Math1220 Midterm 3 Summer, 2012 Kelly MacArthur

Name Key Date ____________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

Every question is worth 10 points.

Part 1: Determine if each series converges absolutely, converges conditionally, or diverges. Show all your work, state which tests you used, and explain your reasoning.

1. $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{3n} = \sum_{n=2}^{\infty} \frac{(-1)^n}{3n} = \frac{1}{3} \underbrace{\sum_{n=2}^{\infty} \frac{(-1)^n}{n}}_{\text{alternating harmonic series}}$

$\left(\cos(n\pi) \to \underset{n=2}{1}, \underset{n=3}{-1}, \underset{n=4}{1}, -1, \ldots \right)$

Converges Absolutely or Converges Conditionally or Diverges (circle one)

Why/Test Used? by LCT and AST (or recognizing alternating harmonic series)

2. $\sum_{n=2}^{\infty} \frac{\sqrt{n}(-4)^n}{(3n-1)!} = \sum_{n=2}^{\infty} \frac{\sqrt{n}(-1)^n 4^n}{(3n-1)!}$

ART:

$$\lim_{n\to\infty} \frac{\sqrt{n+1}\ 4^{n+1}}{(3n+2)!} \cdot \frac{(3n-1)!}{\sqrt{n}\ 4^n}$$

$$= 4 \lim_{n\to\infty} \frac{\sqrt{n+1}\ \cancel{(3n-1)!}}{\sqrt{n}(3n+2)(3n+1)(3n)\cancel{(3n-1)!}}$$

$$= 4(0) = 0 < 1$$

$\Rightarrow$ absolute convergence

Converges Absolutely or Converges Conditionally or Diverges (circle one)

Why? by ART

3. $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{8n^5 - n^2}$

try LCT on $\sum_{n=1}^{\infty} \left| \frac{(-1)^n (n^2+1)}{8n^5 - n^2} \right| = \sum_{n=1}^{\infty} \frac{(n^2+1)}{8n^5-n^2}$

w/ $b_n = \frac{n^2}{n^5} = \frac{1}{n^3} \Rightarrow \sum b_n$ converges, p-series $p = 3 > 1$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2+1}{8n^5-n^2} \cdot \frac{n^3}{1} = \lim_{n\to\infty} \frac{n^5}{8n^5} = \frac{1}{8} < \infty$

$\Rightarrow \sum |a_n|$ also converges

Converges Absolutely or Converges Conditionally or Diverges (circle one)

Why/Test Used? by LCT

4. $\sum_{n=1}^{\infty} \frac{9n}{csc(n^2)}$ $= \sum_{n=1}^{\infty} 9n \sin(n^2)$

$\lim_{n\to\infty} |9n \sin(n^2)| \to \infty$ (and its not an alternating series)

Converges Absolutely or Converges Conditionally or Diverges (circle one)

Why/Test Used? n^{th} term goes to some kind of ∞

5. $\sum_{m=1}^{\infty} 9m^2 e^{-m^3+5}$

$f(x) = \frac{9x^2}{e^{x^3-5}}$

① positive
② continuous
③ nonincreasing

IT $\int_1^{\infty} 9x^2 e^{-x^3+5}\,dx$

$u = -x^3+5$

$du = -3x^2\,dx$

$-3\,du = 9x^2\,dx$

$x=1,\ u = -1+5 = 4$

$x \to \infty,\ u \to -\infty$

$= -3\int_4^{-\infty} e^u\,du$

$= -3 \lim_{b\to -\infty} e^u \Big|_4^b$

$= -3\left(\lim_{b\to-\infty} e^b - e^4\right)$

0

$= 3e^4 < \infty$

$\Rightarrow$ our series also converges (and it's all positive)

Converges Absolutely or Converges Conditionally or Diverges (circle one)

Why/Test Used? by IT

Part 2: Rewrite each sum as an infinite series in summation notation, and then determine if each series converges or diverges. **If it converges, find its sum.**

6. $\ln\left(\frac{3}{7}\right)+\ln\left(\frac{7}{11}\right)+\ln\left(\frac{11}{15}\right)+\ln\left(\frac{15}{19}\right)+\ldots = \sum_{n=1}^{\infty} \ln\left(\frac{4n-1}{4n+3}\right)$

$n=1 \quad n=2 \quad n=3 \quad n=4$

(series representation)

$$= \sum_{n=1}^{\infty}\left(\ln(4n-1) - \ln(4n+3)\right)$$

$$S_p = \sum_{n=1}^{p}\left(\ln(4n-1) - \ln(4n+3)\right)$$

$$= (\ln 3 - \cancel{\ln 7}) + (\cancel{\ln 7} - \cancel{\ln 11}) + (\cancel{\ln 11} - \cancel{\ln 15}) + \dots + (\cancel{\ln(4p-5)} - \cancel{\ln(4p-1)}) + (\cancel{\ln(4p-1)} - \ln(4p+3))$$

$$S_p = \ln 3 - \ln(4p+3)$$

$$\Rightarrow S_\infty = \lim_{p\to\infty} S_p = \lim_{p\to\infty}\left(\ln 3 - \underbrace{\ln(4p+3)}_{\to\infty}\right)$$

$\Rightarrow$ series diverges

Converges or Diverges (circle one)

If it converges, sum = ____________________.

7. 0.135353535 .... = $\frac{1}{10} + \sum_{n=1}^{\infty} \frac{35}{10^{2n+1}}$

(series representation)

$= 0.1 + 0.035 + 0.00035 + 0.0000035 + \dots$

$= \frac{1}{10} + \underset{n=1}{\frac{35}{1000}} + \underset{n=2}{\frac{35}{10^5}} + \underset{n=3}{\frac{35}{10^7}} + \dots$

$= \frac{1}{10} + \sum_{n=1}^{\infty} \frac{35}{10^{2n+1}} = \frac{1}{10} + \sum_{n=1}^{\infty} \frac{35}{100^n(10)}$

$= \frac{1}{10} + \frac{35}{10}\left(\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^n\right)$

$= \frac{1}{10} + \frac{35}{10}\left(\frac{1/100}{1-1/100}\right) = \frac{1}{10} + \frac{35}{10}\left(\frac{1}{99}\right)$

$= \frac{99+35}{990}$

$= \frac{134}{990}$

$= \frac{67}{495}$

Converges or Diverges (circle one)

If it converges, sum **(in exact fraction form)** = 67/495.

Part 3: Answer each question.

8. Find a power series that represents $f(x)=\dfrac{4x^2}{1-5x}+3x^2-3$ and state its radius of convergence.

$$\frac{4x^2}{1-5x} = 4x^2\left(\frac{1}{1-5x}\right) = 4x^2\sum_{n=0}^{\infty}(5x)^n$$

$$= \sum_{n=0}^{\infty} 4(5^n)\,x^{n+2}$$

$$\Rightarrow f(x) = \left(\sum_{n=0}^{\infty} 4(5^n)\,x^{n+2}\right) + 3x^2 - 3$$

$$= \overset{n=0}{4(1)x^2} + \left(\sum_{n=1}^{\infty} 4(5^n)x^{n+2}\right) + 3x^2 - 3$$

$$= x^2 - 3 + \sum_{n=1}^{\infty} 4(5^n)x^{n+2}$$

$|5x| < 1$

$|x| < 1/5$

$\Rightarrow$ radius of convergence $R = 1/5$

Power Series: $x^2 - 3 + \sum_{n=1}^{\infty} 4(5^n)x^{n+2}$

Radius of convergence: $1/5$

9. For the sequence given by $a_n = \dfrac{n^2+2n+1}{\sqrt{2n^4-1}}$

(a) List the first three terms of the sequence.

n	a_n
1	4
2	$9/\sqrt{31}$
3	$\dfrac{16}{\sqrt{161}}$

$\dfrac{1+2+1}{\sqrt{2-1}} = \dfrac{4}{1}$

$\dfrac{4+4+1}{\sqrt{31}}$

$\dfrac{9+6+1}{\sqrt{2(81)-1}} = \dfrac{16}{\sqrt{161}}$

(b) Determine whether $\{a_n\}$ converges or diverges. If it converges, find $\lim_{n\to\infty} a_n$.

$$\lim_{n\to\infty} \frac{n^2+2n+1}{\sqrt{2n^4-1}} = \lim_{n\to\infty} \frac{n^2}{\sqrt{2n^4}} = \frac{1}{\sqrt{2}} \lim_{n\to\infty} \frac{n^2}{n^2}$$

$$= \frac{1}{\sqrt{2}}$$

Converges or Diverges (circle one)

If it converges, $\lim_{n\to\infty} a_n =$ $1/\sqrt{2}$

10. Find the convergence set for the power series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^{n+1}(2n+5)}$.

ART:

$$\lim_{n\to\infty} \frac{|x|^{n+1}}{3^{n+2}(2(n+1)+5)} \frac{3^{n+1}(2n+5)}{|x|^n}$$

$$= \frac{|x|}{3} \lim_{n\to\infty} \frac{2n+5}{2n+7} = \frac{|x|}{3}(1) = \frac{|x|}{3} < 1$$

for convergence

$$|x| < 3$$

$$-3 < x < 3$$

test endpts:

$x = -3$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-3)^n}{3^{n+1}(2n+5)} = \sum_{n=0}^{\infty} \frac{3^n(-1)^{2n}}{3^{n+1}(2n+5)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3(2n+5)}$$

by LCT this diverges (basically a harmonic series)

$x = 3$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{3^{n+1}(2n+5)} = \frac{1}{3}\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+5}$$

by AST, this converges conditionally

convergence set: $(-3, 3]$