} = 0$

$r^2 = A$

$r = \sqrt{A}$



$\Rightarrow r = \sqrt{A}$ and $\theta = \frac{2A}{A} = 2$ (radians) for minimum perimeter

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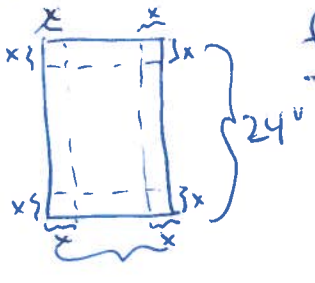
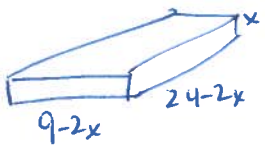
Ex 4 Find volume of largest open box that can be made from a piece of cardboard that is 24" by 9". Find the dimensions of the box that yields max volume.



fold →



$0 < x < 4.5$

$V = x(9-2x)(24-2x)$

$= (9x-2x^2)(24-2x)$

$V' = \frac{dV}{dx} = (9x-2x^2)(-2) + (9-4x)(24-2x)$

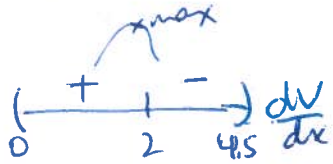
$= -18x + 4x^2 + 216 - 18x - 96x + 8x^2$

$= 12x^2 - 132x + 216$

$= 12(x^2 - 11x + 18)$

$= 12(x-9)(x-2) = 0$

$x = 2, \cancel{9}$ too big



$x = 2$

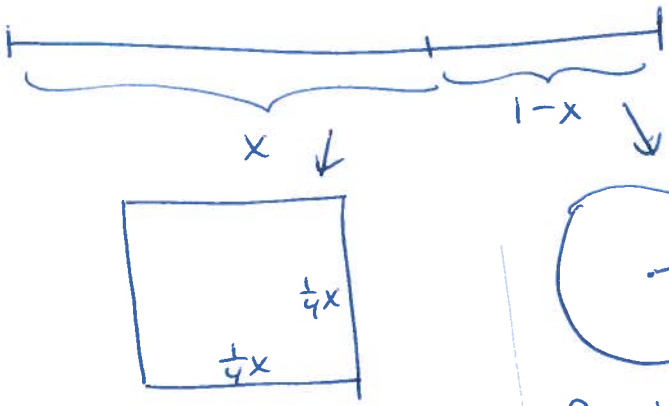
dimensions are: $9-2(2)$ by $24-2(2)$ by 2

$= 5$ in by 20 in by 2 in

max volume $= 5(20)(2) = 200$ in^3

From class today:

$0 < x < 1$



$P_{\square} = x \Rightarrow$ each side is $\frac{1}{4}x$

$\Rightarrow A_{\square} = \left(\frac{1}{4}x\right)^2 = \frac{x^2}{16}$

$P_o = 1-x = 2\pi r$

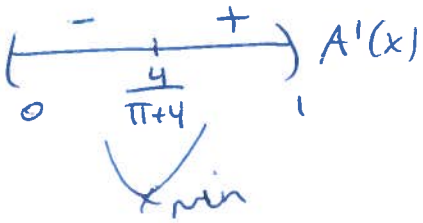
$\Rightarrow r = \frac{1-x}{2\pi}$

$\Rightarrow A_o = \pi\left(\frac{1-x}{2\pi}\right)^2 = \frac{\pi(1-2x+x^2)}{4\pi^2}$

$A_o = \frac{1-2x+x^2}{4\pi}$

total area: $A = \frac{x^2}{16} + \frac{x^2-2x+1}{4\pi}$

$A'(x) = \frac{x}{8} + \frac{2x-2}{4\pi} = 0$



$\frac{x(8\pi)}{8} + \frac{(2x-2)(8\pi)}{4\pi} = 0(8\pi)$

$\pi x + 4x - 4 = 0$

$x(\pi+4) = 4$

$x = \frac{4}{\pi+4}$

one piece of wire is $\frac{\pi}{\pi+4}$ m and other is $\frac{4}{\pi+4}$ m