1.1 Introduction to Limits

"Calculus is the study of limits."

Ex 1: Find the limits.

(a) $\lim_{x \to -2} (x^2 - 2p^2)$

(b) $\lim_{x \to 0} \frac{3x^3 + 2x^2 - x^4}{x^2}$

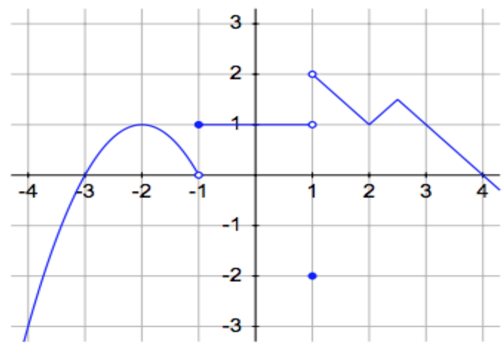
(c) $\lim_{x \to 7^+} \frac{\sqrt{(x-7)^5}}{7 - x}$

$\lim_{x \to c} f(x) = L$ iff

$\lim_{x \to c^-} f(x) = L$ AND $\lim_{x \to c^+} f(x) = L$

Note: iff is not a typo. In mathematics, it means "if and only if", meaning that the implication goes both ways logically.

Ex 2:
If this represents the graph of y = f(x), answer the questions below.



(i) $\lim_{x \to -1^-} f(x)$ = ____________

(ii) $\lim_{x \to -1^+} f(x)$ = ____________

(iii) $\lim_{x \to -1} f(x)$ = ____________

(iv) $f(-1)$ = ____________

(v) $\lim_{x \to 1^+} f(x)$ = ____________

(vi) $\lim_{x \to 1^-} f(x)$ = ____________

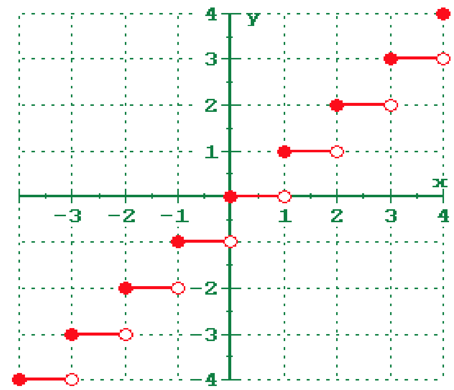
(vii) $\lim_{x \to 1} f(x)$ = ____________

(viii) $f(1)$ = ____________

(ix) $\lim_{x \to -2} f(x)$ = ____________

(x) $f(-2)$ = ____________

Note: The greatest integer function is given by $f(x)=[[x]]$ and its graph looks like this.

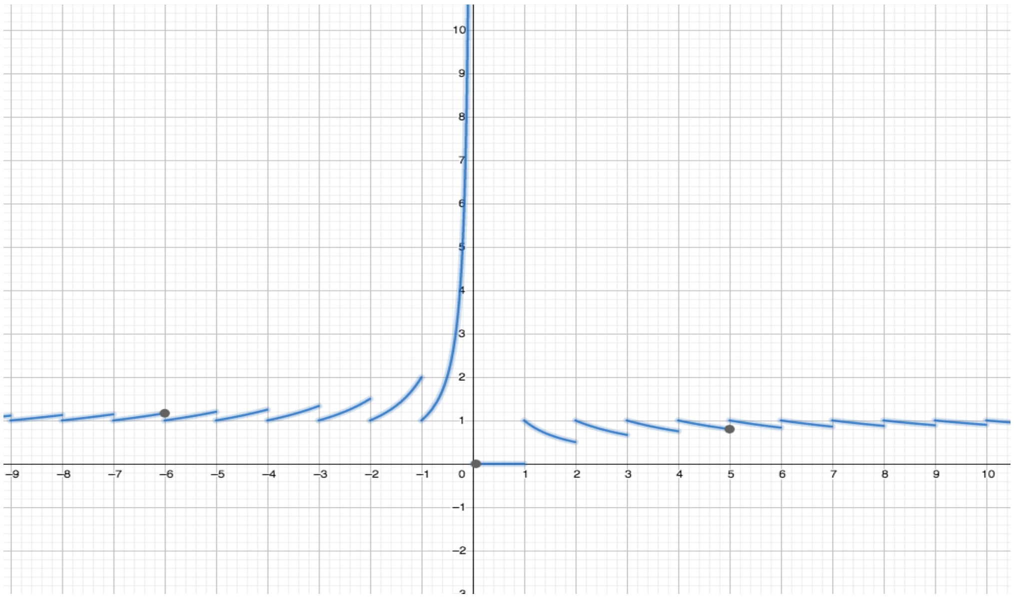


Ex 3: Find the limits.

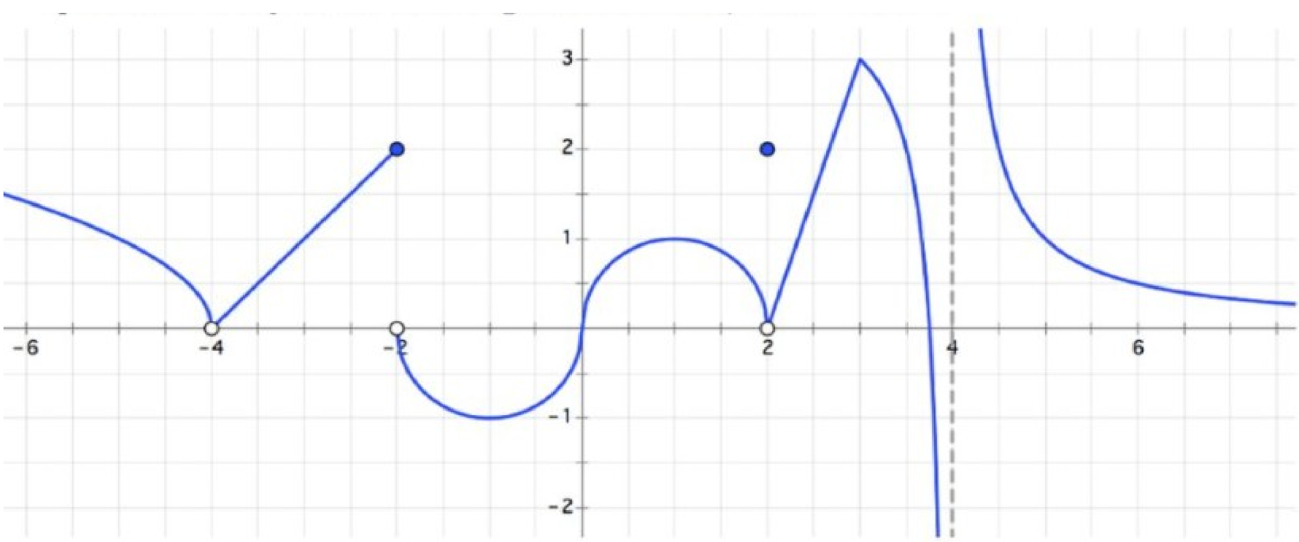
(a) $\lim\limits_{x\to 3} \dfrac{[[x]]}{x}$ RH: LH:	(b) $\lim\limits_{x\to 1.6} \dfrac{[[x]]}{x}$
(c) $\lim\limits_{x\to 0^+} \dfrac{[[x]]}{x}$	(d) $\lim\limits_{x\to 0^-} \dfrac{[[x]]}{x}$
(e) $\lim\limits_{x\to 0} \dfrac{[[x]]}{x}$	(f) If $f(x)=\dfrac{[[x]]}{x}$, what is $f(0)$?

1.1 (continued)

Here's the graph of $y = \frac{[[x]]}{x}$.



Ex 4: For this graph of y=f(x), answer the limit questions.



(a) $\lim_{x \to 2} f(x) =$ ______ (b) $\lim_{x \to 4^-} f(x) =$ ______ (c) $\lim_{x \to 4^+} f(x) =$ ______

(d) $\lim_{x \to -2^+} f(x) =$ ______ (e) $\lim_{x \to -2^-} f(x) =$ ______ (f) $\lim_{x \to -2} f(x) =$ ______

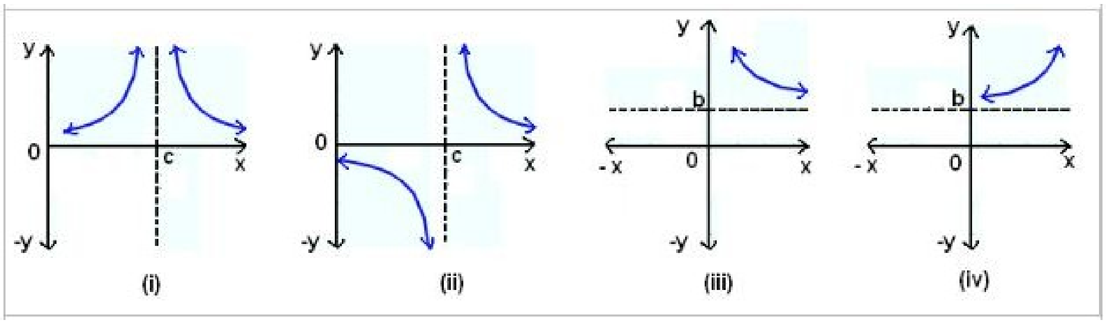
(g) $\lim_{x \to \infty} f(x) =$ ______ (h) $\lim_{x \to -\infty} f(x) =$ ______

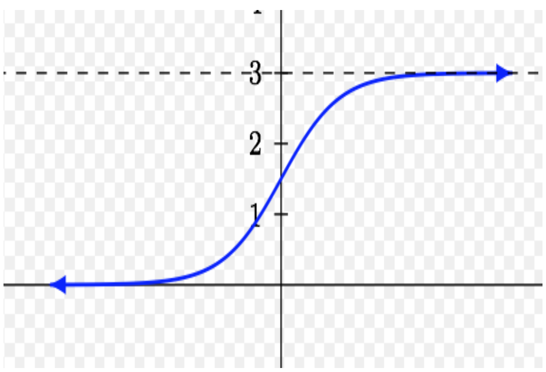
1.3 Limit Theorems

Ex 1: Find these limits. (a) $\lim_{x \to -3} \frac{x^2 - 14x - 51}{x^2 - 4x - 21}$	<u>Limit Theorems</u> (basically limits distribute through everything you'd like them to distribute through as long as the limits of the individual functions exist) Assume $n \in \mathbb{N}$, k is a constant $f(x)$ and $g(x)$ are functions such that $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. Then, (1) $\lim_{x \to c} k = k$ (2) $\lim_{x \to c} x = c$ (3) $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$ (4) $\lim_{x \to c} (f(x) \pm g(x)) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$ (5) $\lim_{x \to c} (f(x) \cdot g(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$ (6) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ as long as $\lim_{x \to c} g(x) \neq 0$ (7) $\lim_{x \to c} (f(x))^n = (\lim_{x \to c} f(x))^n$ (8) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$ as long as $\lim_{x \to c} f(x) > 0$ if n is even.
(b) $\lim_{x \to -1} \frac{x^2 + x}{x^2 + 1}$	
(c) $\lim_{x \to -1^+} \frac{\sqrt{1+x}}{5+5x}$	

Ex 2: Find the limits.	Squeeze Theorem
(a) $\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2}$	Assume $f(x), g(x),$ and $h(x)$ are functions such that $f(x) \leq g(x) \leq h(x)$, $\forall x$ near c, except possibly at c. If $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} g(x) = L$.
(b) $\lim_{x \to -\pi^+} \frac{\sqrt{\pi^3 + x^3}}{x}$	(c) $\lim_{x \to -\pi^+} \frac{x}{\sqrt{\pi^3 + x^3}}$ (segue into next section....)

1.5 Limits at Infinity; Infinite Limits

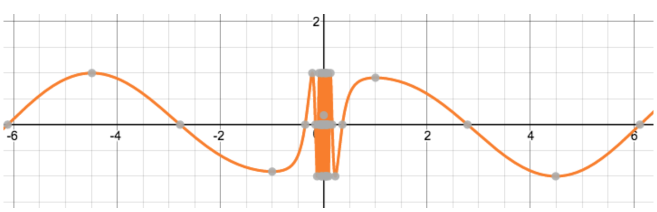




Ex 1: Find the limits. (a) $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$	(b) $\lim_{x\to \infty} x^2 \sin\left(\frac{1}{x}\right)$

1.5 (continued)

Ex 2: Find the limits.

(a) $\lim_{x\to-\infty} \frac{3\sqrt[3]{x^5}+x-7}{\sqrt[3]{5x}+3x+1}$	(b) $\lim_{x\to\infty} \frac{3\sqrt[3]{x^5}+x-7}{\sqrt[3]{5x}+3x+1}$
(c) $\lim_{x\to\infty} \sqrt[5]{\frac{2x^3-5x+1}{\pi x^3+3}}$	(d) $\lim_{x\to-\infty} \sqrt[5]{\frac{2x^3-5x+1}{\pi x^3+3}}$
(e) $\lim_{x\to\infty} \sin\left(x+\frac{1}{x}\right)$ 	(f) $\lim_{x\to\infty} \left(\sqrt{x^2+2x}-x\right)$

1.6 Continuity

<table>
<tr>
<td>Ex 1: (a) Is this function continuous? (b) If not, where is it discontinuous and why does it fail continuity? (c) Can you fix it (patch it) so you remove the discontinuity? If so, do that.
$f(x)=\begin{cases} x \text{ , if } x\le 0 \\ x^2 \text{ , if } 0<x\le 1 \\ 3-x \text{ , if } x>1 \end{cases}$</td>
<td>Continuity at x = c:

(1) $\lim_{x\to c} f(x)$ exists
and
(2) $f(c)$ exists
and
(3) $\lim_{x\to c} f(x)=f(c)$</td>
</tr>
<tr>
<td>Ex 2: (a) Is this function continuous? (b) If not, where is it discontinuous and why does it fail continuity? (c) Can you fix it (patch it) so you remove the discontinuity? If so, do that.
$f(x)=\dfrac{\sqrt{x}-1}{x-1}$</td>
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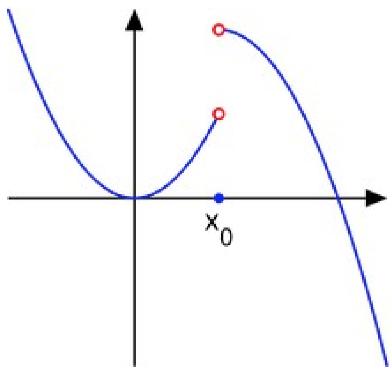
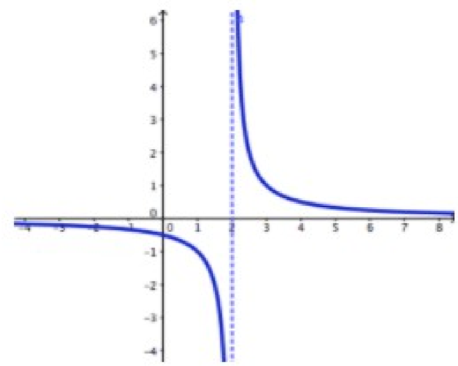
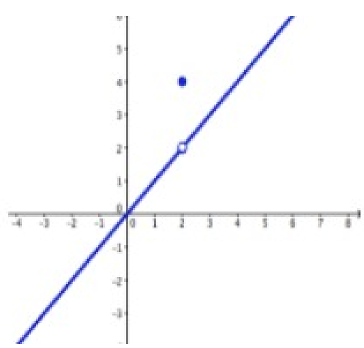
1.6 (continued)

Ex 3: For each graph, answer the following questions:
(a) List all the x-values where the discontinuities occur, if there are any.
(b) For each x-value from (a), what part of the continuity definition does the function fail there (i.e. why is it discontinuous)?
(c) List all the intervals where the function is continuous.

(i) 	(ii) 
(iii) 	(iv)