

## Derivatives

$$\begin{aligned}
 D_x e^x &= e^x \\
 D_x \sin(x) &= \cos(x) \\
 D_x \cos(x) &= -\sin(x) \\
 D_x \tan(x) &= \sec^2(x) \\
 D_x \cot(x) &= -\csc^2(x) \\
 D_x \sec(x) &= \sec(x)\tan(x) \\
 D_x \csc(x) &= -\csc(x)\cot(x) \\
 D_x \sin^{-1} &= \frac{1}{\sqrt{1-x^2}}, x \in [-1, 1] \\
 D_x \cos^{-1} &= \frac{-1}{\sqrt{1-x^2}}, x \in [-1, 1] \\
 D_x \tan^{-1} &= \frac{1}{1+x^2}, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\
 D_x \sec^{-1} &= \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1 \\
 D_x \sinh(x) &= \cosh(x) \\
 D_x \cosh(x) &= \sinh(x) \\
 D_x \tanh(x) &= \operatorname{sech}^2(x) \\
 D_x \coth(x) &= -\operatorname{csch}^2(x) \\
 D_x \operatorname{sech}(x) &= -\operatorname{sech}(x)\tanh(x) \\
 D_x \operatorname{csch}(x) &= -\operatorname{csch}(x)\coth(x) \\
 D_x \sinh^{-1} &= \frac{1}{\sqrt{x^2+1}} \\
 D_x \cosh^{-1} &= \frac{-1}{\sqrt{x^2-1}}, x > 1 \\
 D_x \tanh^{-1} &= \frac{1}{1-x^2}, -1 < x < 1 \\
 D_x \operatorname{sech}^{-1} &= \frac{1}{x\sqrt{1-x^2}}, 0 < x < 1 \\
 D_x \ln(x) &= \frac{1}{x}
 \end{aligned}$$

## Integrals

$$\begin{aligned}
 \int \frac{1}{x} dx &= \ln|x| + c \\
 \int e^x dx &= e^x + c \\
 \int a^x dx &= \frac{1}{\ln a} a^x + c \\
 \int e^{ax} dx &= \frac{1}{a} e^{ax} + c \\
 \int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1}(x) + c \\
 \int \frac{1}{1+x^2} dx &= \tan^{-1}(x) + c \\
 \int \frac{1}{x\sqrt{x^2-1}} dx &= \sec^{-1}(x) + c \\
 \int \sinh(x) dx &= \cosh(x) + c \\
 \int \cosh(x) dx &= \sinh(x) + c \\
 \int \tanh(x) dx &= \ln|\cosh(x)| + c \\
 \int \tanh(x)\operatorname{sech}(x) dx &= -\operatorname{sech}(x) + c \\
 \int \operatorname{sech}^2(x) dx &= \tanh(x) + c \\
 \int \operatorname{csch}(x)\coth(x) dx &= -\operatorname{csch}(x) + c \\
 \int \tan(x) dx &= -\ln|\cos(x)| + c \\
 \int \cot(x) dx &= \ln|\sin(x)| + c \\
 \int \cos(x) dx &= \sin(x) + c \\
 \int \sin(x) dx &= -\cos(x) + c \\
 \int \frac{1}{\sqrt{a^2-u^2}} du &= \sin^{-1}\left(\frac{u}{a}\right) + c \\
 \int \frac{1}{a^2+u^2} du &= \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + c \\
 \int \ln(x) dx &= (x\ln(x)) - x + c
 \end{aligned}$$

## U-Substitution

Let  $u = f(x)$  (can be more than one variable).

Determine:  $du = \frac{f'(x)}{dx} dx$  and solve for dx.

Then, if a definite integral, substitute the bounds for  $u = f(x)$  at each bound. Solve the integral using u.

## Integration by Parts

$$\int u dv = uv - \int v du$$

## Fns and Identities

$$\begin{aligned}
 \sin(\cos^{-1}(x)) &= \sqrt{1-x^2} \\
 \cos(\sin^{-1}(x)) &= \sqrt{1-x^2}
 \end{aligned}$$

$$\begin{aligned}
 \sec(\tan^{-1}(x)) &= \sqrt{1+x^2} \\
 \tan(\sec^{-1}(x)) &= (\sqrt{x^2-1} \text{ if } x \geq 1) \\
 &= (-\sqrt{x^2-1} \text{ if } x \leq -1) \\
 \sinh^{-1}(x) &= \ln x + \sqrt{x^2+1} \\
 \sinh^{-1}(x) &= \ln x + \sqrt{x^2-1}, x > -1 \\
 \tanh^{-1}(x) &= \frac{1}{2} \ln x + \frac{1+x}{1-x}, 1 < x < -1 \\
 \operatorname{sech}^{-1}(x) &= \ln\left[\frac{1+\sqrt{1-x^2}}{x}\right], 0 < x \leq -1 \\
 \sinh(x) &= \frac{e^x - e^{-x}}{2} \\
 \cosh(x) &= \frac{e^x + e^{-x}}{2}
 \end{aligned}$$

## Trig Identities

$$\begin{aligned}
 \sin^2(x) + \cos^2(x) &= 1 \\
 1 + \tan^2(x) &= \sec^2(x) \\
 1 + \cot^2(x) &= \csc^2(x) \\
 \sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\
 \cos(x \pm y) &= \cos(x)\cos(y) \pm \sin(x)\sin(y) \\
 \tan(x \pm y) &= \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)} \\
 \sin(2x) &= 2\sin(x)\cos(x) \\
 \cos(2x) &= \cos^2(x) - \sin^2(x) \\
 \cosh(n^2x) - \sinh^2(x) &= 1 \\
 1 + \tan^2(x) &= \sec^2(x) \\
 1 + \cot^2(x) &= \csc^2(x) \\
 \sin^2(x) &= \frac{1 - \cos(2x)}{2} \\
 \cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
 \tan^2(x) &= \frac{1 - \cos(2x)}{1 + \cos(2x)} \\
 \sin(-x) &= -\sin(x) \\
 \cos(-x) &= \cos(x) \\
 \tan(-x) &= -\tan(x)
 \end{aligned}$$

## Calculus 3 Concepts

### Cartesian coords in 3D

given two points:

$(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ ,

Distance between them:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Midpoint:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Sphere with center (h,k,l) and radius r:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

### Vectors

Vector:  $\vec{u}$

Unit Vector:  $\hat{u}$

$$\text{Magnitude: } \|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

Unit Vector:  $\hat{u} = \frac{\vec{u}}{\|\vec{u}\|}$

### Dot Product

$\vec{u} \cdot \vec{v}$

Produces a Scalar

(Geometrically, the dot product is a vector projection)

$$\vec{u} = \langle u_1, u_2, u_3 \rangle$$

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$

$\vec{u} \cdot \vec{v} = 0$  means the two vectors are

Perpendicular  $\theta$  is the angle between them.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos(\theta)$$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

NOTE:

$$\hat{u} \cdot \hat{v} = \cos(\theta)$$

$$\|\vec{u}\|^2 = \vec{u} \cdot \vec{u}$$

$$\vec{u} \cdot \vec{v} = 0 \text{ when } \perp$$

Angle Between  $\vec{u}$  and  $\vec{v}$ :

$$\theta = \cos^{-1}\left(\frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \|\vec{v}\|}\right)$$

Projection of  $\vec{u}$  onto  $\vec{v}$ :

$$pr_{\vec{v}}\vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2}\right)\vec{v}$$

### Cross Product

$\vec{u} \times \vec{v}$

Produces a Vector

(Geometrically, the cross product is the

area of a parallelogram with sides  $\|\vec{u}\|$

and  $\|\vec{v}\|$ )

$$\vec{u} = \langle u_1, u_2, u_3 \rangle$$

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$\vec{u} \times \vec{v} = \vec{0}$  means the vectors are parallel

## Lines and Planes

### Equation of a Plane

$(x_0, y_0, z_0)$  is a point on the plane and

$\langle A, B, C \rangle$  is a normal vector

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$\langle A, B, C \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$Ax + By + Cz = D \text{ where}$$

$$D = Ax_0 + By_0 + Cz_0$$

### Equation of a line

A line requires a Direction Vector

$\vec{u} = \langle u_1, u_2, u_3 \rangle$  and a point

$(x_1, y_1, z_1)$

then,

a parameterization of a line could be:

$$x = u_1 t + x_1$$

$$y = u_2 t + y_1$$

$$z = u_3 t + z_1$$

### Distance from a Point to a Plane

The distance from a point  $(x_0, y_0, z_0)$  to

a plane  $Ax + By + Cz = D$  can be expressed

by the formula:

$$d = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

## Coord Sys Conv

### Cylindrical to Rectangular

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$z = z$$

### Rectangular to Cylindrical

$$r = \sqrt{x^2 + y^2}$$

$$\tan(\theta) = \frac{y}{x}$$

$$z = z$$

### Spherical to Rectangular

$$x = \rho \sin(\phi) \cos(\theta)$$

$$y = \rho \sin(\phi) \sin(\theta)$$

$$z = \rho \cos(\phi)$$

### Rectangular to Spherical

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\tan(\theta) = \frac{y}{x}$$

$$\cos(\phi) = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

### Spherical to Cylindrical

$$r = \rho \sin(\phi)$$

$$\theta = \theta$$

$$z = \rho \cos(\phi)$$

### Cylindrical to Spherical

$$\rho = \sqrt{r^2 + z^2}$$

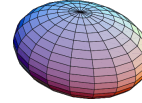
$$\theta = \theta$$

$$\cos(\phi) = \frac{z}{\sqrt{r^2 + z^2}}$$

## Surfaces

### Ellipsoid

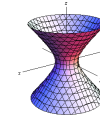
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



### Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

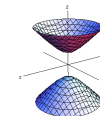
(Major Axis: z because it follows -)



### Hyperboloid of Two Sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

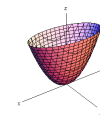
(Major Axis: Z because it is the one not subtracted)



### Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

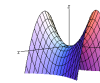
(Major Axis: z because it is the variable NOT squared)



### Hyperbolic Paraboloid

(Major Axis: Z axis because it is not squared)

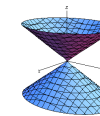
$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$



### Elliptic Cone

(Major Axis: Z axis because it's the only one being subtracted)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



### Cylinder

1 of the variables is missing

OR

$$(x-a)^2 + (y-b)^2 = c$$

(Major Axis is missing variable)

## Partial Derivatives

Partial Derivatives are simply holding all other variables constant (and act like constants for the derivative) and only taking the derivative with respect to a given variable.

Given  $z=f(x,y)$ , the partial derivative of z with respect to x is:

$$f_x(x, y) = z_x = \frac{\partial z}{\partial x} = \frac{\partial f(x, y)}{\partial x}$$

likewise for partial with respect to y:

$$f_y(x, y) = z_y = \frac{\partial z}{\partial y} = \frac{\partial f(x, y)}{\partial y}$$

### Notation

For  $f_{xyy}$ , work "inside to outside"  $f_x$

then  $f_{xy}$ , then  $f_{xyy}$

$$f_{xyy} = \frac{\partial^3 f}{\partial^2 y \partial x}$$

For  $\frac{\partial^3 f}{\partial^2 y \partial x}$ , work right to left in the denominator

## Gradients

The Gradient of a function in 2 variables

$$\text{is } \nabla f = \langle f_x, f_y \rangle$$

The Gradient of a function in 3 variables

$$\text{is } \nabla f = \langle f_x, f_y, f_z \rangle$$

## Chain Rule(s)

Take the Partial derivative with respect to the first-order variables of the function times the partial (or normal) derivative of the first-order variable to the ultimate variable you are looking for summed with the same process for other first-order variables this makes sense for. Example:

let  $x = x(s, t)$ ,  $y = y(t)$  and  $z = z(x, y)$ .

z then has first partial derivative:

$$\frac{\partial z}{\partial x} \text{ and } \frac{\partial z}{\partial y}$$

x has the partial derivatives:

$$\frac{\partial x}{\partial s} \text{ and } \frac{\partial x}{\partial t}$$

and y has the derivative:

$$\frac{dy}{dt}$$

In this case (with z containing x and y as well as x and y both containing s and t), the chain rule for  $\frac{\partial z}{\partial s}$  is  $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s}$

The chain rule for  $\frac{\partial z}{\partial t}$  is

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

Note: the use of "d" instead of "∂" with the function of only one independent variable

## Limits and Continuity

### Limits in 2 or more variables

Limits taken over a vectorized limit just evaluate separately for each component of the limit.

### Strategies to show limit exists

1. Plug in Numbers, Everything is Fine

2. Algebraic Manipulation

-factoring/dividing out

-use trig identities

3. Change to polar coords

$$if(x, y) \rightarrow (0, 0) \Leftrightarrow r \rightarrow 0$$

### Strategies to show limit DNE

1. Show limit is different if approached

from different paths

( $x=y$ ,  $x=y^2$ , etc.)

2. Switch to Polar coords and show the

limit DNE.

### Continuity

A fn,  $z = f(x, y)$ , is continuous at (a, b) if

$$f(a, b) = \lim_{(x, y) \rightarrow (a, b)} f(x, y)$$

Which means:

- The limit exists
- The fn value is defined
- They are the same value

## Directional Derivatives

Let  $z=f(x,y)$  be a function,  $(a,b)$  a point in the domain (a valid input point) and  $\hat{u}$  a unit vector (2D).

The Directional Derivative is then the derivative at the point  $(a,b)$  in the direction of  $\hat{u}$  or:

$$D_{\hat{u}}f(a,b) = \hat{u} \cdot \nabla f(a,b)$$

This will return a scalar. 4-D version:

$$D_{\hat{u}}f(a,b,c) = \hat{u} \cdot \nabla f(a,b,c)$$

## Tangent Planes

let  $F(x,y,z) = k$  be a surface and  $P = (x_0, y_0, z_0)$  be a point on that surface. Equation of a Tangent Plane:

$$\nabla F(x_0, y_0, z_0) \cdot \langle x - x_0, y - y_0, z - z_0 \rangle <$$

## Approximations

let  $z = f(x, y)$  be a differentiable function total differential of  $f = dz$   
 $dz = \nabla f \cdot \langle dx, dy \rangle >$

This is the *approximate* change in  $z$  The actual change in  $z$  is the difference in  $z$  values:

$$\Delta z = z - z_1$$

## Maxima and Minima

### Internal Points

1. Take the Partial Derivatives with respect to X and Y ( $f_x$  and  $f_y$ ) (Can use gradient)
2. Set derivatives equal to 0 and use to solve system of equations for x and y
3. Plug back into original equation for z. Use Second Derivative Test for whether points are local max, min, or saddle

### Second Partial Derivative Test

1. Find all  $(x,y)$  points such that  $\nabla f(x, y) = \vec{0}$
2. Let  $D = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}^2(x, y)$  IF (a)  $D > 0$  AND  $f_{xx} < 0$ ,  $f(x,y)$  is local max value  
 (b)  $D > 0$  AND  $f_{xx}(x, y) > 0$   $f(x,y)$  is local min value  
 (c)  $D < 0$ ,  $(x,y,f(x,y))$  is a saddle point  
 (d)  $D = 0$ , test is inconclusive
3. Determine if any boundary point gives min or max. Typically, we have to parametrize boundary and then reduce to a Calc 1 type of min/max problem to solve.

**The following only apply only if a boundary is given**

1. check the corner points
2. Check each line ( $0 \leq x \leq 5$  would give  $x=0$  and  $x=5$ )  
 On Bounded Equations, this is the global min and max...second derivative test is not needed.

## Lagrange Multipliers

Given a function  $f(x,y)$  with a constraint  $g(x,y)$ , solve the following system of equations to find the max and min points on the constraint (NOTE: may need to also find internal points.):

$$\nabla f = \lambda \nabla g$$

$$g(x, y) = 0(\text{orkifgiven})$$

## Double Integrals

With Respect to the xy-axis, if taking an integral,

$\int \int dydx$  is cutting in vertical rectangles,  
 $\int \int dx dy$  is cutting in horizontal rectangles

### Polar Coordinates

When using polar coordinates,  
 $dA = r dr d\theta$

## Surface Area of a Curve

let  $z = f(x,y)$  be continuous over  $S$  (a closed Region in 2D domain)  
 Then the surface area of  $z = f(x,y)$  over  $S$  is:

$$SA = \int \int_S \sqrt{f_x^2 + f_y^2 + 1} dA$$

## Triple Integrals

$$\int \int \int_S f(x, y, z) dv =$$

$$\int_{a_1}^{a_2} \int_{\phi_1(x)}^{\phi_2(x)} \int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x, y, z) dz dy dx$$

Note:  $dv$  can be exchanged for  $dx dy dz$  in any order, but you must then choose your limits of integration according to that order

## Jacobian Method

$$\int \int_G f(g(u, v), h(u, v)) |J(u, v)| du dv = \int \int_R f(x, y) dx dy$$

$$J(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Common Jacobians:

Rect. to Cylindrical:  $r$

Rect. to Spherical:  $\rho^2 \sin(\phi)$

## Vector Fields

let  $f(x, y, z)$  be a scalar field and

$$\vec{F}(x, y, z) =$$

$$M(x, y, z)\hat{i} + N(x, y, z)\hat{j} + P(x, y, z)\hat{k} \text{ be a vector field,}$$

Gradient of  $f = \nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle >$

Divergence of  $\vec{F}$ :

$$\nabla \cdot \vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

Curl of  $\vec{F}$ :

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

## Line Integrals

C given by  $x = x(t), y = y(t), t \in [a, b]$

$$\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) ds$$

where  $ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$

$$\text{or } \sqrt{1 + (\frac{dy}{dx})^2} dx$$

$$\text{or } \sqrt{1 + (\frac{dx}{dy})^2} dy$$

To evaluate a Line Integral,

- get a parametrized version of the line (usually in terms of  $t$ , though in exclusive terms of  $x$  or  $y$  is ok)
- evaluate for the derivatives needed (usually  $dy, dx,$  and/or  $dt$ )
- plug in to original equation to get in terms of the independent variable
- solve integral

## Work

Let  $\vec{F} = M\hat{i} + \hat{j} + \hat{k}$  (force)

$$M = M(x, y, z), N = N(x, y, z), P = P(x, y, z)$$

(Literally)  $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\text{Work } w = \int_C \vec{F} \cdot d\vec{r}$$

(Work done by moving a particle over curve C with force  $\vec{F}$ )

## Independence of Path

### Fund Thm of Line Integrals

C is curve given by  $\vec{r}(t), t \in [a, b]$ ;  $\vec{r}'(t)$  exists. If  $f(\vec{r})$  is continuously differentiable on an open set containing C, then  $\int_C \nabla f(\vec{r}) \cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$

### Equivalent Conditions

$\vec{F}(\vec{r})$  continuous on open connected set D. Then,

(a)  $\vec{F} = \nabla f$  for some fn  $f$ . (if  $\vec{F}$  is conservative)

$\Leftrightarrow$  (b)  $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$  indep. of path in D

$\Leftrightarrow$  (c)  $\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = 0$  for all closed paths in D.

### Conservation Theorem

$\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$  continuously differentiable on open, simply connected set D.

$\vec{F}$  conservative  $\Leftrightarrow \nabla \times \vec{F} = \vec{0}$

(in 2D  $\nabla \times \vec{F} = \vec{0}$  iff  $M_y = N_x$ )

## Green's Theorem

(method of changing line integral for double integral - Use for Flux and Circulation across 2D curve and line integrals over a closed boundary)

$$\oint M dy - N dx = \int \int_R (M_x + N_y) dx dy$$

$$\oint M dx + N dy = \int \int_R (N_x - M_y) dx dy$$

Let:

· R be a region in xy-plane

· C is simple, closed curve enclosing R (w/ parameterization  $\vec{r}(t)$ )

·  $\vec{F}(x, y) = M(x, y)\hat{i} + N(x, y)\hat{j}$  be continuously differentiable over RUC.

### Form 1: Flux Across Boundary

$\vec{n}$  = unit normal vector to C

$$\oint_C \vec{F} \cdot \vec{n} = \int \int_R \nabla \cdot \vec{F} dA$$

$$\Leftrightarrow \oint M dy - N dx = \int \int_R (M_x + N_y) dx dy$$

### Form 2: Circulation Along Boundary

$$\oint_C \vec{F} \cdot d\vec{r} = \int \int_R \nabla \times \vec{F} \cdot \hat{u} dA$$

$$\Leftrightarrow \oint M dx + N dy = \int \int_R (N_x - M_y) dx dy$$

### Area of R

$$A = \oint (-\frac{1}{2} y dx + \frac{1}{2} x dy)$$

## Gauss' Divergence Thm

(3D Analog of Green's Theorem - Use for Flux over a 3D surface) Let:

·  $\vec{F}(x, y, z)$  be vector field continuously differentiable in solid S

· S is a 3D solid ·  $\partial S$  boundary of S (A Surface)

·  $\hat{n}$  unit outer normal to  $\partial S$

Then,

$$\int \int \partial S \vec{F}(x, y, z) \cdot \hat{n} dS = \int \int \int_S \nabla \cdot \vec{F} dV$$

$$(dV = dx dy dz)$$

## Surface Integrals

Let

· R be closed, bounded region in xy-plane

·  $f$  be a fn with first order partial derivatives on R

· G be a surface over R given by

$$z = f(x, y)$$

·  $g(x, y, z) = g(x, y, f(x, y))$  is cont. on R  
 Then,

$$\int \int_G g(x, y, z) dS =$$

$$\int \int_R g(x, y, f(x, y)) dS$$

where  $dS = \sqrt{f_x^2 + f_y^2 + 1} dy dx$

### Flux of $\vec{F}$ across G

$$\int \int_G \vec{F} \cdot \vec{n} dS =$$

$$\int \int_R [-M f_x - N f_y + P] dx dy$$

where:

$$\cdot \vec{F}(x, y, z) =$$

$$M(x, y, z)\hat{i} + N(x, y, z)\hat{j} + P(x, y, z)\hat{k}$$

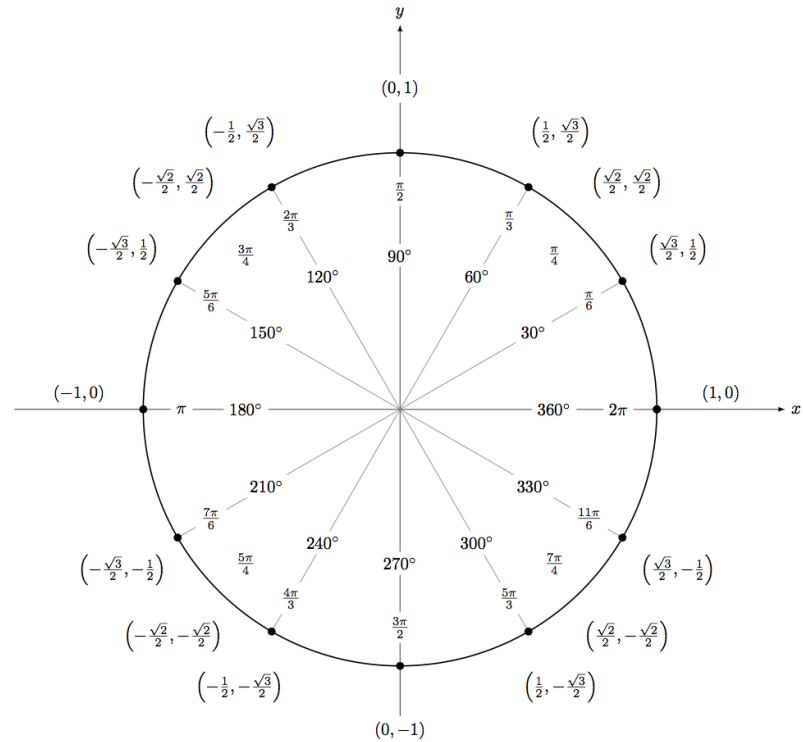
· G is surface  $f(x,y)=z$

·  $\vec{n}$  is upward unit normal on G.

·  $f(x,y)$  has continuous 1<sup>st</sup> order partial derivatives

## Unit Circle

(cos, sin)



## Other Information

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Where a Cone is defined as  
 $z = \sqrt{a(x^2 + y^2)}$ ,

In Spherical Coordinates,

$$\rho = \cos^{-1}\left(\sqrt{\frac{a}{1+a}}\right)$$

Right Circular Cylinder:

$$V = \pi r^2 h, SA = \pi r^2 + 2\pi r h$$

$$\lim_{n \rightarrow \infty} (1 + \frac{m}{n})^{pn} = e^{mp}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc(\cos(\theta))$$

## Stokes Theorem

Let:

· S be a 3D surface

$$\cdot \vec{F}(x, y, z) =$$

$$M(x, y, z)\hat{i} + N(x, y, z)\hat{j} + P(x, y, z)\hat{k}$$

· M,N,P have continuous 1<sup>st</sup> order partial derivatives

· C is piece-wise smooth, simple, closed, curve, positively oriented

·  $\vec{T}$  is unit tangent vector to C.

Then,

$$\oint \vec{F}_c \cdot \vec{T} dS = \int \int_S (\nabla \times \vec{F}) \cdot \hat{n} dS =$$

$$\int \int_R (\nabla \times \vec{F}) \cdot \vec{n} dx dy$$

Remember:

$$\oint \vec{F} \cdot \vec{T} ds = \int_C (M dx + N dy + P dz)$$

Originally Written By Daniel Kenner for MATH 2210 at the University of Utah.

Source code available at

[https://github.com/keytotime/Calc3\\_CheatSheet](https://github.com/keytotime/Calc3_CheatSheet)

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