Description of some of Professor Milton's Achievements

Developing the theory of composite materials. Prof. Milton is one of the world's leaders working in the theory of composites. This is exemplified by his book "The theory of composites", which is a virtual encyclopedia on the subject. While an undergraduate at Sydney University he developed the first comprehensive set of bounds on the complex dielectric constant of composite materials- these essentially bound the energy absorption and refractive index (how much the composite bends light) [1,2,3,6,7]. They are now known as the Bergman-Milton bounds (Bergman independently and at the same time derived some of the bounds). Prof. Milton with Drs. Berryman and Gibiansky and Prof. Lakes later generalized the bounds to the complex elastic moduli of two-phase composites [49,51,63,69].

Prof. Milton gave the first mathematical proof that isotropic materials which get fatter as they are stretched can exist [46]. With Prof. Cherkaev he addressed the grand question of what elasticity tensors are realizable [60]. Elasticity tensors, which determine the elastic response of bodies, can be represented by symmetric positive 6 by 6 matrices- Milton and Cherkaev showed that every such matrix can be realized by an appropriate composite of a very stiff material and a very compliant material. A key to the construction was introducing a new class of materials, called pentamodes (google pentamodes) which are like fluids in that they only resist one loading (which is compression for fluids), but unlike fluids this loading can be a mixture of compression and shear. Pentamodes have been built by the group of Martin Wegener, and are a key element in some of the cloaking designs proposed by the group of Andrew Norris. In 2012, before they built pentamodes the group of Wegener in the paper, "Tailored 3D Mechanical Metamaterials Made by Dip-in Direct-Laser-Writing Optical Lithography" wrote: "In addition, the "holy grail" of mechanical materials, namely pentamode materials, that can be seen as the mother of all materials, might become accessible as well. Pentamodes, suggested by Milton and Cherkaev in 1995, are solids that behave like fluids with a very small effective shear modulus."

In elementary physics textbooks it is mentioned that the sign of the "Hall coefficient" (which measures the transverse voltage when a current flows through a material, with a magnetic field applied perpendicular to the current) determines the sign of the charge carrier. This is based on simple minded reasoning based on the direction of the force on a free electron moving in a magnetic field. Profs. Briane and Milton obtained a counterexample [100,108]: one could combine three materials, all with positive Hall coefficient, to get a composite with negative Hall coefficient. Recently the group of Martin Wegener simplified the design to one material plus void, and have physically constructed the world's first "Hall effect reversal material".

. It is well known that convex sets can be represented as intersections of half planes (in two-dimensions) or half spaces (in higher dimensions). This is the famous Legendre transform. Profs. Francfort and Milton [52] showed that the set which is the range of effective tensors as the geometry varies over all configurations, can similarly be characterized by minimums of energies and complementary energies.

The new transform is called a "W-transform" and is key to finding the possible range of elasticity tensors of 3d-printed materials [157].

Prof. Milton obtained a complete characterization, for two-dimensional, two-phase composites of two isotropic phases, of the set of all possible functions that the (matrix valued) effective conductivity can have as a function of the component conductivities [24]. He showed that any such composite can be mimicked by an appropriate hierarchical laminate composite. Prof. Milton and Ms. Clark (then his graduate student) showed that one could also characterize the possible functions of two-dimensional polycrystals as functions of the matrix valued conductivity of the single crystal from which the polycrystal is built [54].

There is a long history of exact (microstructure independent) relations satisfied by the effective tensors of composites. Prof. Milton and collaborators derived some [33,44,47,48 58,62,64,66,67], but a comprehensive theory, which encompassed all exact relations, was missing. A key advance was made by Grabovsky who obtained algebraic conditions for an exact relation to hold for layered composites. This enabled Prof. Milton, and (then) Drs. Grabovsky and Sage to obtain algebraic conditions which ensured a candidate exact relation held for all composite geometries, not just laminate ones [68,72].

Development of new types of continued fractions and hierarchical decompositions of subspace collections. While seeking bounds on the effective tensors of multicomponent materials, Prof. Milton realized that the analytic technique of using fractional linear transformations to obtain bounds on Stieltjes functions had a natural counterpart at the level of the subspaces of fields involved in the problem. This led to an entirely new method for obtaining bounds, the field equation recursion method [25,26,42], and associated with this was an expansion of the effective tensor as a new type of continued fraction, involving matrices of varying dimension at each level in the fraction.

Accelerated algorithm for computing the fields in periodic composites using Fast Fourier Transforms. In 1994 Moulinec and Suquet found an efficient scheme for computing the fields in periodic composites. It was based on series expansions for the fields and fast Fourier transforms. Using an alternative series expansion Prof. Milton and Dr. Eyre [71] obtained a vastly accelerated Fast Fourier transform scheme for computing the fields. The enhancement was particularly great when the contrast in the phases was high.

Solving long standing conjectures. In 1961 Eshelby made a famous conjecture- that among all inclusion shapes only the ellipsoid has the property that the elasticity field is uniform inside the inclusion when the applied field is uniform. It is unclear whether he meant for all uniform applied fields (the "weak conjecture") or for a single uniform applied field (the "strong conjecture"). In 2006 Professors Milton and Kang [92,101] and (independently) Dr. Liping Liu proved the weak conjecture. In these papers Professors Milton and Kang also proved the related conjecture 1951 conjecture of Polya and Szego that the inclusion whose polarization tensor has minimal trace for a given volume would necessarily take the shape of a sphere. Profs. Ammari, Capdeboscq, Kang, Lee, Milton and Zribi also made progress on the Eshelby strong conjecture [117].

In 1985 Mortola and Steffe conjectured a complicated formula for the effective conductivity of a four phase checkerboard. In 2001 (using quite different derivations) Prof. Milton [77] and (independently) Profs. Craster and Obnosov proved this conjecture. In 1980 Prof. Milton conjectured a phase interchange inequality [2] that applied to the effective conductivity of an isotropic mixture of two isotropic phases. An almost complete proof of this inequality was obtained in 1989 by Profs. Avellaneda, Cherkaev, Lurie, and Milton [34]. (A small error was later corrected by Profs. Nesi and Zhikov in 1991, completing the proof).

Correlating the response of materials at different frequencies. The Kramers Kronig relations are well known relations that determine how much a material can "bend light" given one knows much it absorbs light over all frequencies. An experimentalist Dr. Mantese, working at General Motors, had mentioned to Professor Milton that he thought that the absorption could always be adjusted outside the measured frequency interval to obtain agreement with the Kramers Kronig relations, for any set of measured data in the frequency interval. Prof. Milton with Drs. Mantese and Eyre [65] showed this is not the case: there exist rigorous bounds that correlate the measurements in any given frequency window and which provide a natural generalization of the Kramers-Kronig relations to frequency band limited experimental data. Profs. Milton and Lakes with Dr. Eyre also correlated the viscoelastic moduli at a set of frequencies [78].

Characterizing the response of networks. It is important to know what responses of discrete networks are possible- the classic example is the characterization of Foster of the responses of two-terminal electrical networks, which is an important tool in electrical synthesis and design. Prof. Milton, with then postdocs Dr. Guevara Vasquez and Onofrei obtained a complete characterization (within linear elasticity) of the dynamic response of discrete multiterminal networks of springs and masses [123]. Profs. Milton and Seppecher introduced an entirely new type of network (with the elements being triangles joined by cylinders) that is appropriate for Maxwell's equations at fixed frequency, and they completely characterized the possible responses of such networks [113,118]. Profs. Milton and Seppecher also completely characterized the response at fixed frequency of multiterminal electrical, acoustic, and elastdynamic discrete networks [105].

Pioneering work on superlensing. In 1993 and 1994, Profs. McPhedran and Milton and Dr. Nicorovici obtained some striking results [50,56, see also 89]. For the two-dimensional quastistatic equations of electromagnetism (quasistatic meaning that the wavelength is long compared to the body) they found that a circular shell (annulus) having a dielectric constant of -1, surrounding a circular core, would be invisible to any applied fields if the core was empty and if the core was not empty would respond the same way as a solid circle of material of much larger radius: the shell acts to magnify the core. In the method of images it is well known that if a point source is placed outside a circle of material then the field outside the inclusion is exactly that of the point source plus an "image source" located inside the circle of material. But if this correspondence held, then the "image source" would appear *outside* the annulus. This seemed interesting to us, so in that paper we explored this, taking the physical limit as the loss in the shell tends to zero, and found that on one side of the "image source" it appeared as if there was an actual source there, but that on the other side of the "image source" there appeared oscillations, now called anomalous localized resonance. These results *are evident* from the equations and numerical simulations in [56], but more attention should have been drawn to them: the phenomena--point image sources with regions of anomalous resonance on one side of them—is the key mechanism that explains the bold claim of Sir John Pendry in 2000 that a slab having dielectric constant -1 (and magnetic permeability -1) would be a perfect lens…not subject to the diffraction limit. *Later we recognized that Pendry's paper, despite receiving over 10,000 citations and being the basis for the award of the million dollar Kavli prize, is fundamentally flawed. For dipole sources less than a distance d/2 from the lens, where d is the lens thickness, the transmission is zero not one [96]. For sources at distances between d/2 and d from the lens the anomalously resonant fields obscure the image [89,96]. See the slides http://www.maths.dur.ac.uk/events/Meetings/LMS/104/talks/1072milt.pdf for more historical remarks.*

Work on neutral inclusions and cloaking. Neutral inclusions are "invisible" to applied fields in certain directions: one can insert them in a homogeneous medium without disturbing the surrounding field. Prof. Milton and Dr. Serkov [76] obtained a variety of interesting neutral inclusions in two-dimensions of one conducting material surrounded by an appropriately shaped shell. In a different direction, as mentioned in the preceding section Profs. McPhedran, Milton and Dr. Nicorovici found that an annulus having a dielectric constant of -1 could be invisible to all applied fields [56, see also 89].

More stunning was the result of Prof. Milton and Dr. Nicorovici [91] that any finite collection of polarizable dipoles, or indeed dipole sources producing a finite amount of power would become invisible if they were appropriately close to this annulus, no matter what the applied field. The same held true for a polarizable dipole (or dipole power source) appropriately close to the Pendry superlens. This "cloaking due to anomalous resonance" attracted considerable attention. A beautiful numerical demonstration of it was given in [97]. Profs. Ammari, Ciralo, Kang, Lee, and Milton [135,142] showed that this cloaking occurs for a wide variety of sources, not just discrete ones.

Profs. Briane, Milton, and Willis, investigated [93] whether the "transformation optics" approach to cloaking could be carried through to elastic waves. They found that the equations transformed to the Willis equations (coupling momentum with strain, and stress with acceleration). Materals with anisotropic effective mass density would be required and they obtained simple models exhibiting this surprising behavior. Later Prof. Milton found an explicit model exhibiting a Willis type behavior [98].

A different sort of cloaking was devised by Prof. Milton and (then) Drs. Guevara Vasquez and Onofrei [111,112,124,130]. This was active exterior cloaking. A few appropriately designed sources (designed according to incident wave, and designed so they did not radiate into the far field) generated a quiet zone, within which the object to be cloaked could be placed. Many interesting generalizations by many authors followed.

Novel variational principles. Energy minimization variational principles have proved to be an important tool in analysis, dating back to pioneering work of Dirichlet and Thompson (and are particularly useful for bounding the response of bodies, among other things). Building upon the work of Cherkaev and Gibiansky (who treated the "quasistatic case"- when the wavelength is long compared to the body), Profs. Bouchitte, Milton, Seppecher and Willis [107,119] derived for the first time power dissipation minimization principles appropriate to the equations of acoustics, electromagnetism and elastodynamics (wave equations in elastic media) at fixed frequency in an inhomogeneous body, when at least one of the component material is at least slightly lossy (absorbs energy), or alternatively when the frequency is complex (so the fields grow with time). Prof. Milton also generalized the variational principles of Cherkaev and Gibiansky to other non self-adjoint problems (non-symmetric in a sense) such as conduction in a magnetic field where the conductivity matrix becomes non-symmetric [37], which Profs. Briane and Milton [121,126] used to obtain bounds.

Work on inverse problems. In 1982 Prof. Milton working with Profs. McKenzie and McPhedran and Phan Thien [13,12] realized that bounds on the effective properties of composites could be used in an inverse way to find information about the geometry of the composite, in particular the volume fractions of the phases. More recently Prof. Milton and collaborators have used many of the techniques developed to bound the response of composites, to now bound the response of inhomgeneous bodies. In turn these bounds can be used in an inverse fashion to bound the volume fractions of the phases from measurements of the fields at the boundary [128,129,131,132,140,144,148]. This could have useful medical applications, for example to determine the size of a breast cancer.

Extending the theory of compostes to other areas of science. This is the title of my new book where many breakthrough ideas are presented. These include rewriting the equations of Physics in a canonical form that illuminates the connection with the theory of composites. In particular, Schrodinger's equation can be desymmetrized, and this desymmetrized form has the advantage that iterative Fast Fourier Transform (FFT) methods for solving it can be done by just doing the FFT on the coordinates of 2 electrons, rather than all electrons.

New sorts of function, superfunctions, are introduced. These generalize the normal concept of function and the basic objects are subspace collections. There are natural rules for addition, subtraction, multiplication, division and substitution of these. They have practical uses: substitution at the subspace level can lead to faster iterative Fast Fourier Transform methods for the fields in a composite.

It is recognized that the problem of determining the response of bodies, for conductivity, Maxwell's equations, elastic and elastodynamic equations, and acoustic equations, among others, can be reformulated as a problem in the abstract theory of composites, and therefore many the tools of composites can be mapped over to the response of bodies. This response is governed by the Dirichlet to Neumann (DtN) map, that maps surface potentials to surface fluxes (suitably interpreted for the different physical problems). It is the analog of the effective tensor in composites, and one consequence of the abstract theory of compostes is that there is an integral representation formula for the DtN map as a function of the moduli of the component materials in the body. This leads to new prospective methods for imaging what is inside bodies. It has obvious importance to medical imaging, geophysical prospecting, and homeland security.

The book also generalizes the notion of conservation laws, to what are called boundary field equalities and inequalities. Subject to some constraints on the moduli inside a body, boundary equalities and inequalities are identities or inequalities satisfied by the fields and fluxes at the surface.

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