Metamaterials: Composite Materials with Striking Properties

Graeme Milton, University of Utah

The Lycurgus Cup (4th Century Roman, British Museum)

Lit from in front

Lit from behind

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Communicated by Professor J. LARMOR, *Sec. R.S.*

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Introduction.

§ 1. THE present paper contains a discussion of some optical properties of a medium containing minute metal spheres. The discussion is divided into two Parts: the first Part dealing with colours in metal glasses, in which the proportion of volume occupied by metal is small; the second Part dealing with metal films, in which this proportion may have any value from zero to unity.

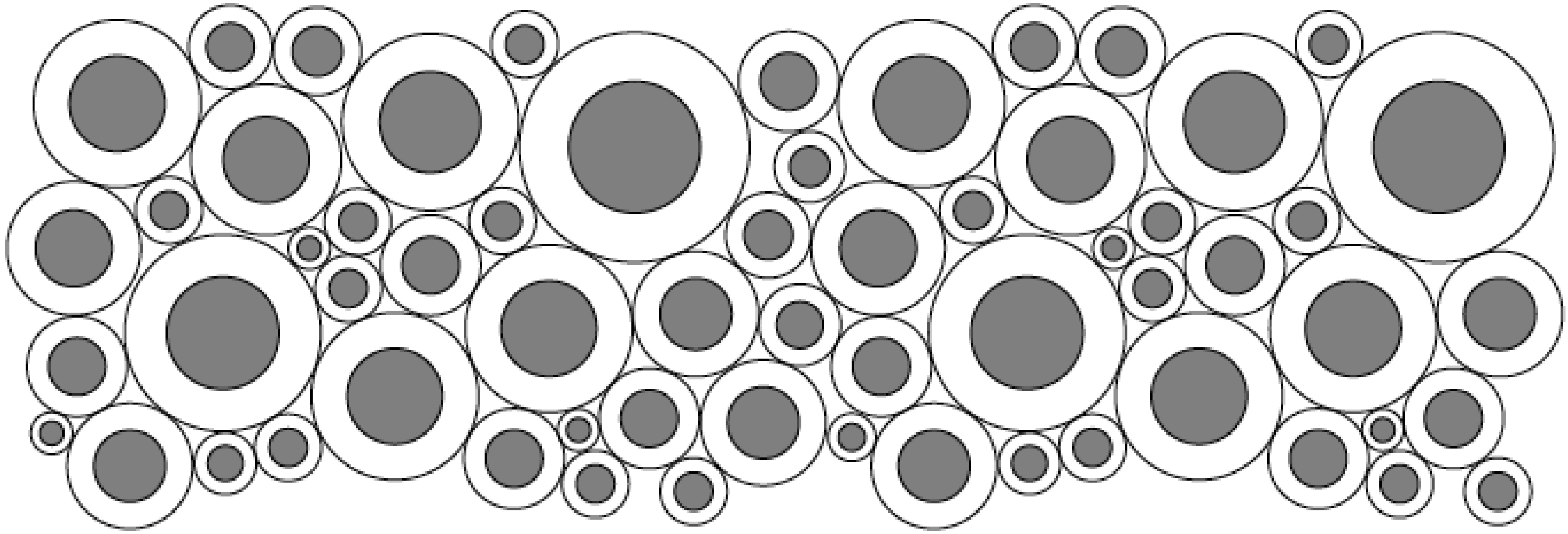
$$\epsilon_* = \epsilon_0 + \frac{3f_1\epsilon_0(\epsilon_1-\epsilon_0)}{3\epsilon_0+f_2(\epsilon_1-\epsilon_0)}$$

Formula of Clausius (1879) and Mossotti (1846-1850),
Lorenz (1869, 1875, 1880), Lorentz (1870)
Maxwell (1873) and
Maxwell Garnett (1904)

Dilute periodic array of spheres: Rayleigh (1892).

History – see Landauer (1978), 1st ETOPIM conference proceedings

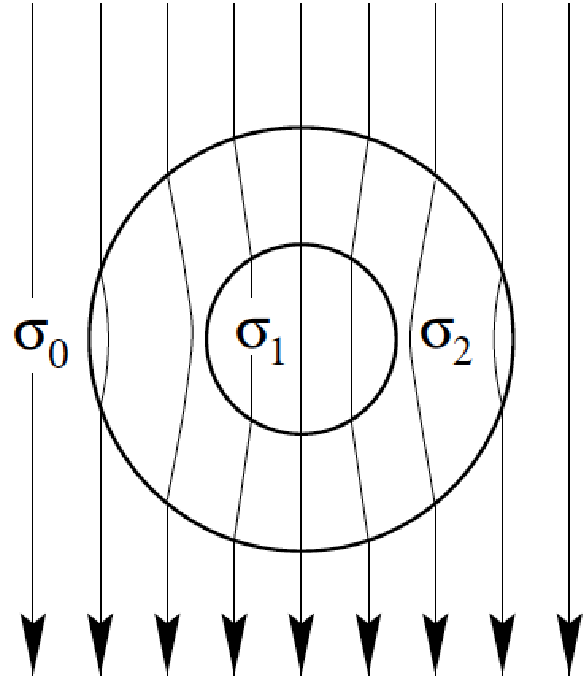
Exact Formula:



Hashin-Shtrikman Sphere Assemblage (1962)

$$\sigma_0 = \sigma_* = \sigma_2 + \frac{3 f_1 \sigma_2 (\sigma_1 - \sigma_2)}{3\sigma_2 + f_2(\sigma_1 - \sigma_2)},$$

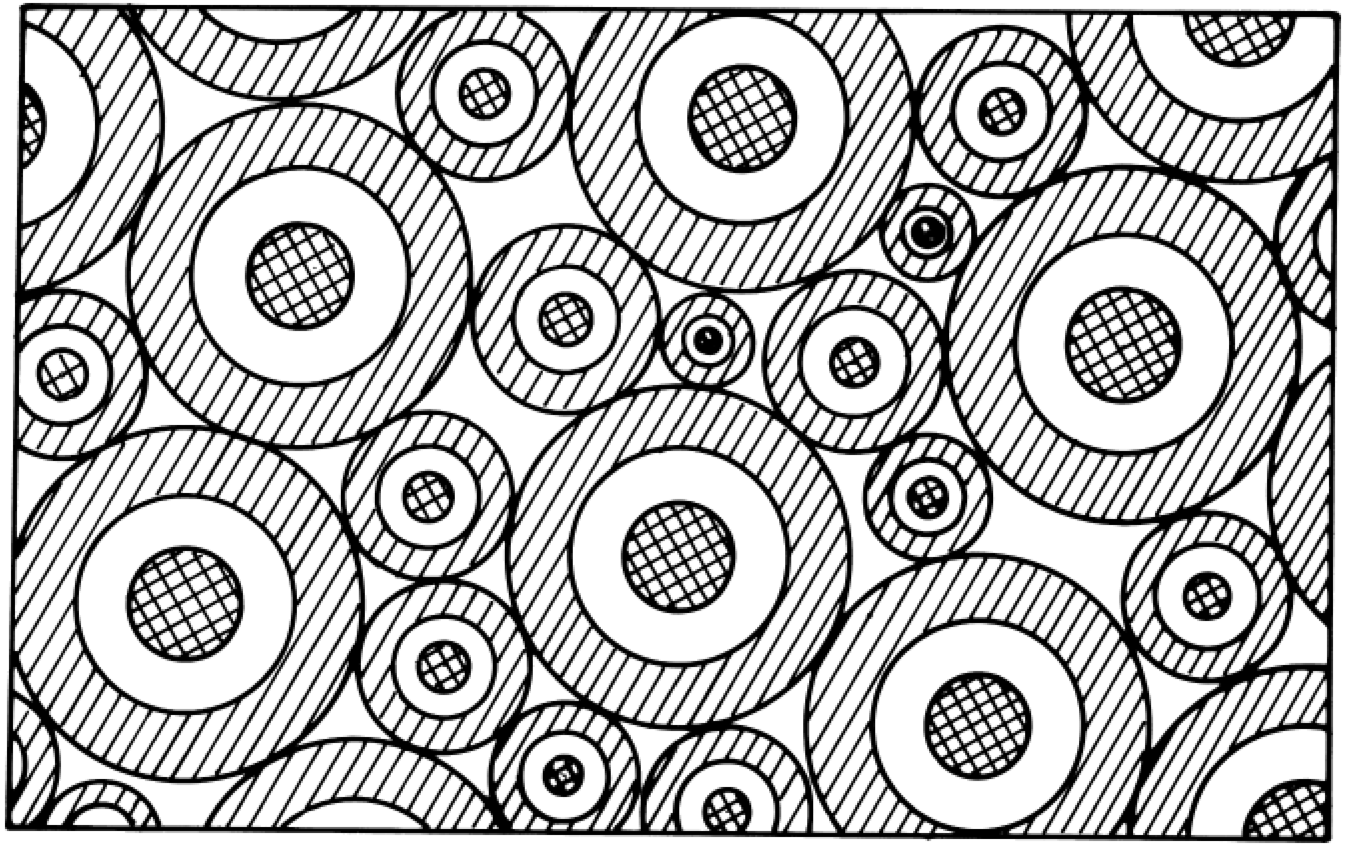
Neutral Inclusion



Naomi Halas's Group (2002)

Rice University

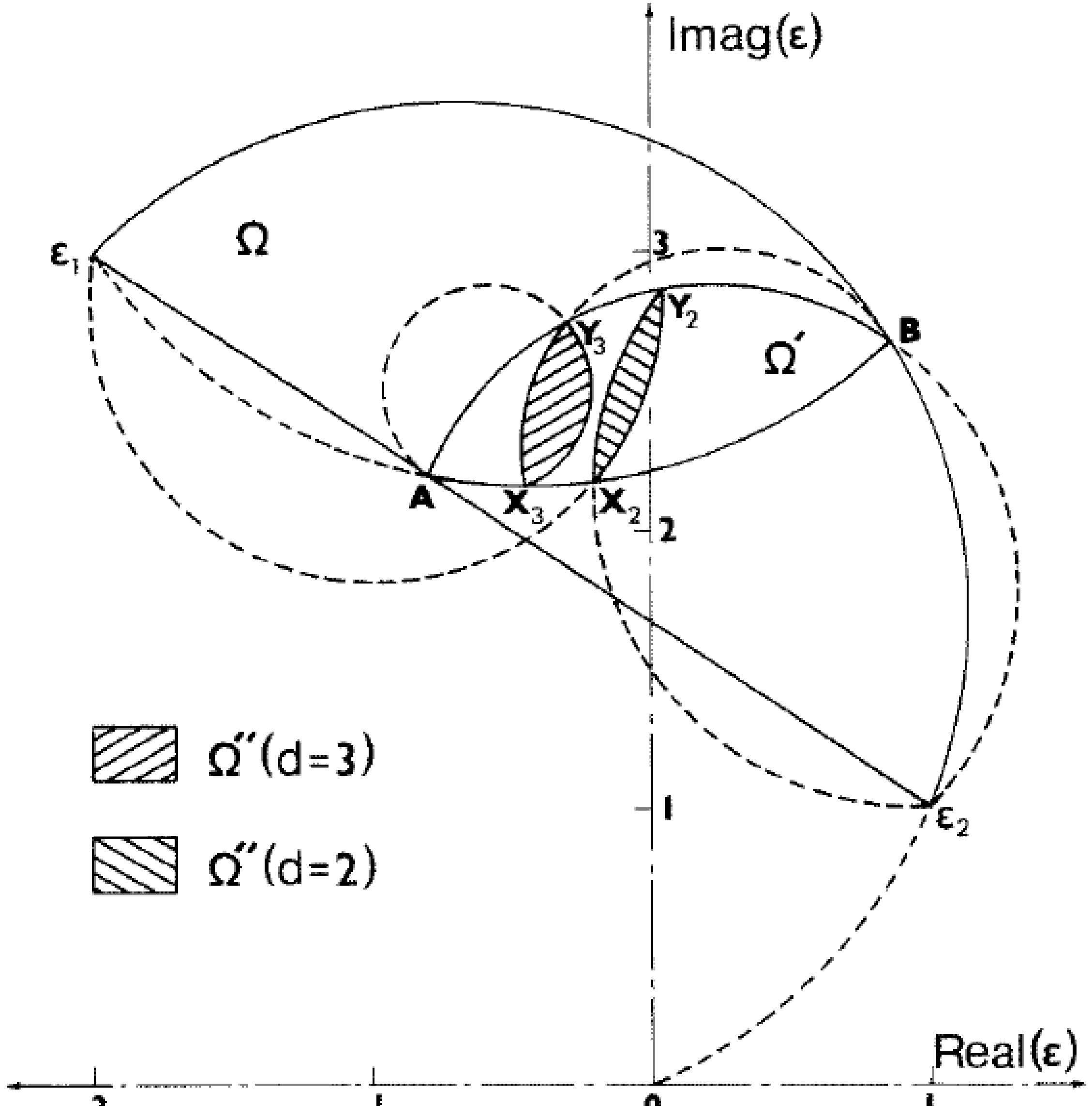
Gold Nanoshells



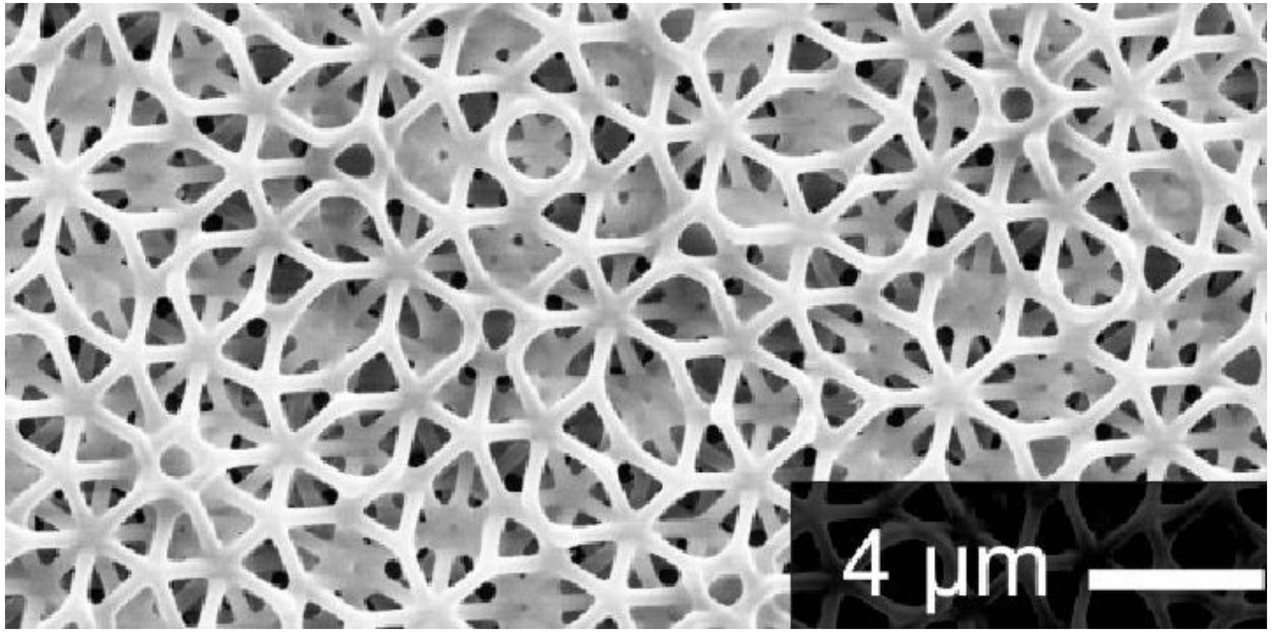
Doubly coated sphere assemblages (Schulgasser 1977)

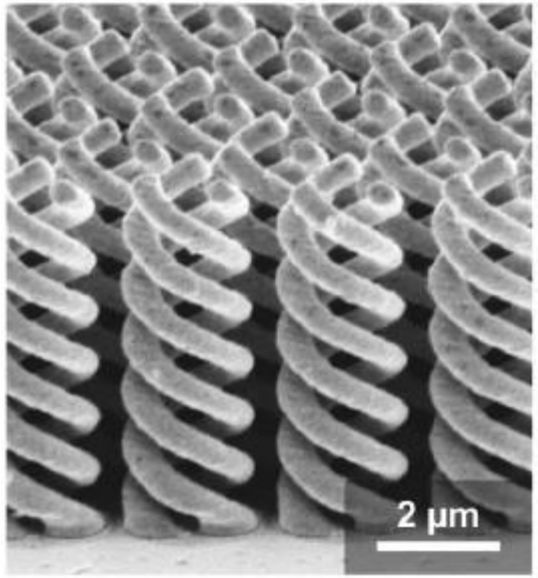
$$\sigma_* = \sigma_3 + \cfrac{3(1-f_3)\sigma_3}{f_3 - \cfrac{3\sigma_3}{\sigma_3 - \sigma_2 - \cfrac{3f_1\sigma_2}{f_2 - \cfrac{3(1-f_3)\sigma_2}{\sigma_2 - \sigma_1}}}},$$

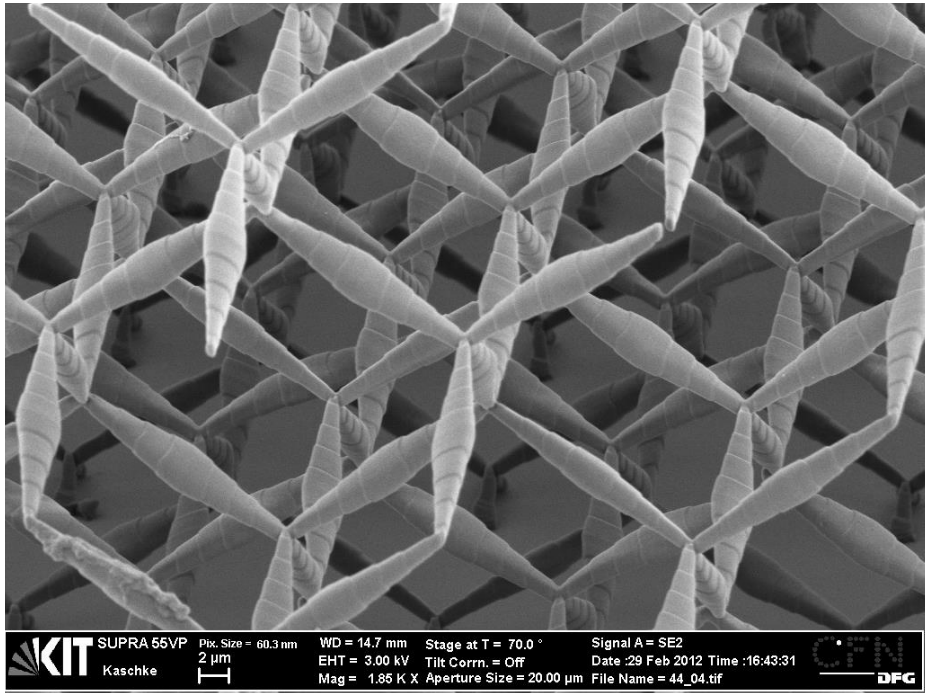
Model independent bounds:



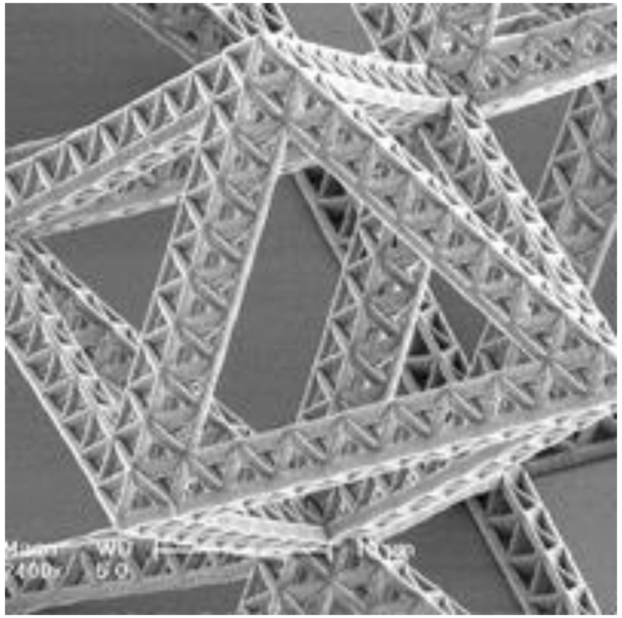
Bergman-Milton Bounds (1980)

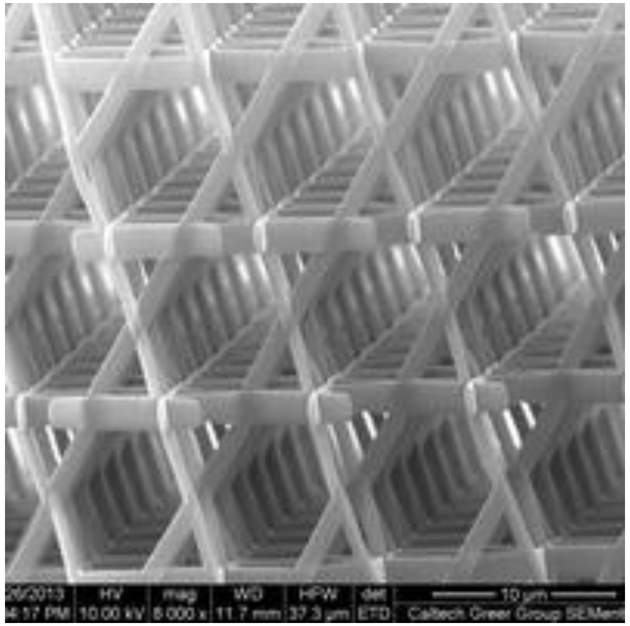


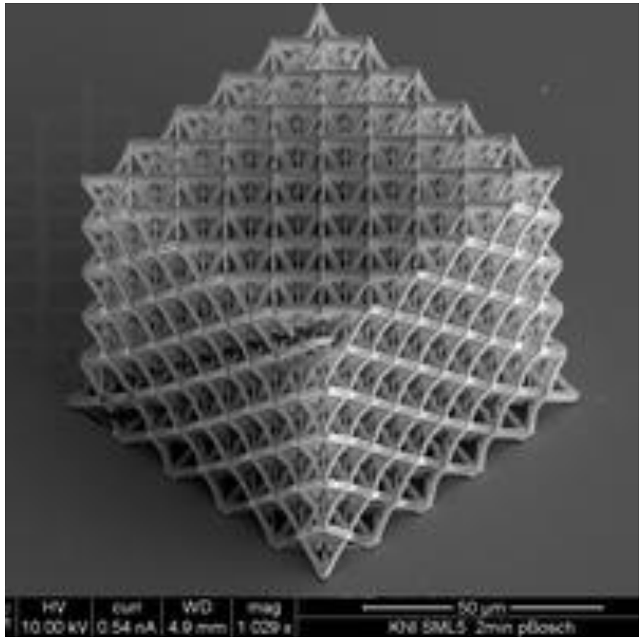




Group of
Martin Wegener







Group of Julia Greer

Walser 1999:

Macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of *two or more responses* to specific excitation.

Browning and Wolf 2001:

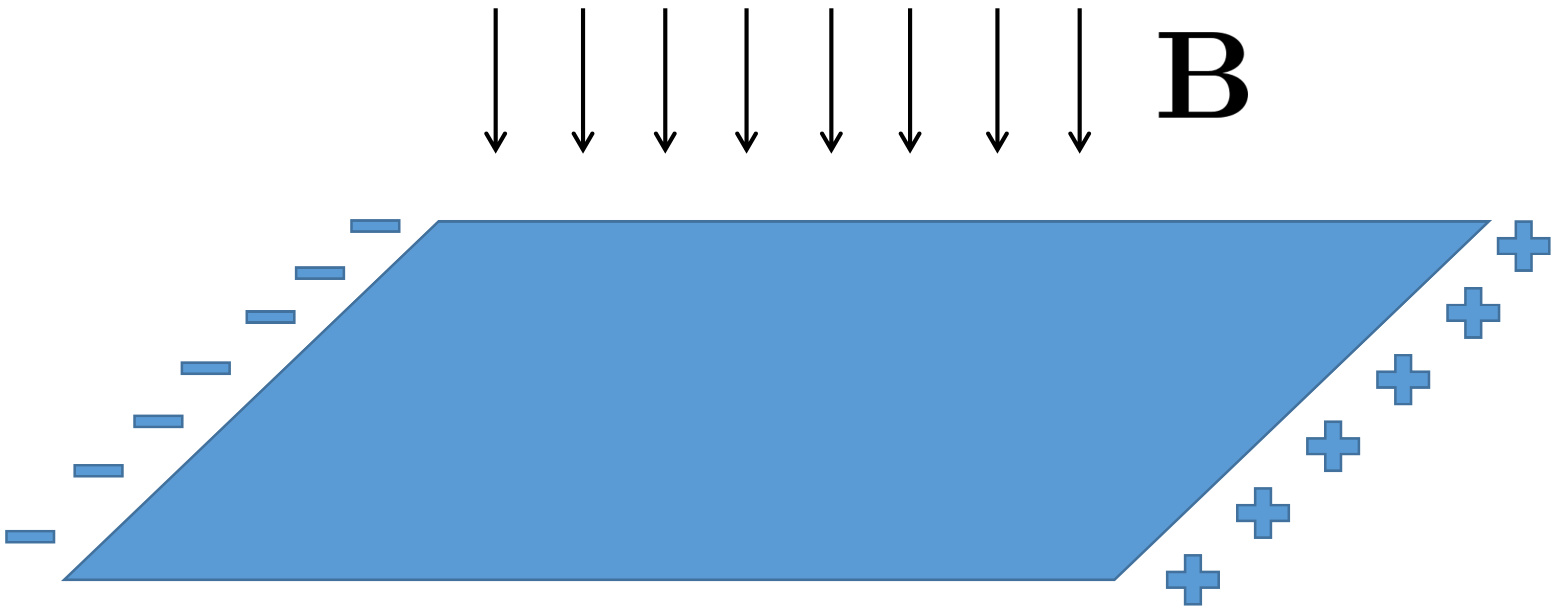
Metamaterials are a new class of ordered composites that exhibit exceptional properties not readily observed in nature.

With Kadic, Van Hecke and Wegener 2019

Metamaterials are rationally designed composites made of tailored building blocks that are composed of one or more constituent bulk materials. The metamaterial properties go beyond those of the ingredient materials, qualitatively or quantitatively.

With the addendum that ‘the properties of the metamaterial can be mapped onto effective-medium parameters’, It's constantly a surprise to find what properties a composite can exhibit.

One interesting example:



Hall Voltage

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

In elementary physics textbooks one is told that in classical physics the sign of the Hall coefficient tells one the sign of the charge carrier.

However there is a counterexample!

Mathematically: Find a conducting periodic composite with say cubic symmetry, where the matrix-valued electric field has negative trace of its cofactor matrix in some regions.

Geometry suggested by artist Dylon Whyte

Picture Courtesy Dylon Whyte

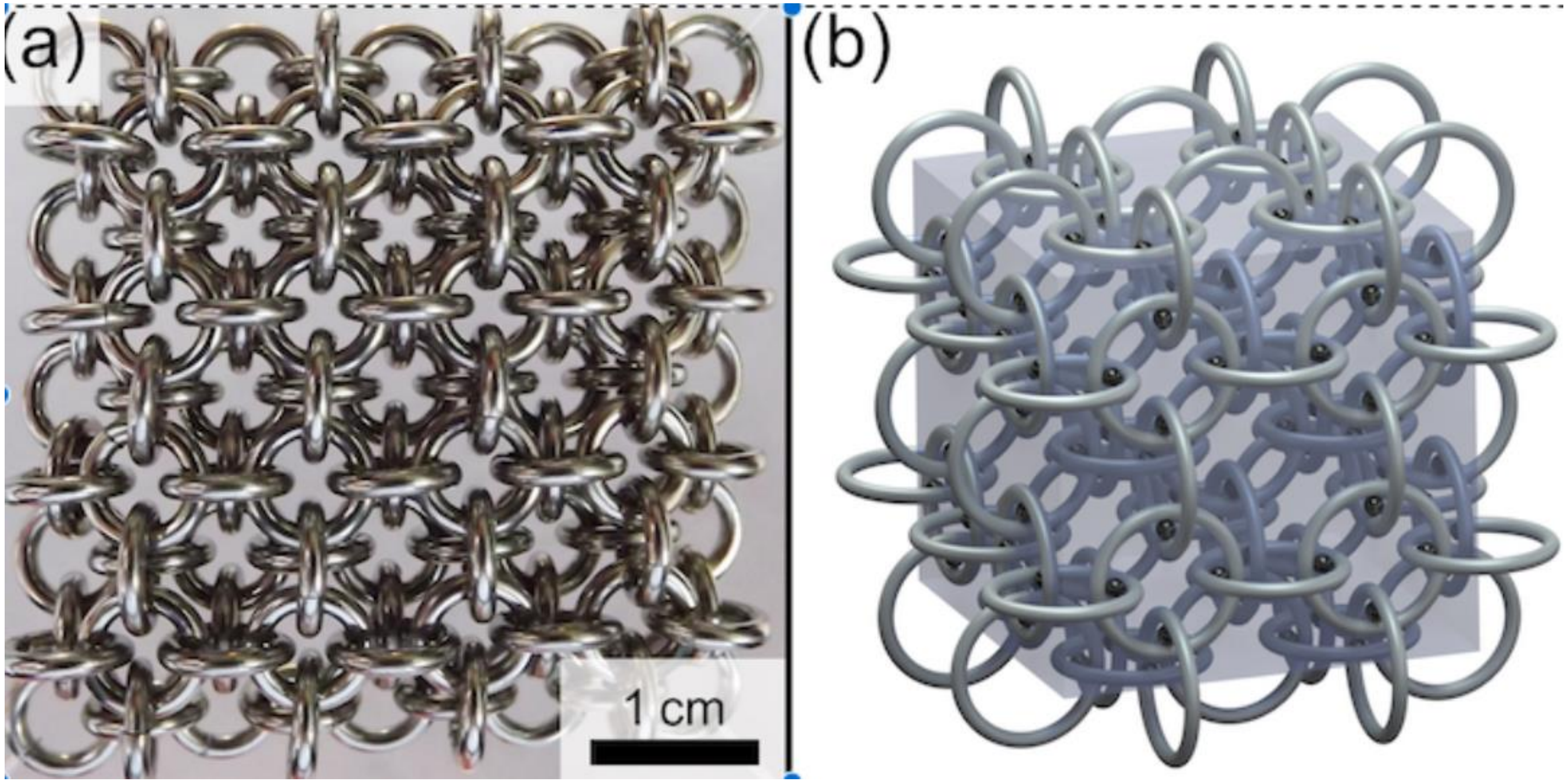
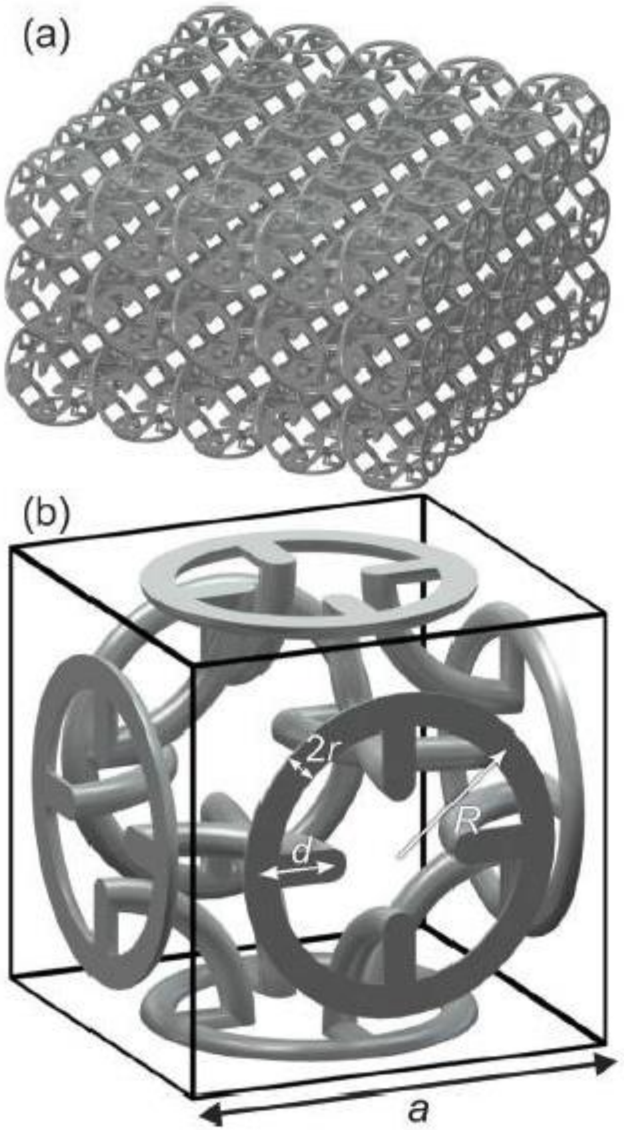
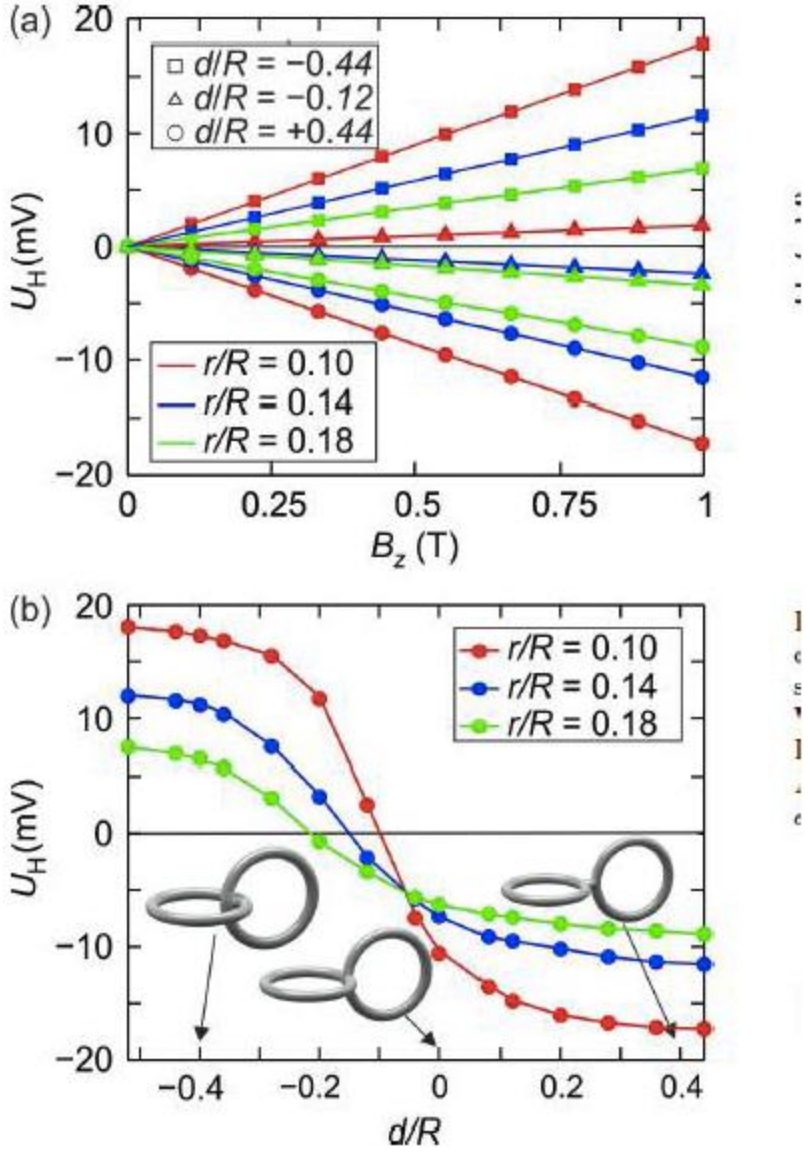


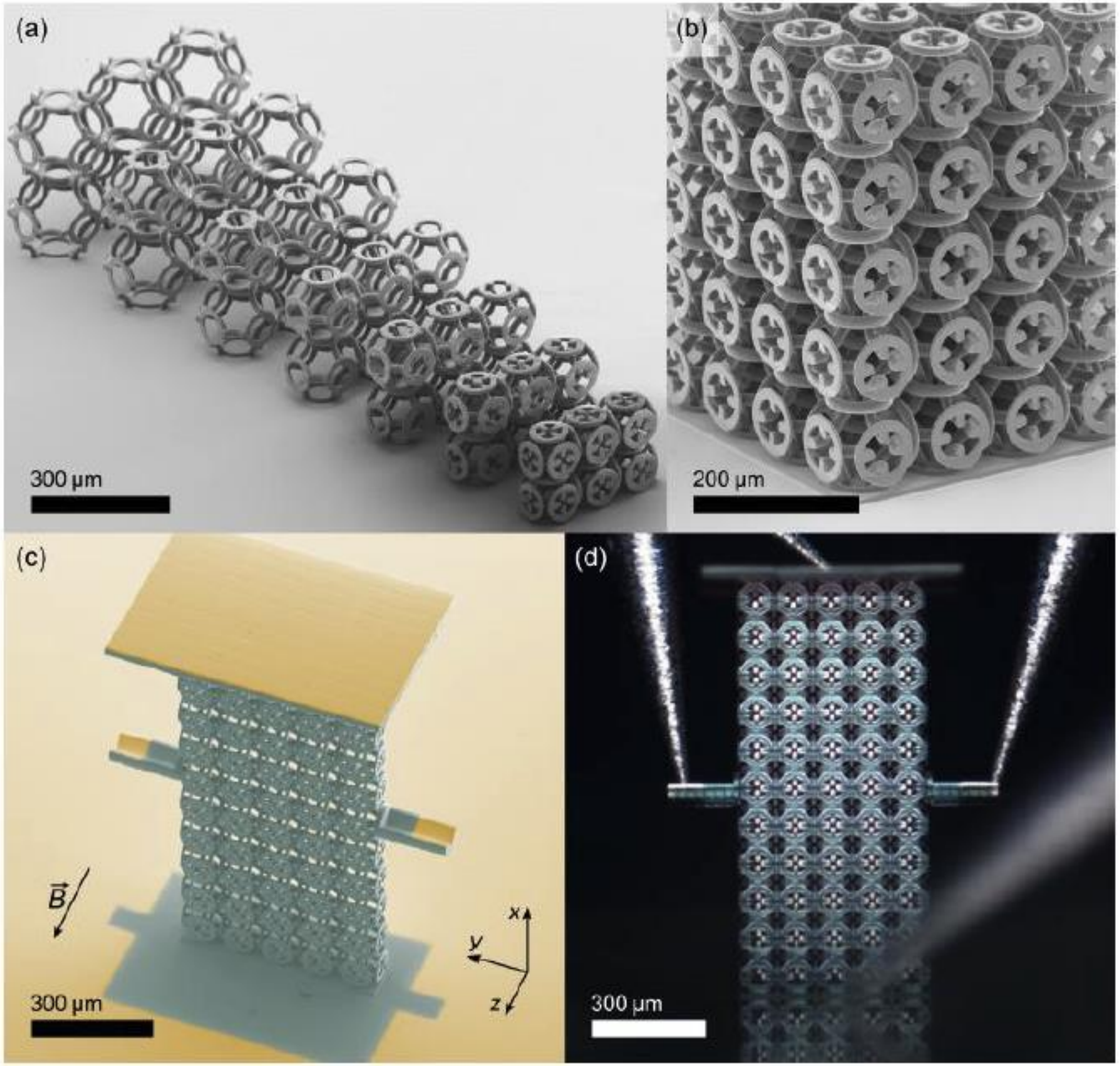
Image Courtesy Christian Kern

A material with cubic symmetry having a Hall Coefficient opposite to that of the constituents (with Marc Briane)

Simplification of Kadic et.al. (2015)

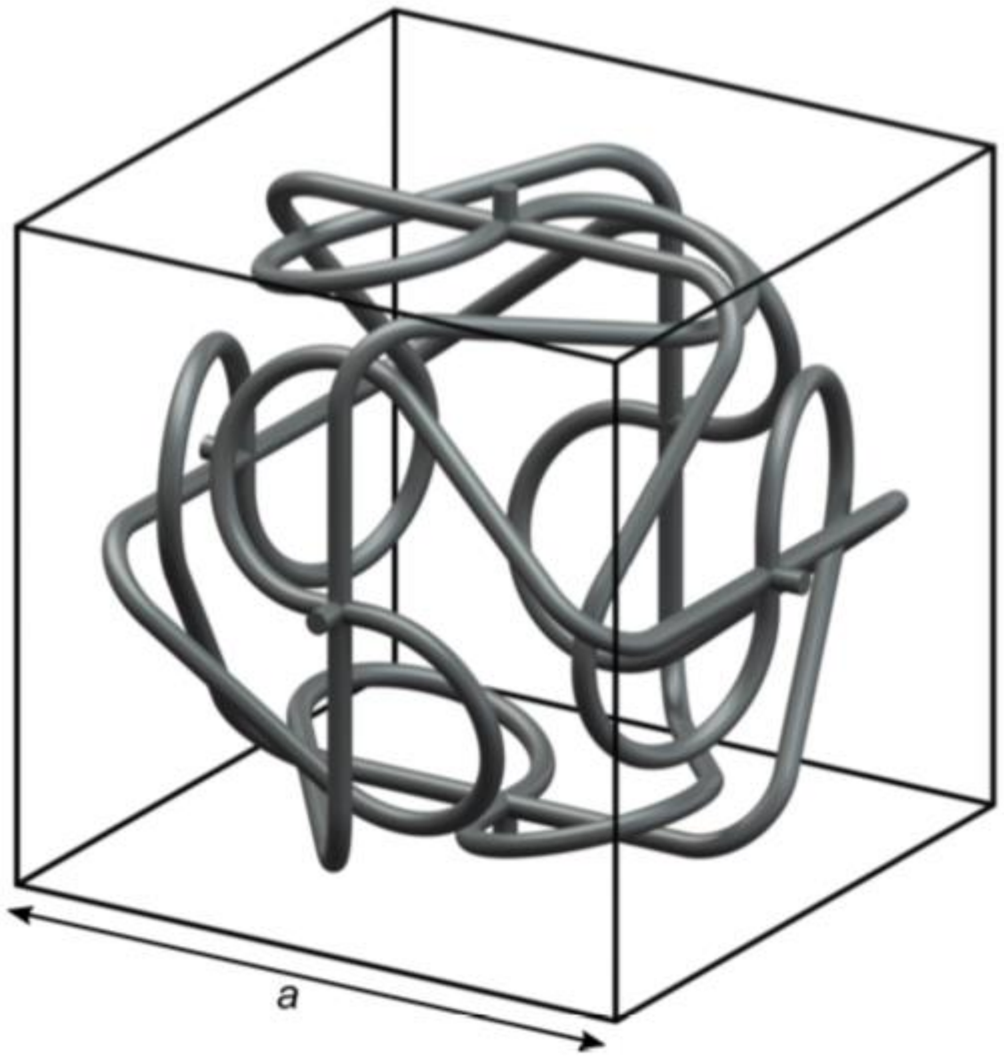




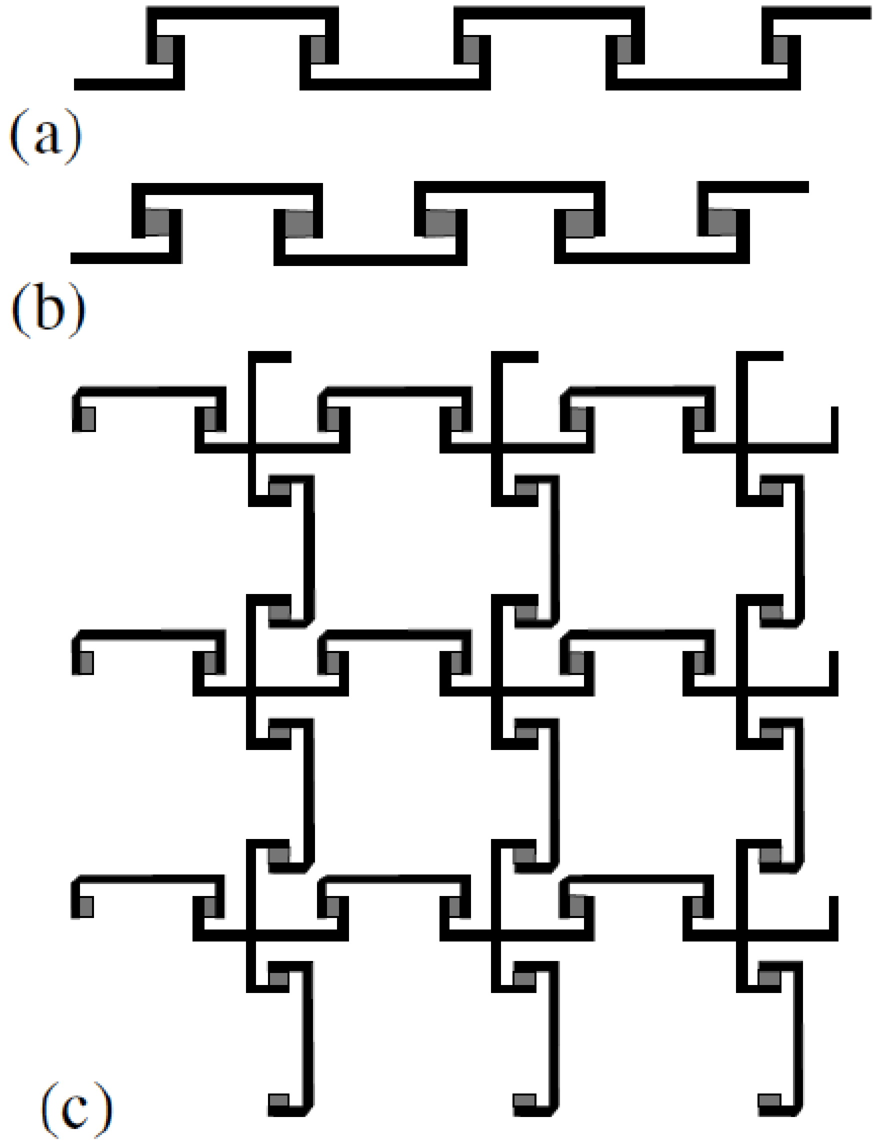


Experimental realization of Kern, Kadic, and Wegener

Alternate Structure of Christian Kern:



Another example: negative expansion from positive expansion



Original designs: Lakes (1996); Sigmund & Torquato (1996, 1997)

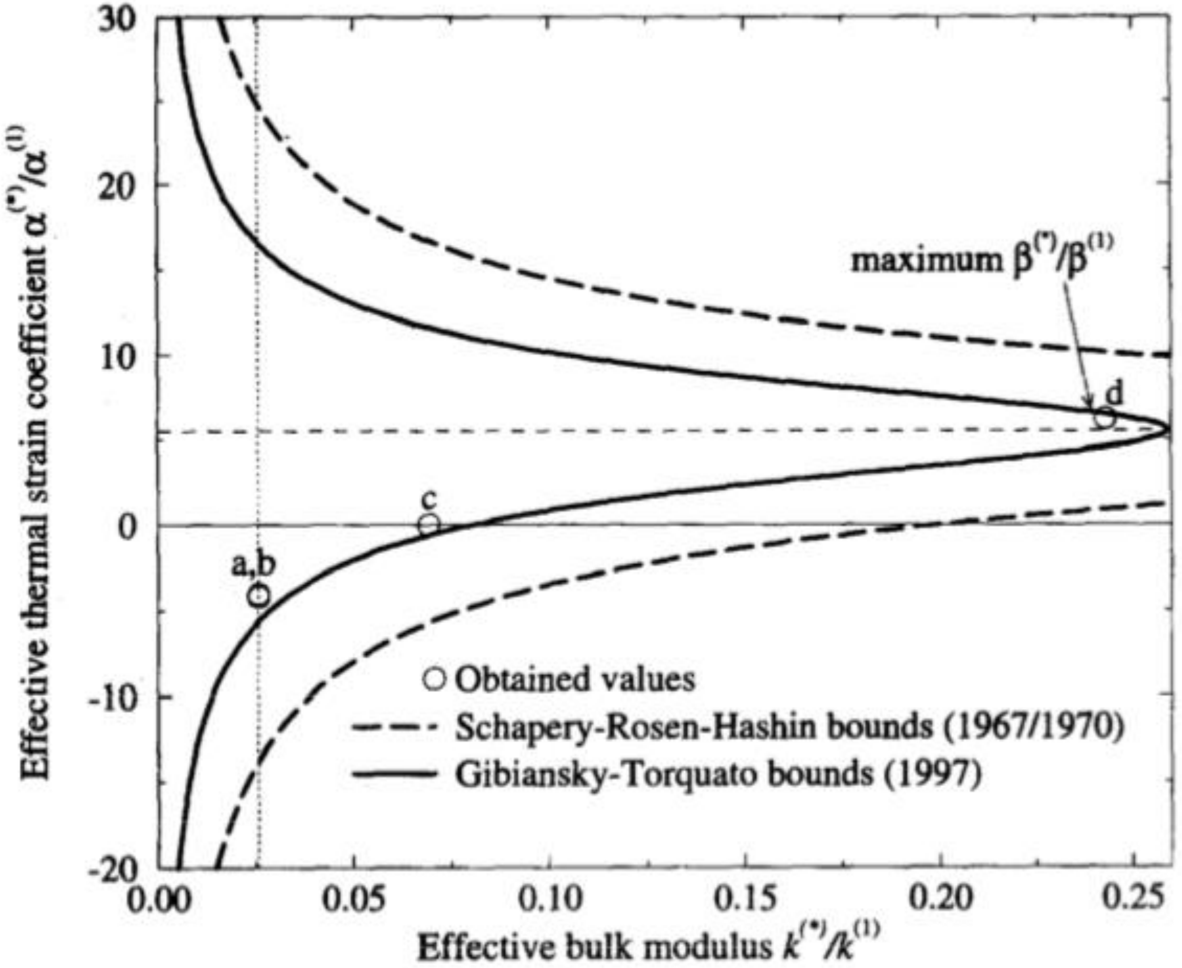
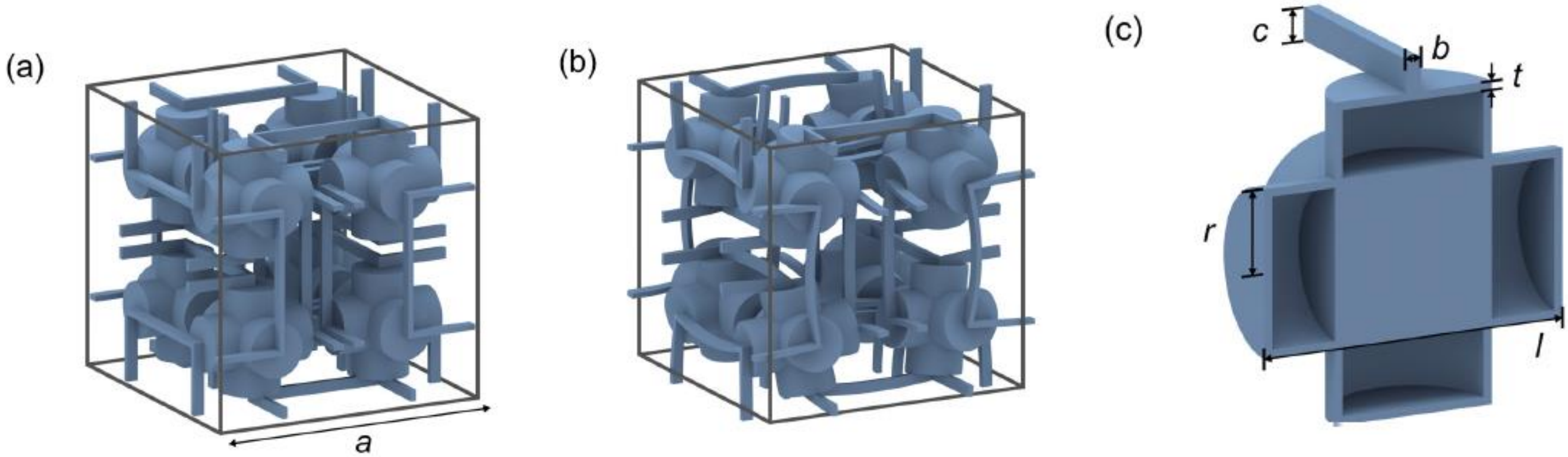


Fig. 4. Bounds for three-phase design example. The circles with letters a–d denote the obtained values for the microstructures shown in Figs 5 and 6.

Metamaterial Mantra: what is not obviously forbidden may actually be possible

One can get a similar effect for poroelasticity



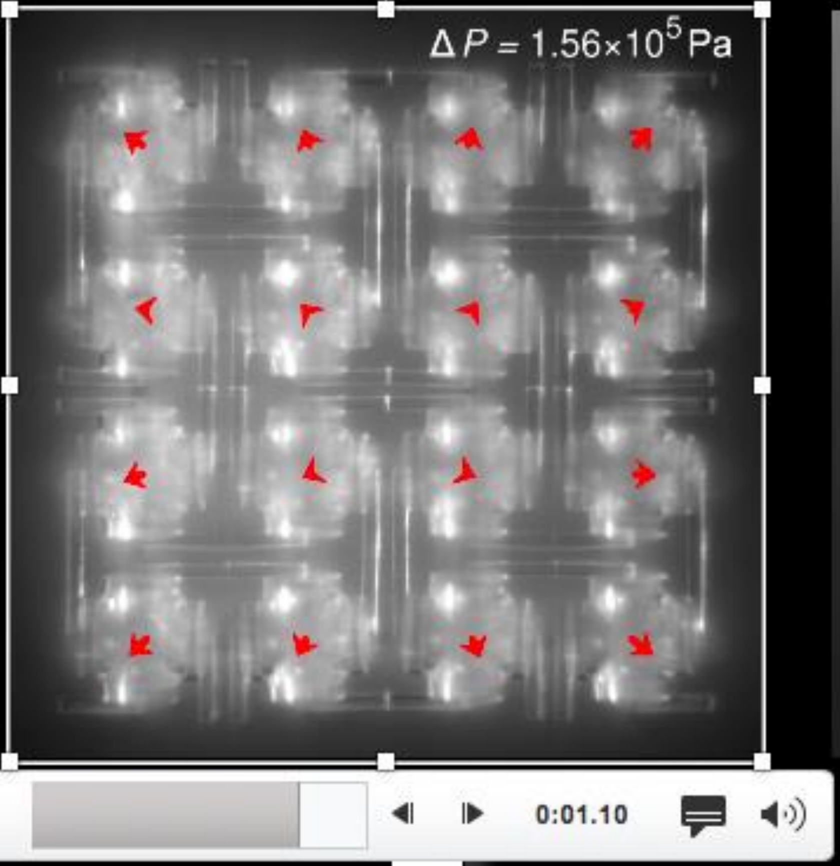
Qu, et.al 2017

Same equation,
but different physics

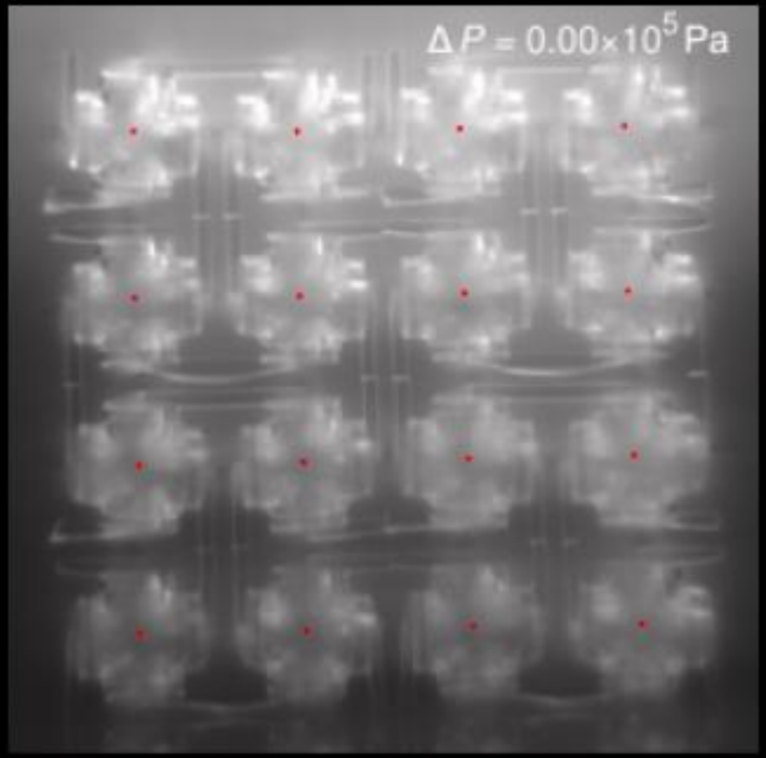
Displacement-Vector Fields

(Courtesy of Martin Wegener)

top view (*xy*-plane)



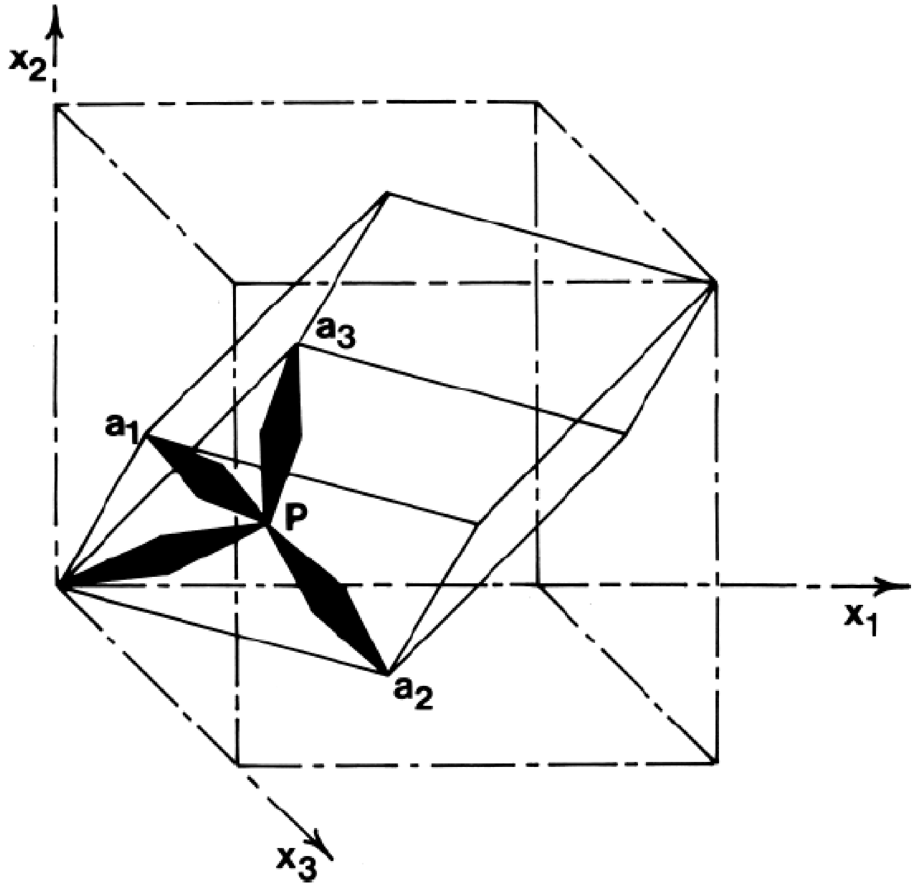
side view (*xz*-plane)



a = 150 μm, all displacement vectors ×15, from image cross-correlation analysis

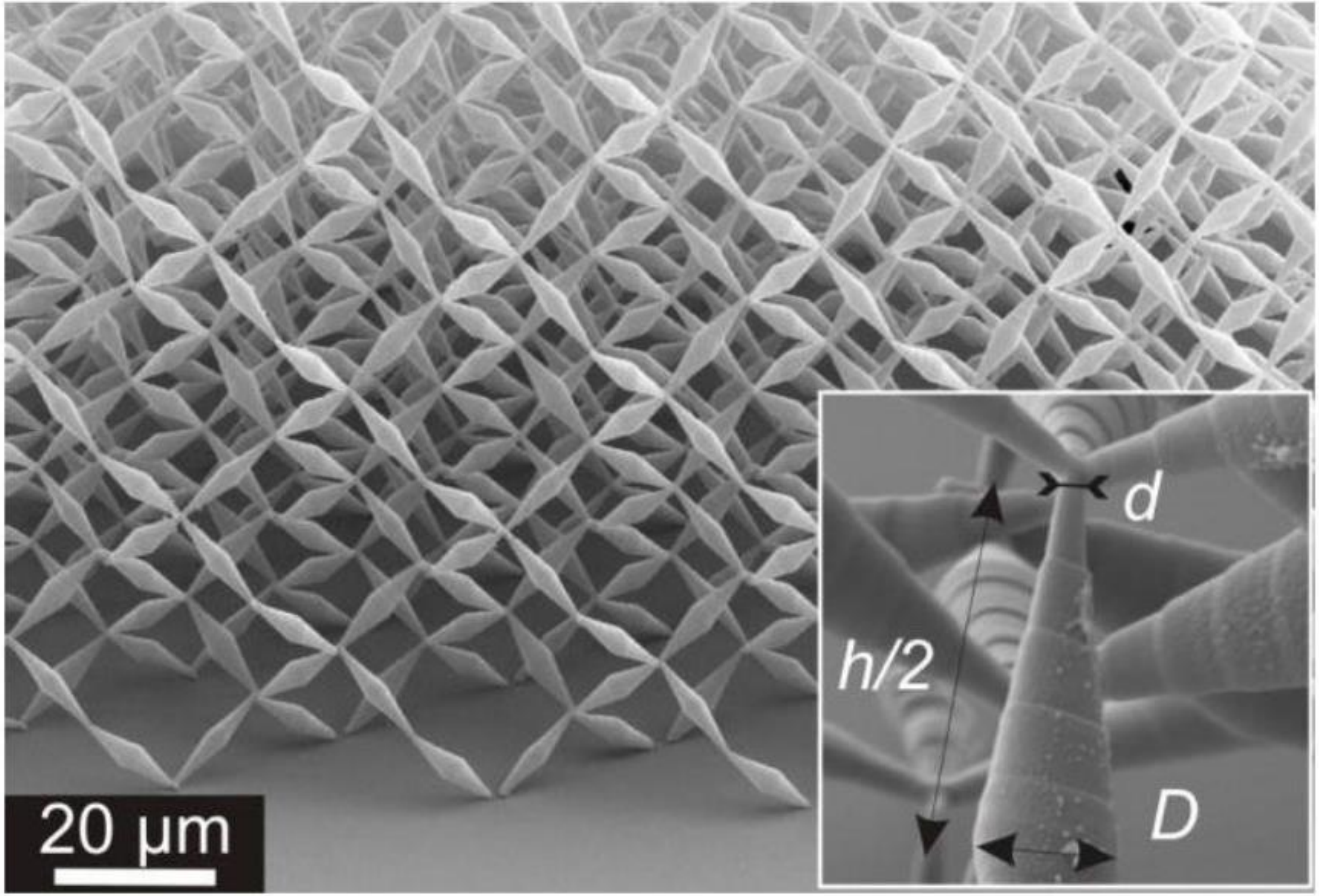
New classes of elastic materials (with Cherkaev, 1995)

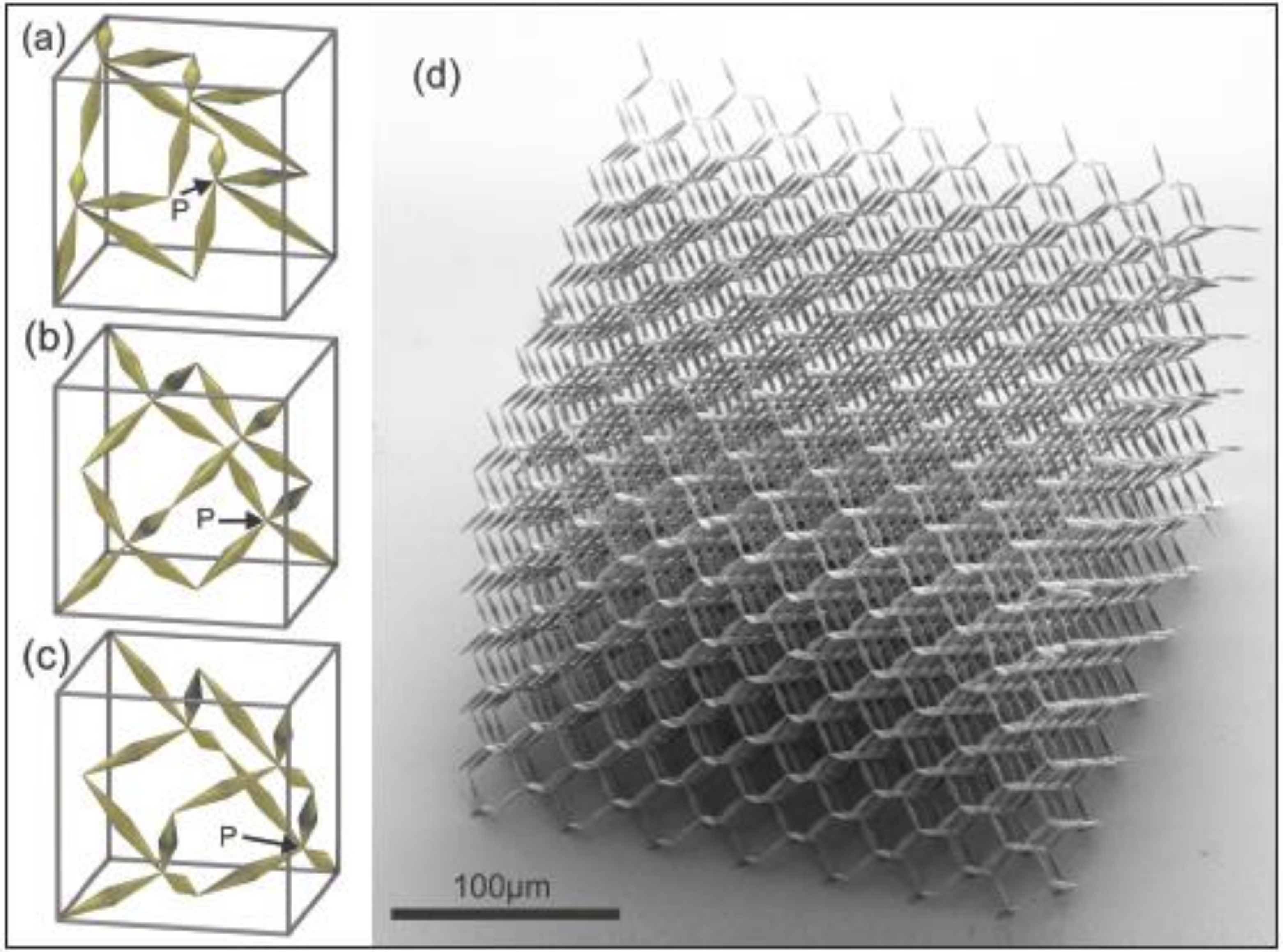
A three dimensional pentamode material which can support any prescribed loading



Like a fluid it only supports one loading, unlike a fluid that loading may be anisotropic

Realization of Kadic et.al. 2012

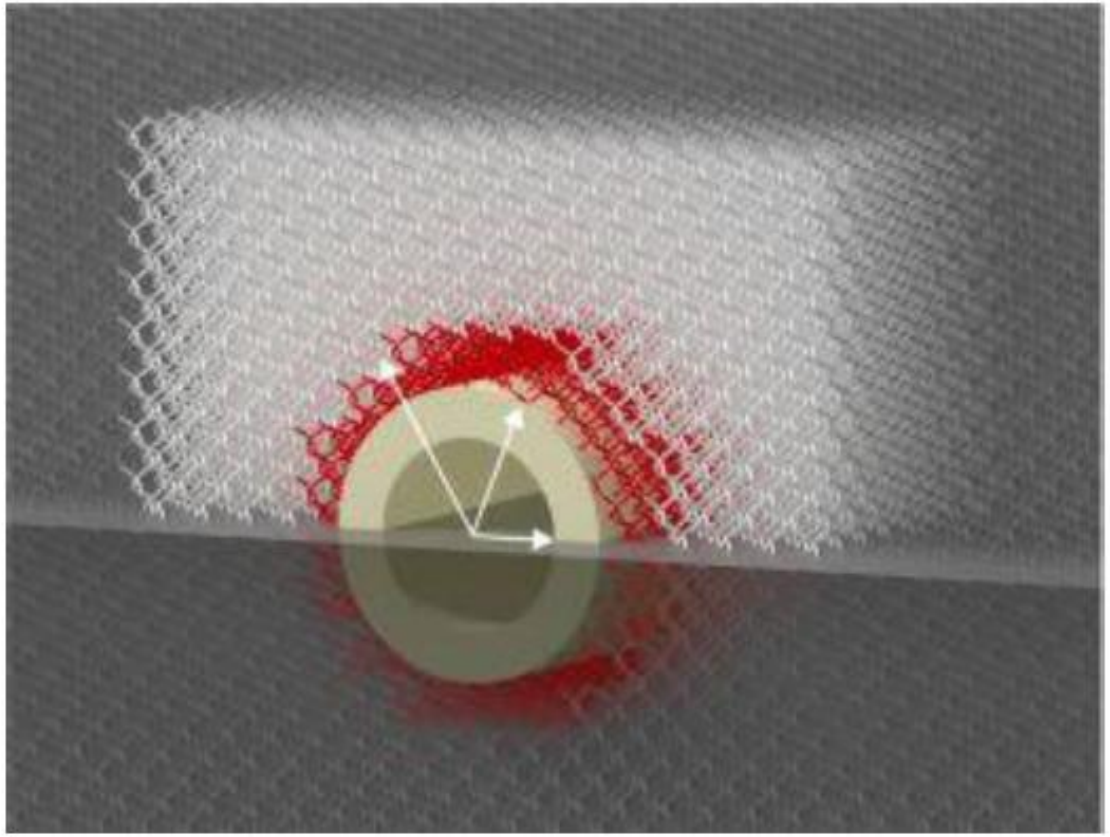


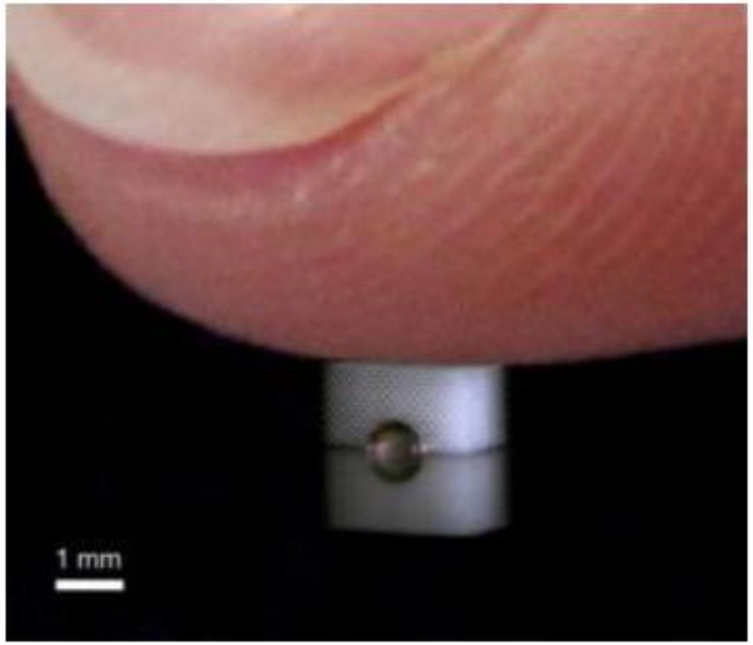


Key Feature: At any junction only 4 tips meet

Application of Pentamodes:

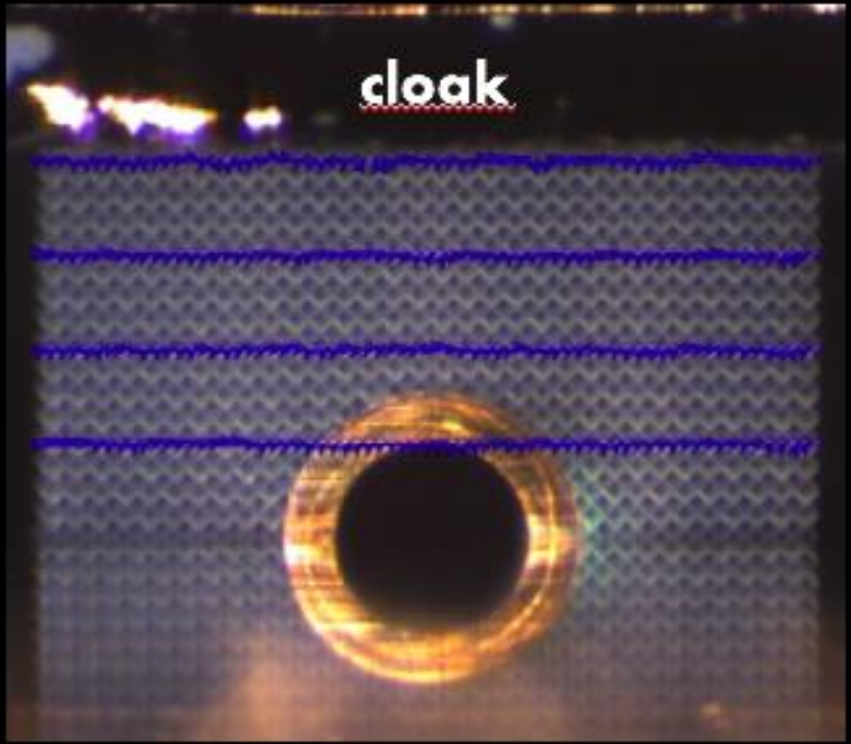
Cloak making an object “unfeelable”:
Buckmann et. al. (2014)

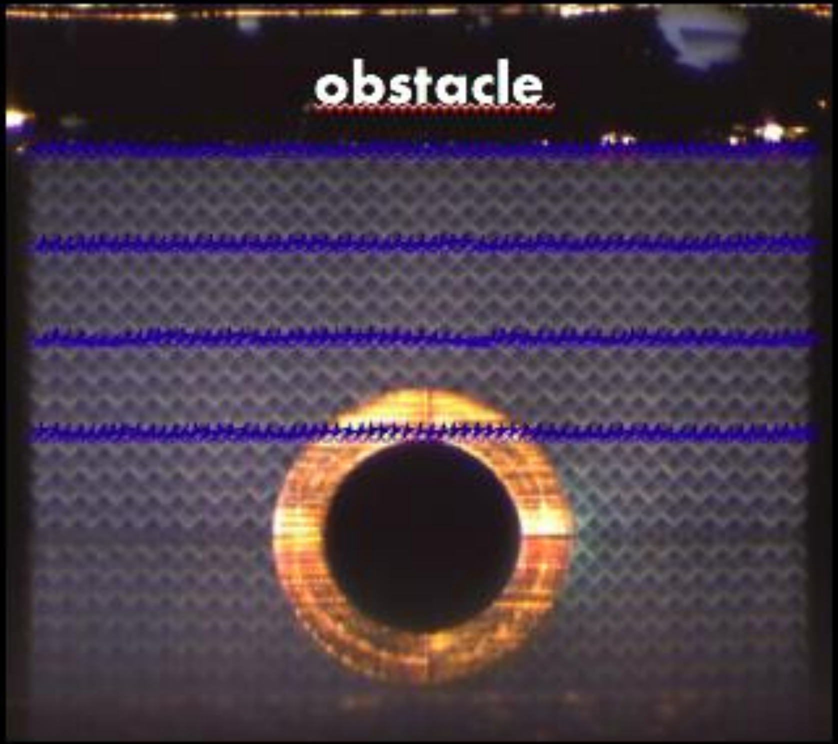




Measured Movies

(Courtesy of Martin Wegener)





= 4 × displacement vector

reference has 16×8×8 = 1024 extended fcc unit cells, total volume $V = 2\ mm^3$

More generally, as observed by Norris (2008) , pentamodes can be used for acoustic cloaking. They can guide the stress field around the object to be cloaked.

A pentamode has effective elasticity tensor $\mathbf{C}_* = \mathbf{A} \otimes \mathbf{A}$

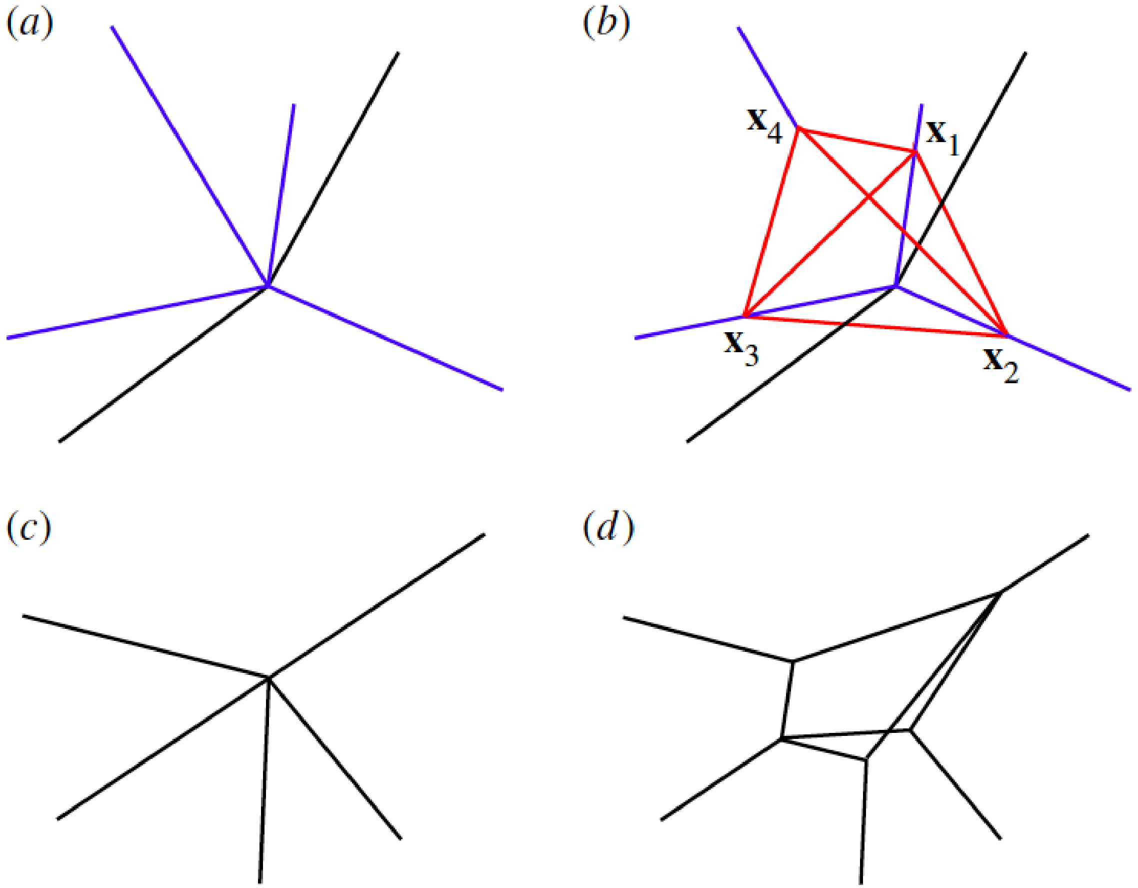
On a macroscopic scale with smoothly varying pentamode structure we can get an elasticity tensor field $\mathbf{C}_*(\mathbf{x}) = \mathbf{A}(\mathbf{x}) \otimes \mathbf{A}(\mathbf{x})$

The associated stress field is

$$\sigma(\mathbf{x}) = \mathbf{C}_*(\mathbf{x})\epsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathrm{Tr}[\mathbf{A}(\mathbf{x})\epsilon(\mathbf{x})]$$

($\mathbf{A}(\mathbf{x})$ must be chosen so this can have zero divergence).

A network under given tension having internal nodes where more than 4 wires meet can be replaced by a network where at most 4 wires meet at any internal node:

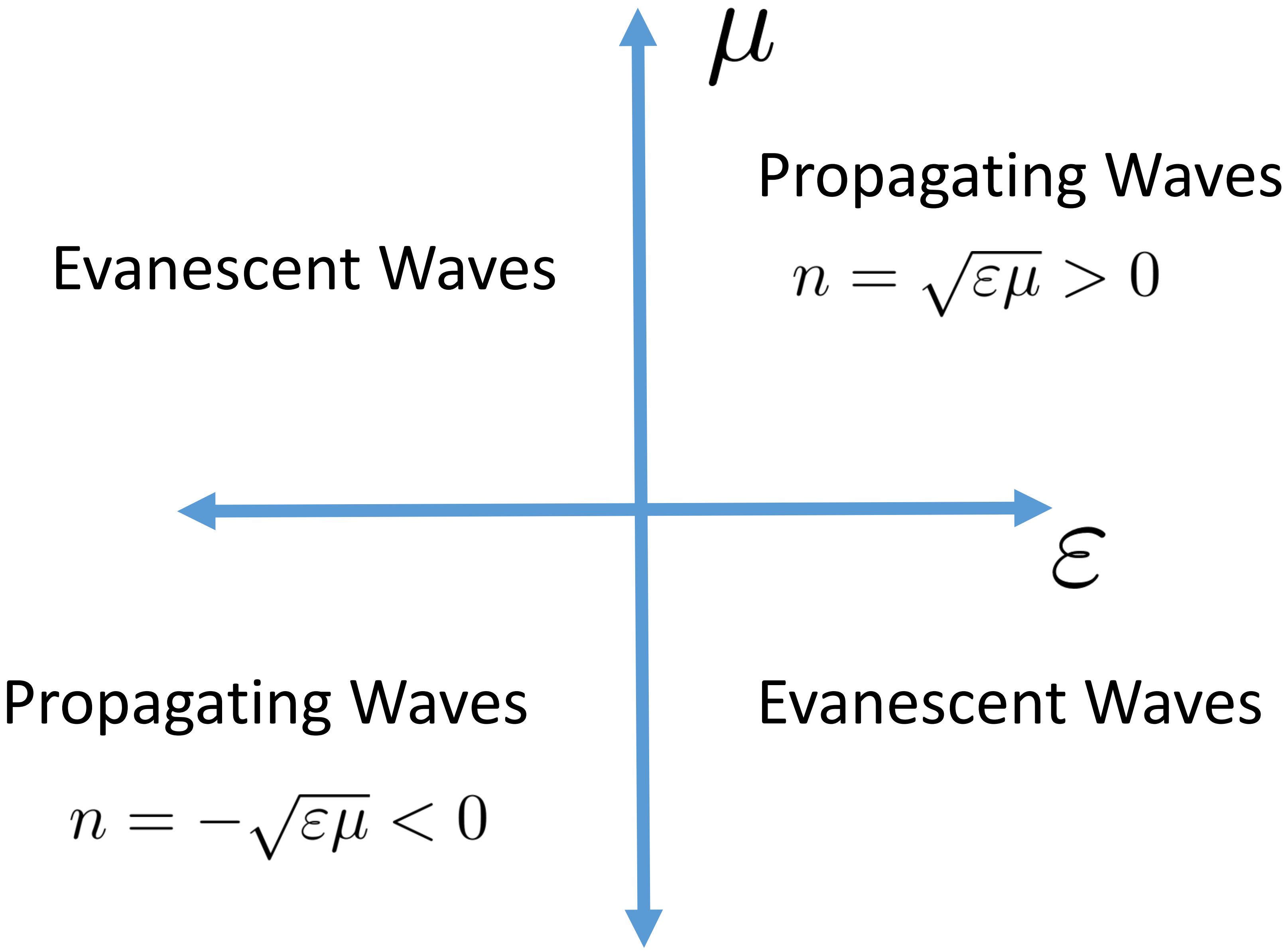


(Joint work with Bouchitte, Mattei, and Seppecher, 2019)

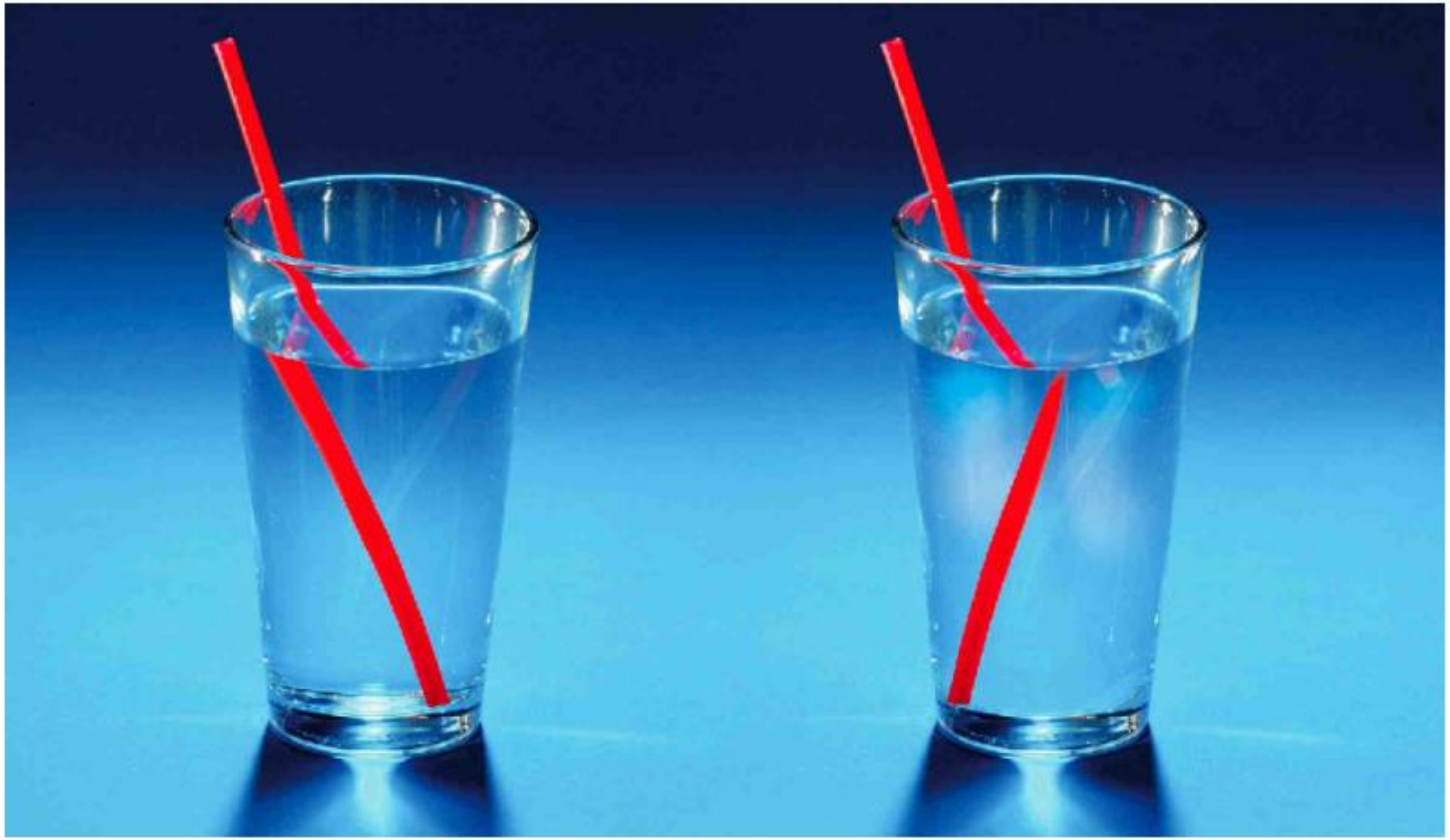
Excitement in the early 2000's: negative refractive index

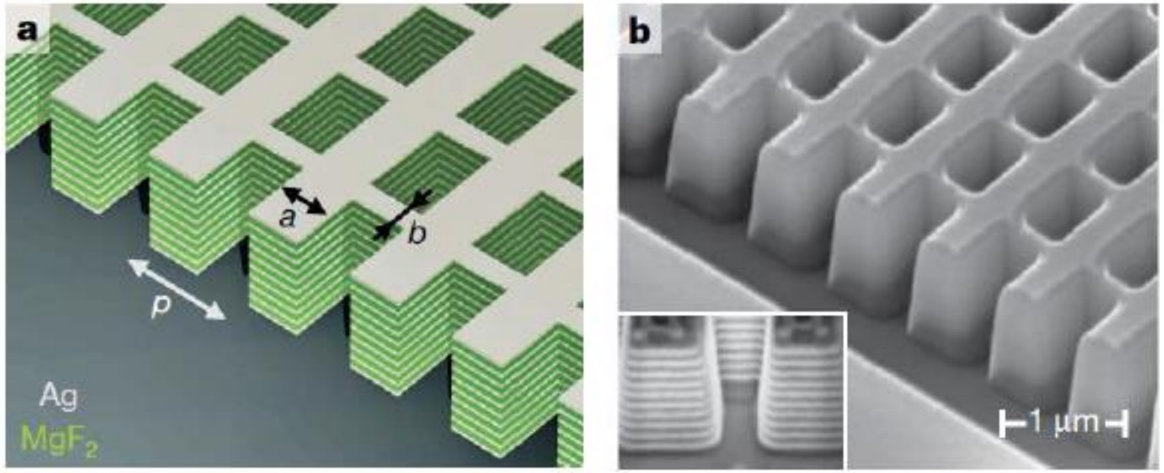
Wave Equation:

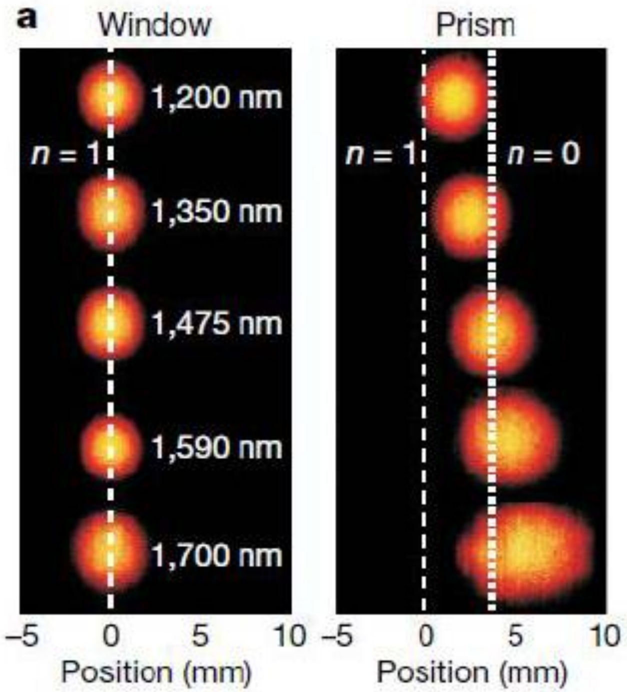
$$\nabla^2 \mathbf{E} + \omega^2 \epsilon \mu \mathbf{E} = 0$$

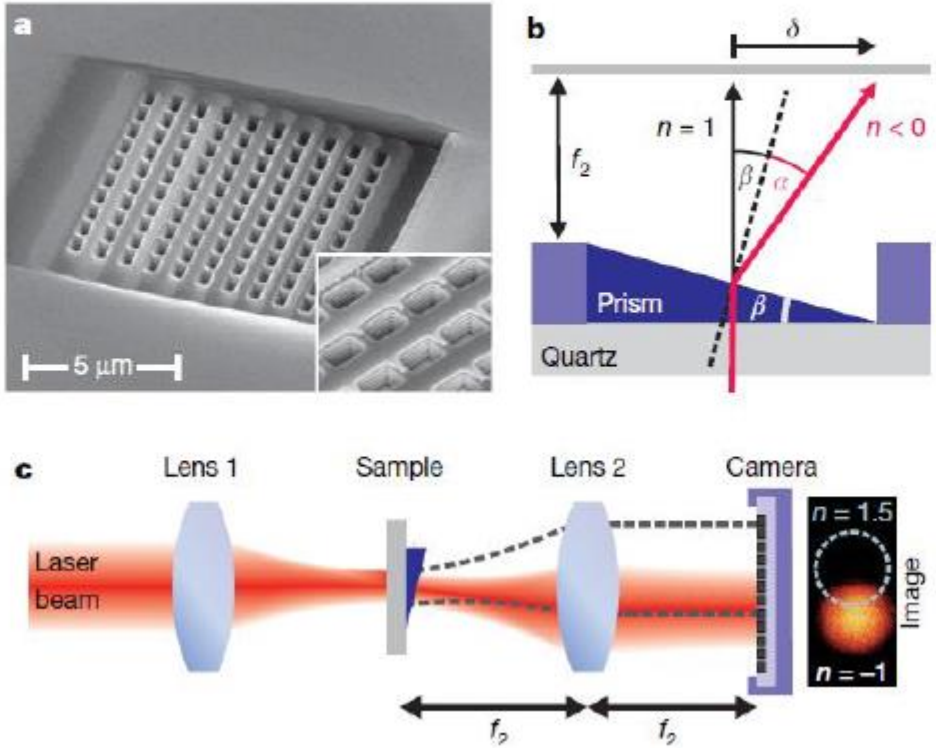


Negative Refraction Simulation: Hess 2008

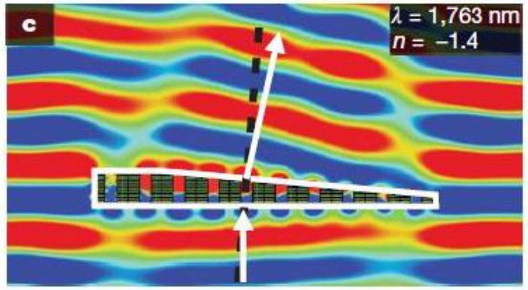




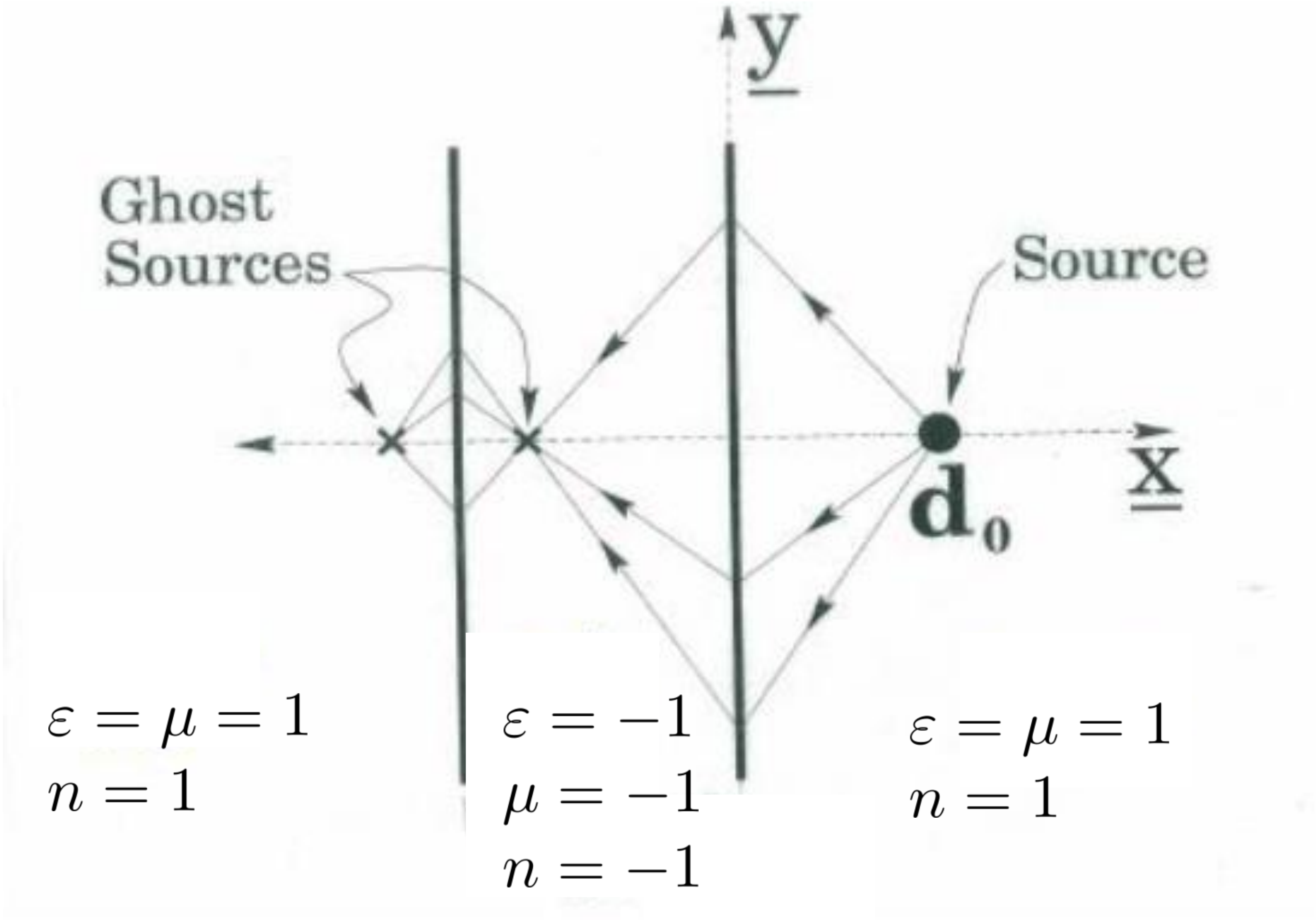


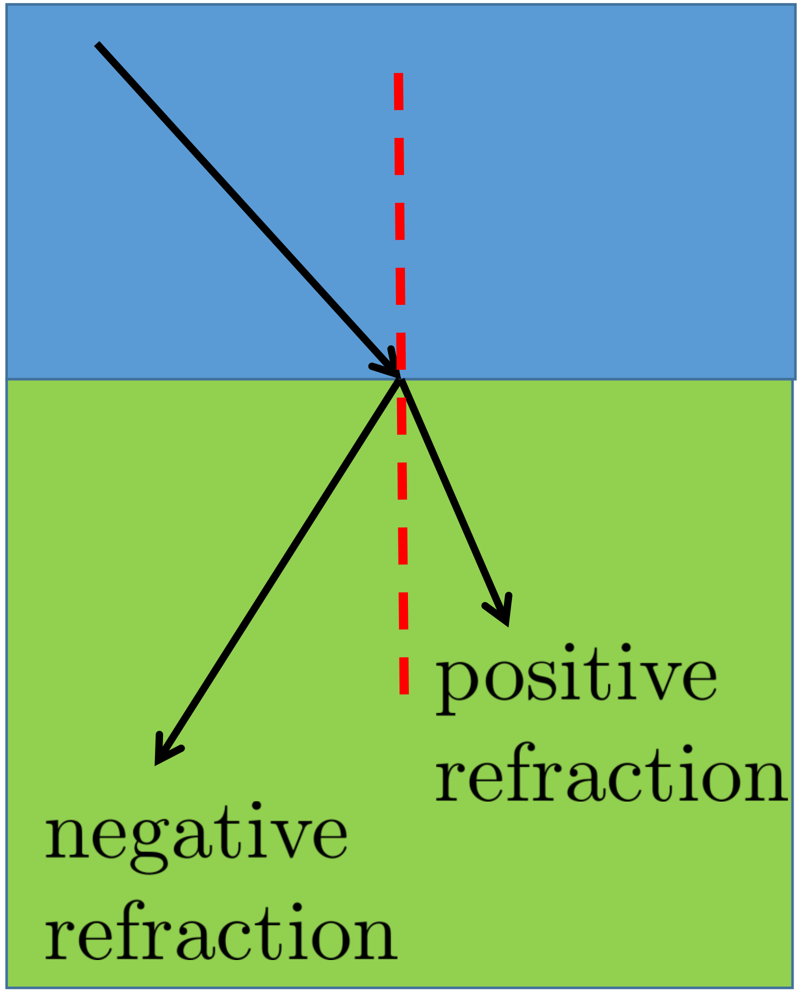


Negative refraction at optical frequencies: Valentine et. al.(2008)



Focusing beyond the diffraction limit: the superlens (Pendry, 2000)





Widespread, but incorrect………

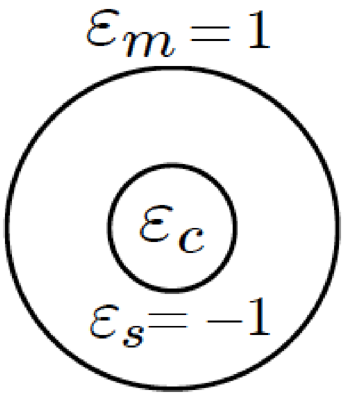
Answer to how superlenses image a point source comes from an earlier paper:

PHYSICAL REVIEW B VOLUME 49, NUMBER 12 15 MARCH 1994-II

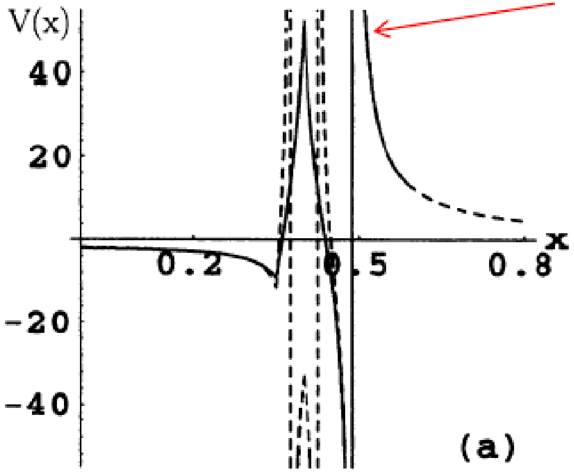
Optical and dielectric properties of partially resonant composites

N. A. Nicorovici and R. C. McPhedran
Department of Theoretical Physics, School of Physics, University of Sydney, Sydney, New South Wales 2006, Australia

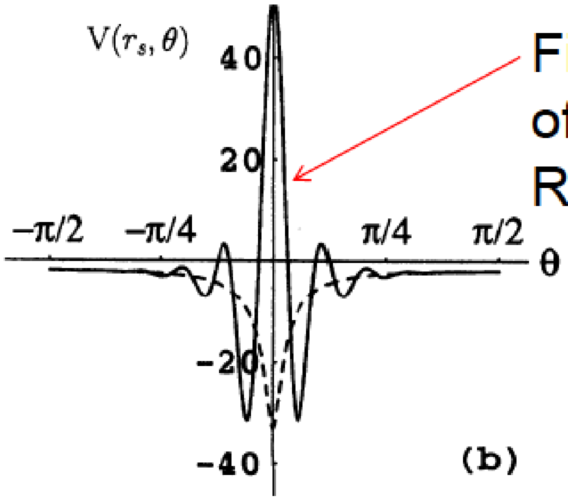
G. W. Milton*
Department of Mathematics, University of Utah, Salt Lake City, Utah 84112
(Received 2 November 1993)



First Discovery of a Ghost Source

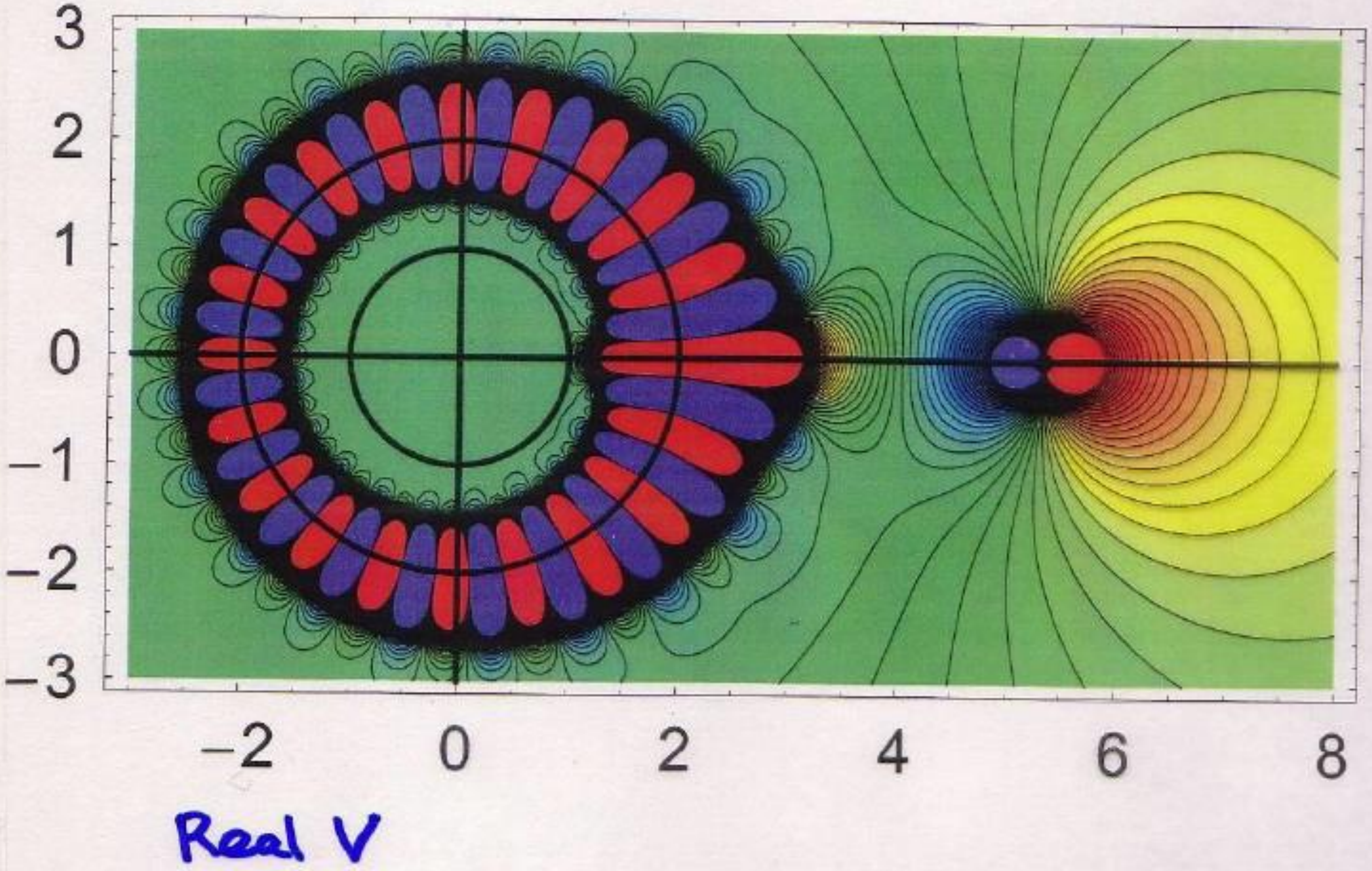


(Shell radius=0.4, Core radius=0.35)



First Discovery of Anomalous Resonance

Ghost sources and anomalous resonance are the essential mechanisms that explain superlensing.

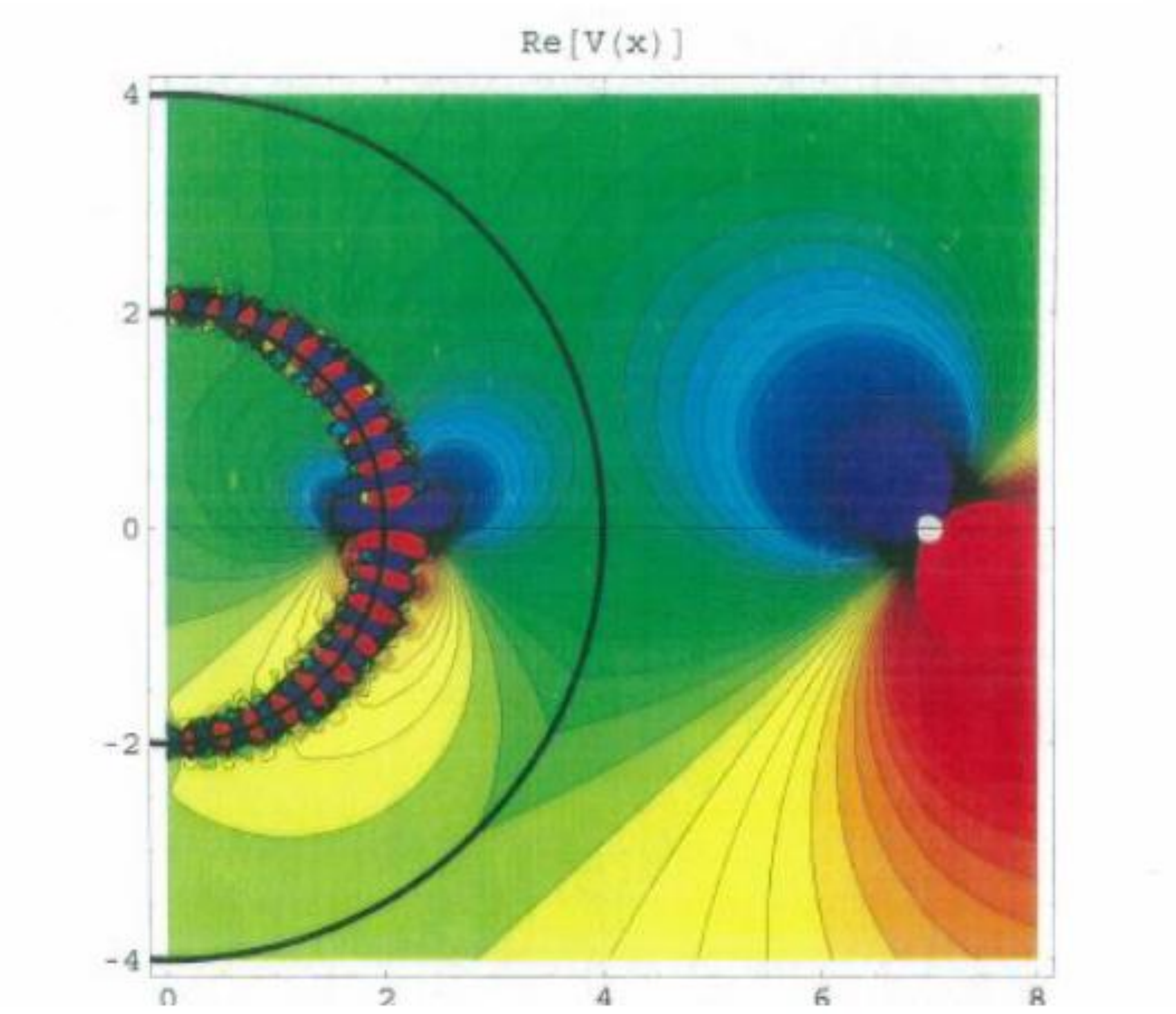


$\varepsilon_c = 100$

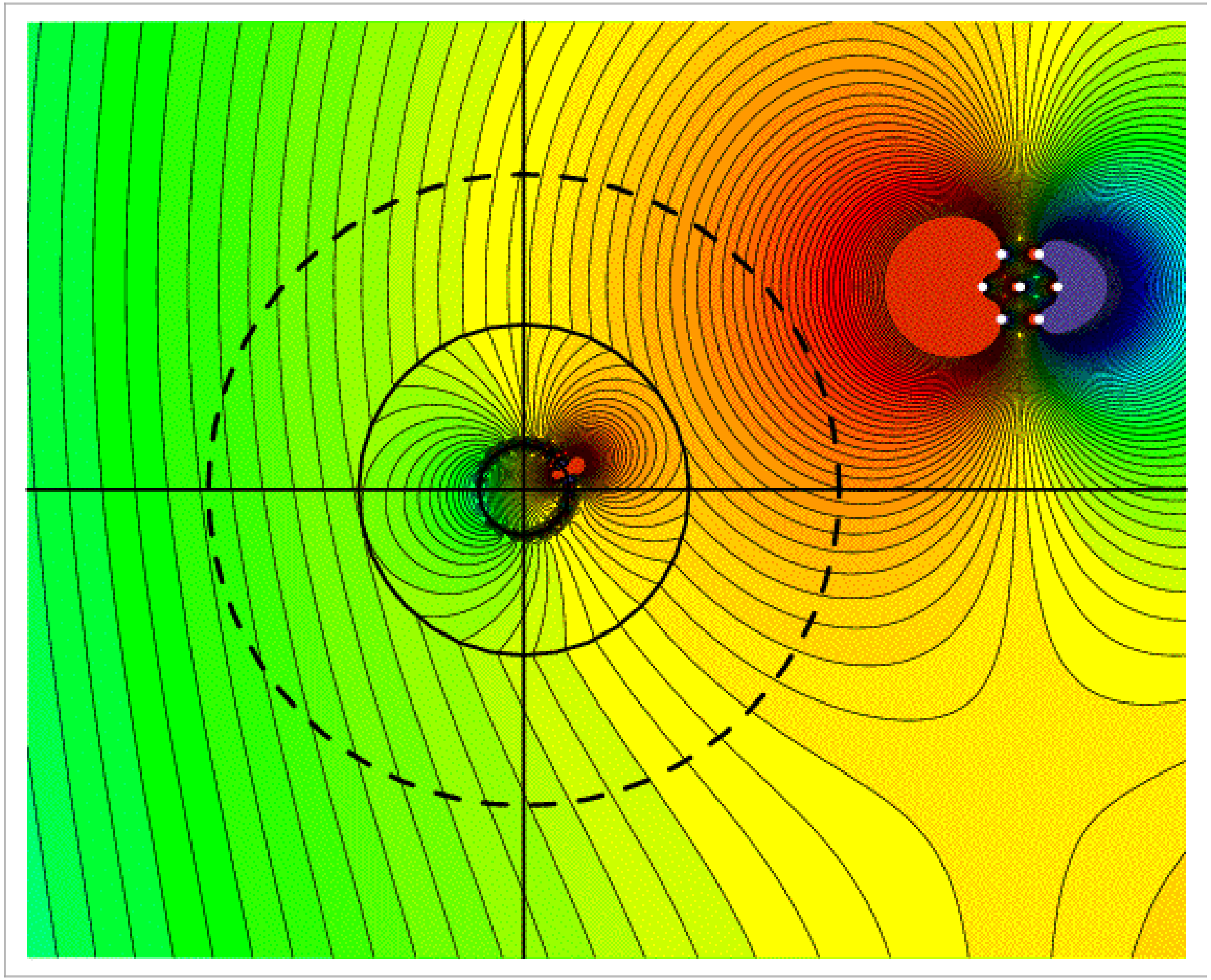
$\varepsilon_s = -1 + 10^{-12} i$

$\varepsilon_m = 1$

Later Simulation



When the shell was hollow we found it was completely invisible to any applied field

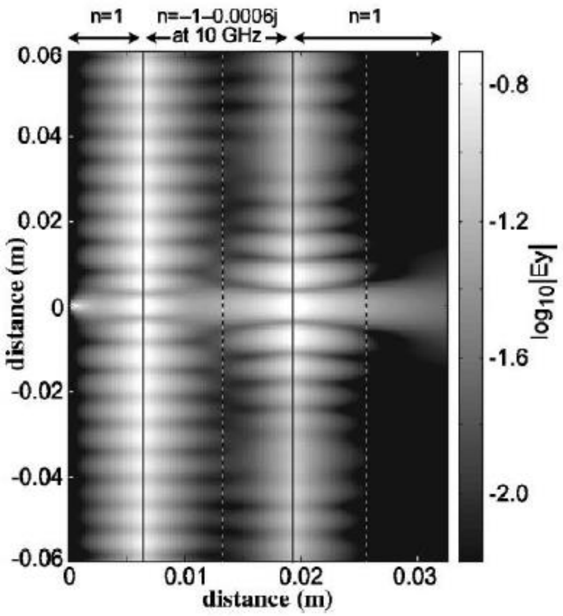


Cloaking due to anomalous resonance
With Botten, McPhedran
Nicorovici 2006,2007

Many other works
in particular
by Hoai Minh Nguyen

Similarly for the "perfect lens" there are anomalously resonant regions:

Work by Garcia and Nieto-Vesperinas (2002) and Pokrovsky and Efros (2002) indicated large fields between the ghost sources.



Correct Picture

Numerical Results of Cummer (2003) showing the anomalously resonant regions on both sides of the lens

In fact instead of getting perfect transmission sometimes the transmission is zero! The lens "cloaks" the dipole energy source if it is close enough to the lens.

An important parallel:

Maxwell's Equations:

$$\frac{\partial}{\partial x_i}\left(C_{ijk\ell}\frac{\partial E_\ell}{\partial x_k}\right) = \{\omega^2 \boldsymbol{\varepsilon} \mathbf{E}\}_j$$

$$C_{ijk\ell} = e_{ijm}e_{k\ell n}\{\boldsymbol{\mu}^{-1}\}_{mn}$$

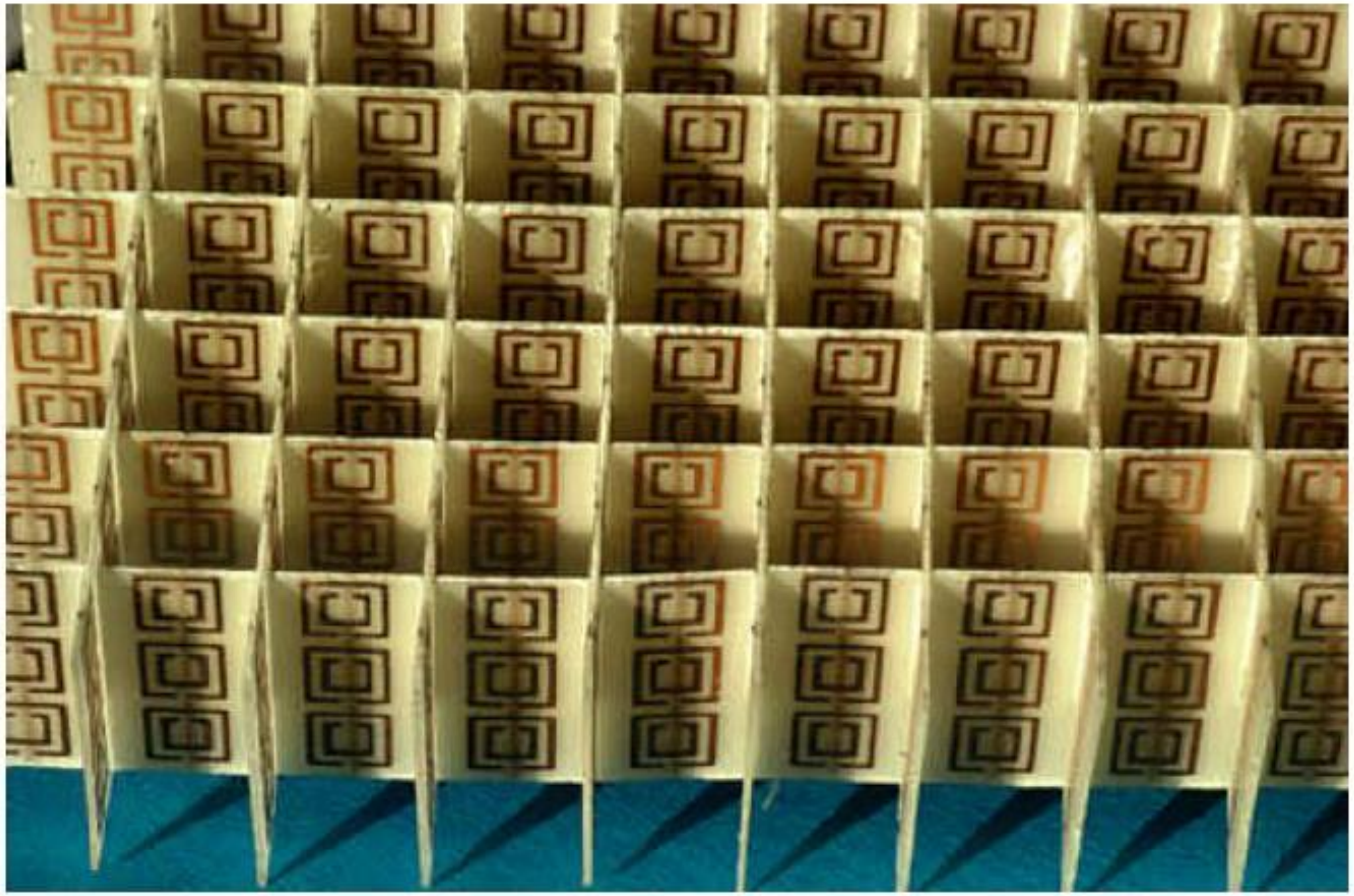
Continuum Elastodynamics:

$$\frac{\partial}{\partial x_i}\left(C_{ijk\ell}\frac{\partial u_\ell}{\partial x_k}\right) = -\{\omega^2 \boldsymbol{\rho} \mathbf{u}\}_j$$

Suggests that $\varepsilon(\omega)$ and $\boldsymbol{\rho}(\omega)$ might have similar properties

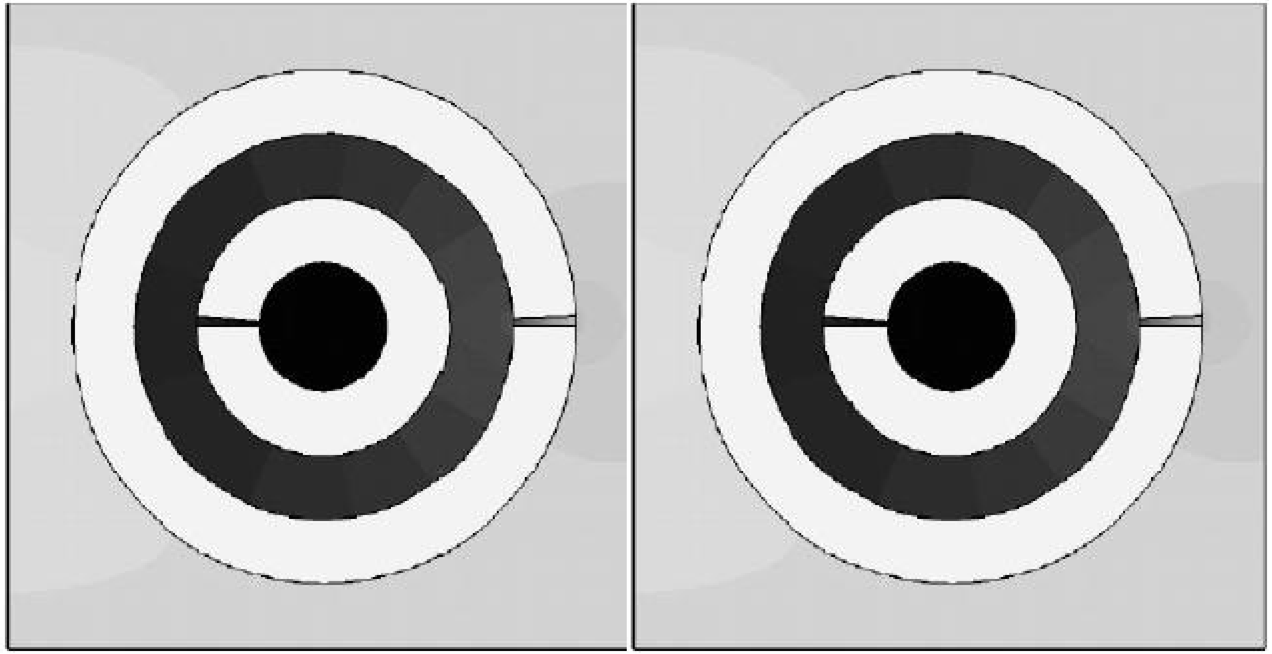
Specifically a similar dependence on frequency

There is a close connection between negative density and negative magnetic permeability



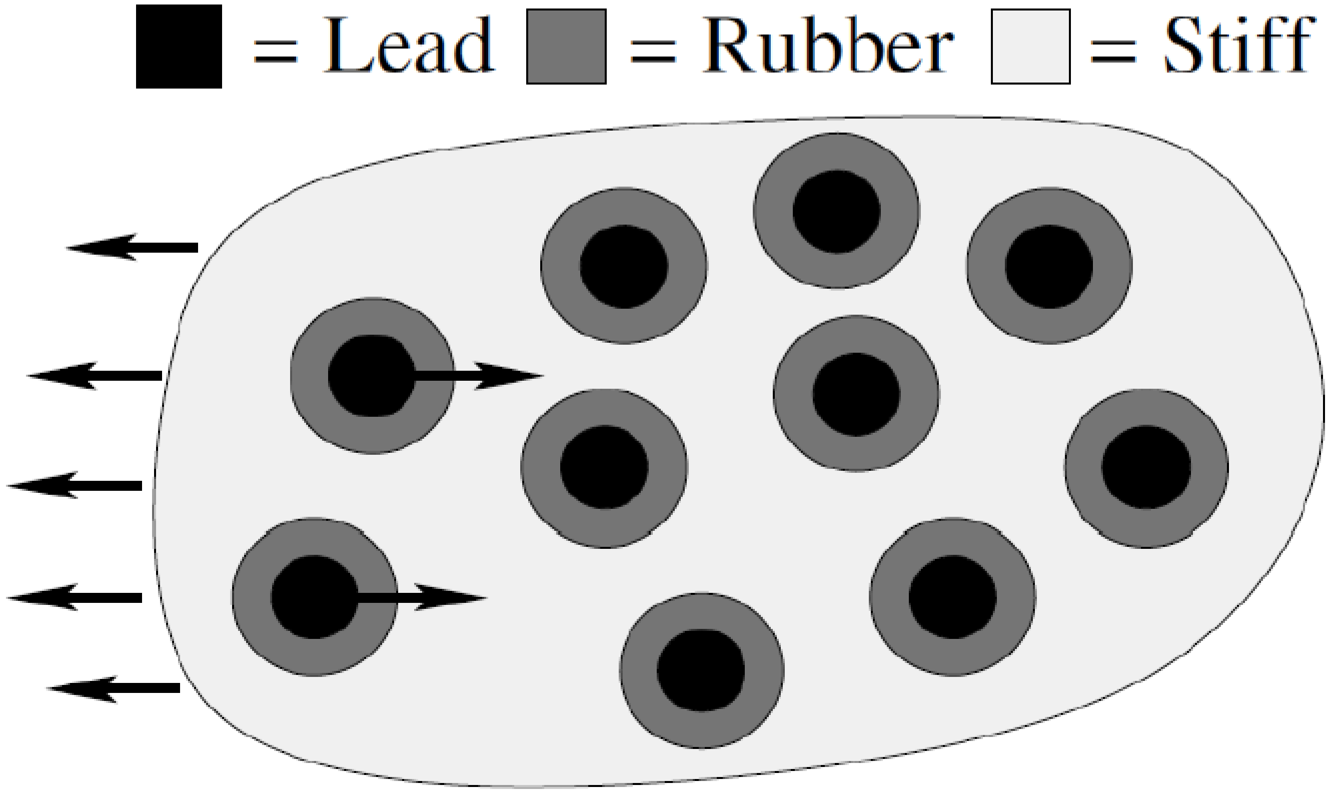
Split ring structure of David Smith

In two dimensions the Helmholtz equation describes both antiplane elastodynamics and TE (or TM) electrodynamics

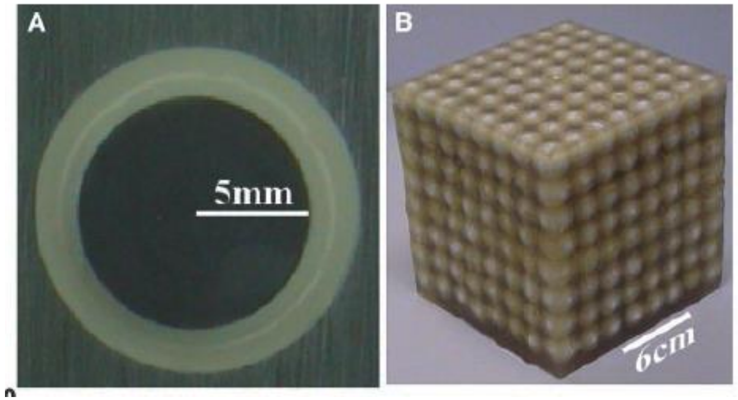


Split ring resonantor structure behaves as an acoustic band gap material (Movchan and Guenneau, 2004)

Sheng, Zhang, Liu, and Chan (2003) found that materials could exhibit a negative effective density over a range of frequencies



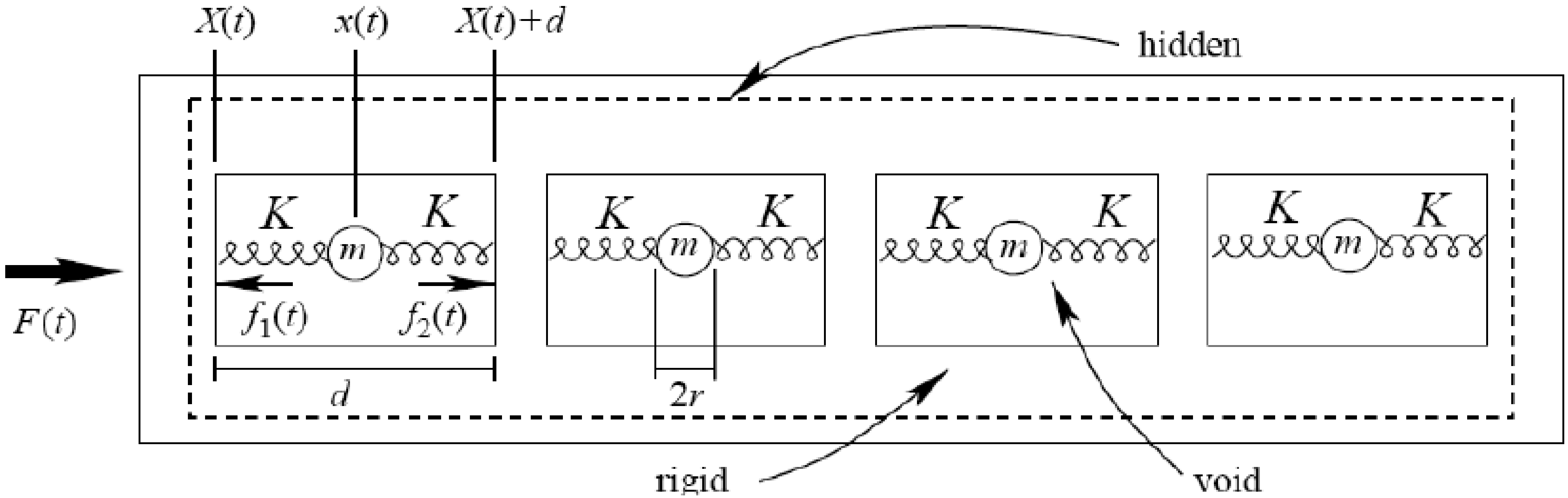
Experiment: Liu et. al (2000)



High Contrast homogenization

Mathematically the observation goes back to Zhikov (2000) also Bouchitte & Felbacq (2004)

A simplified one-dimensional model:



$$\hat{P} = M\hat{V}, \quad \text{with} \quad M = M_0 + \frac{2Knm}{2K - m\omega^2},$$

(With John Willis). Note: the displacement must be small compared to the cell size

Early work recognizing anisotropic and negative effective densities:

Auriault and Bonnet (1985)

Le comportement macroscopique est celui d'un corps élastique monophasique de masse volumique ρ à caractére tensoriel et dépendante de la pulsation ω:

$$\rho = \langle \varrho_1 \rangle + \langle g(\omega) \rangle \varrho_2 .$$

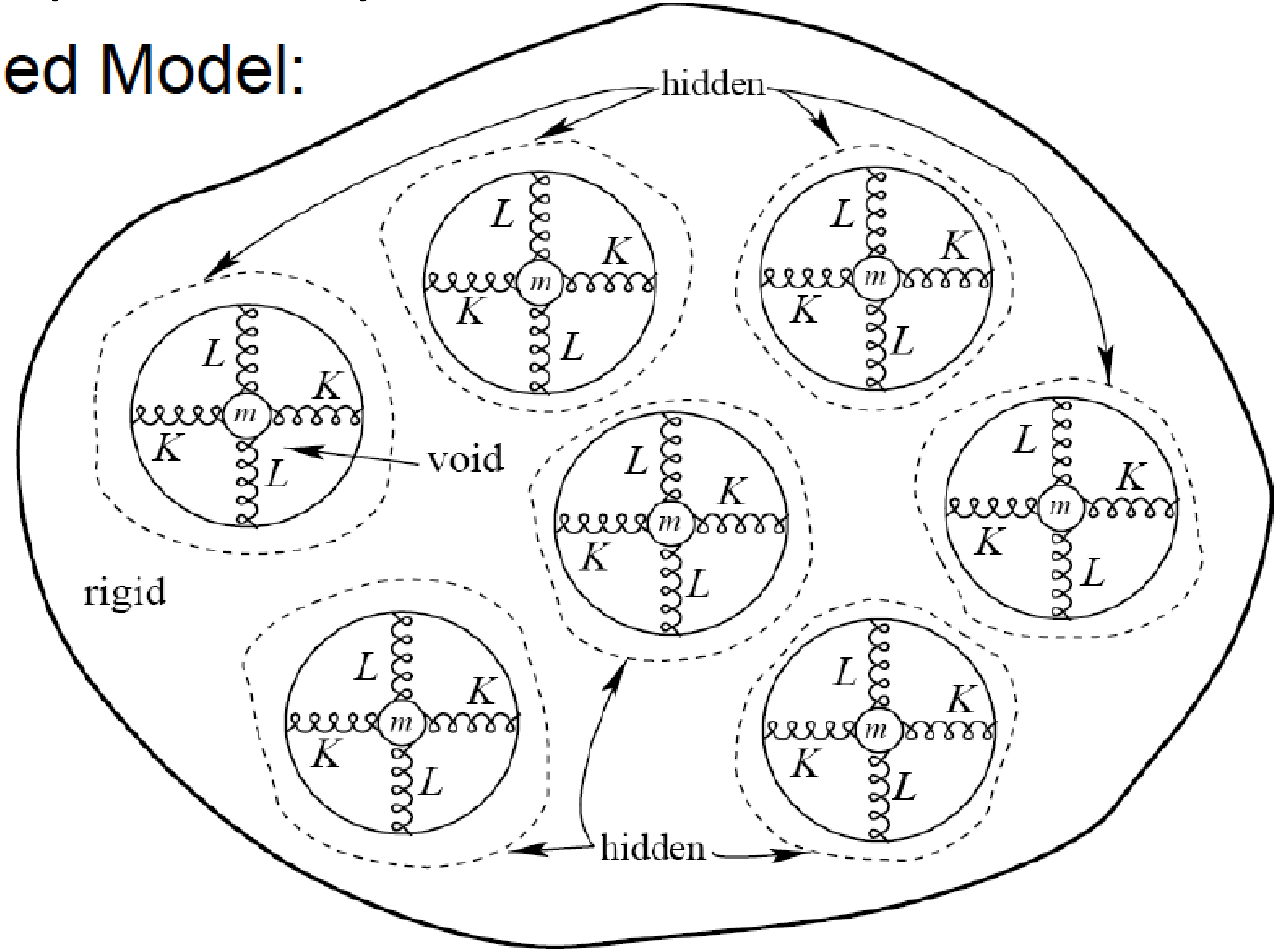
And in english (1995):

"The monochromatic macroscopic behavior is elastic, but with an effective density ϱ^{eff} of tensorial character and depending on the pulsation"

"hatched areas correspond to negative densities ϱ^{eff}, i.e., to stopping bands."

Anisotropic Density

Simplified Model:



Anisotropic density in layered materials: Schoenberg and Sen (1983)

The springs could have some damping in which case the mass will be complex

(With John Willis)

What do we learn?
For materials with microstructure, Newton's law

$$\mathbf{F} = m\mathbf{a}$$

needs to be replaced by

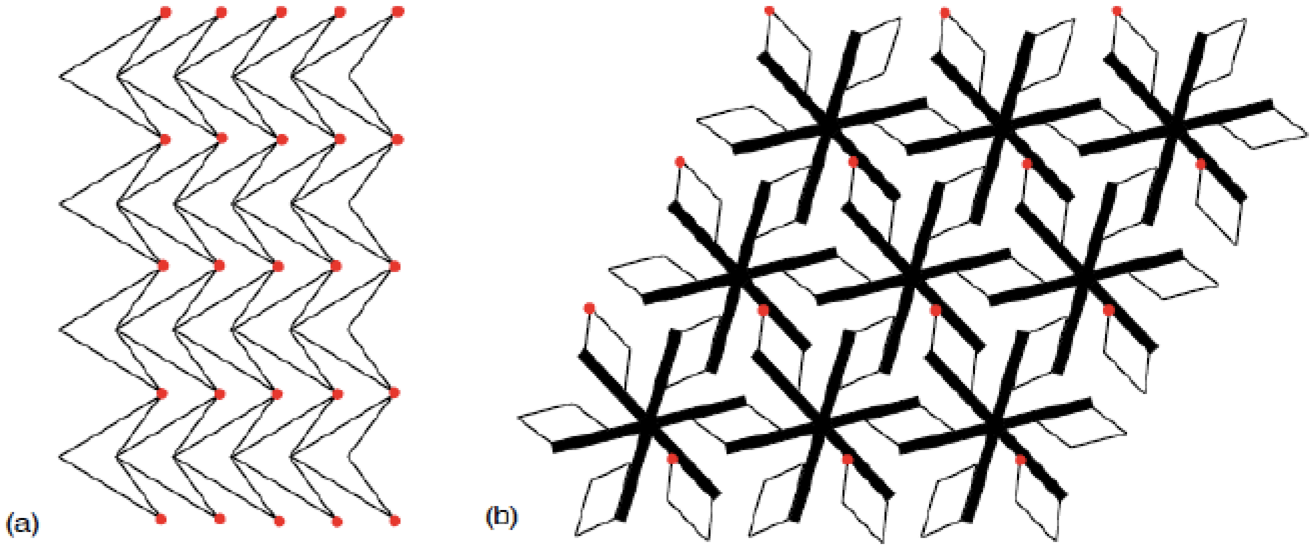
$$\mathbf{F}(t) = \int_{-\infty}^{t} \mathbf{K}(t' - t)\mathbf{a}(t')\,dt'$$

It takes some time for the internal masses to respond to the macroscopically applied force.

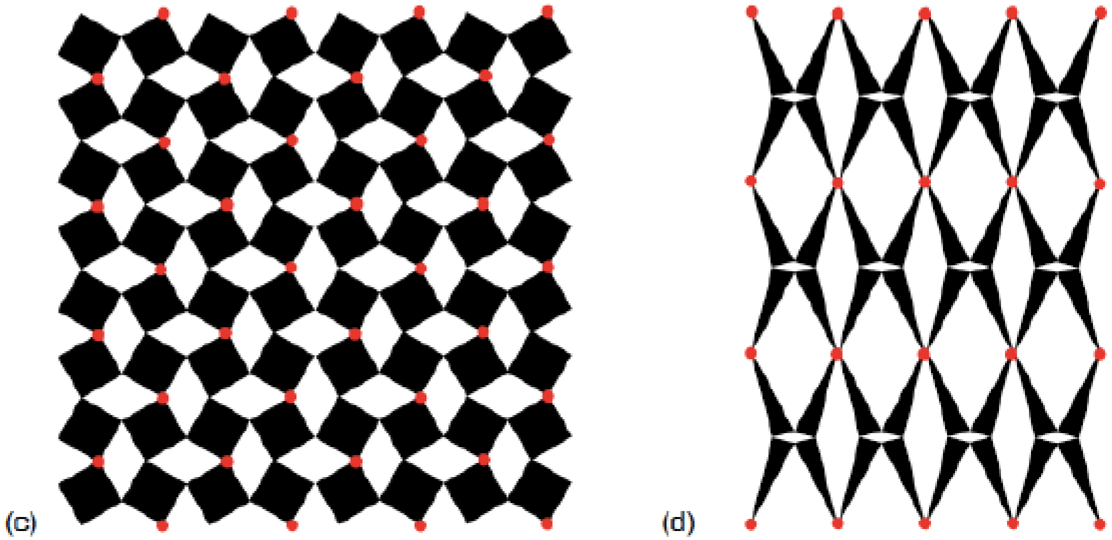
(With John Willis)

Unimode and Bimode Affine Materials

Examples of nonlinear 2d unimode materials

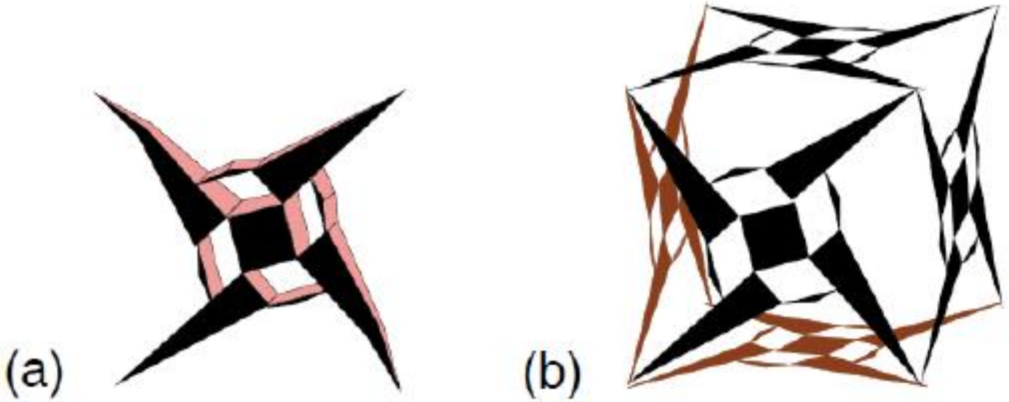


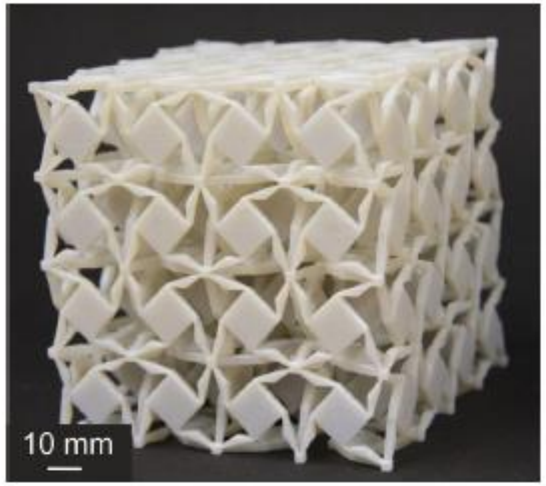
Larsen et. al.   Milton

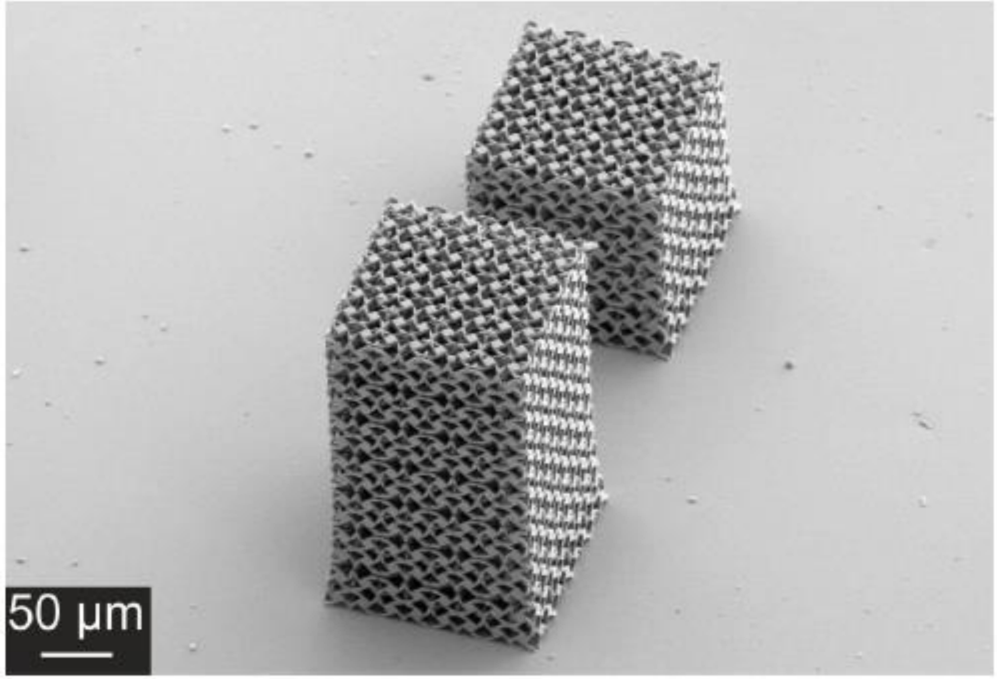


Grima and Evans

Three Dimensional Dilational materials

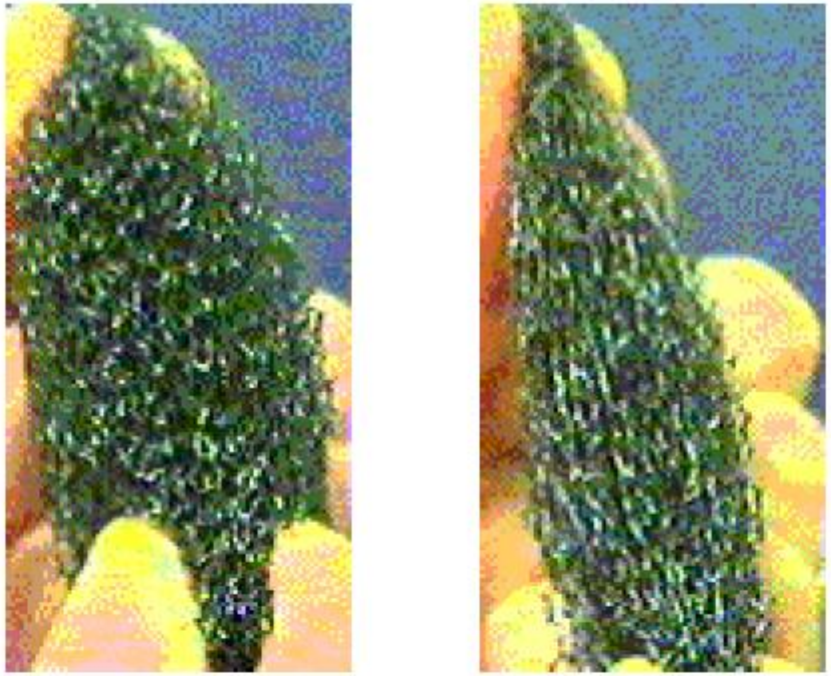




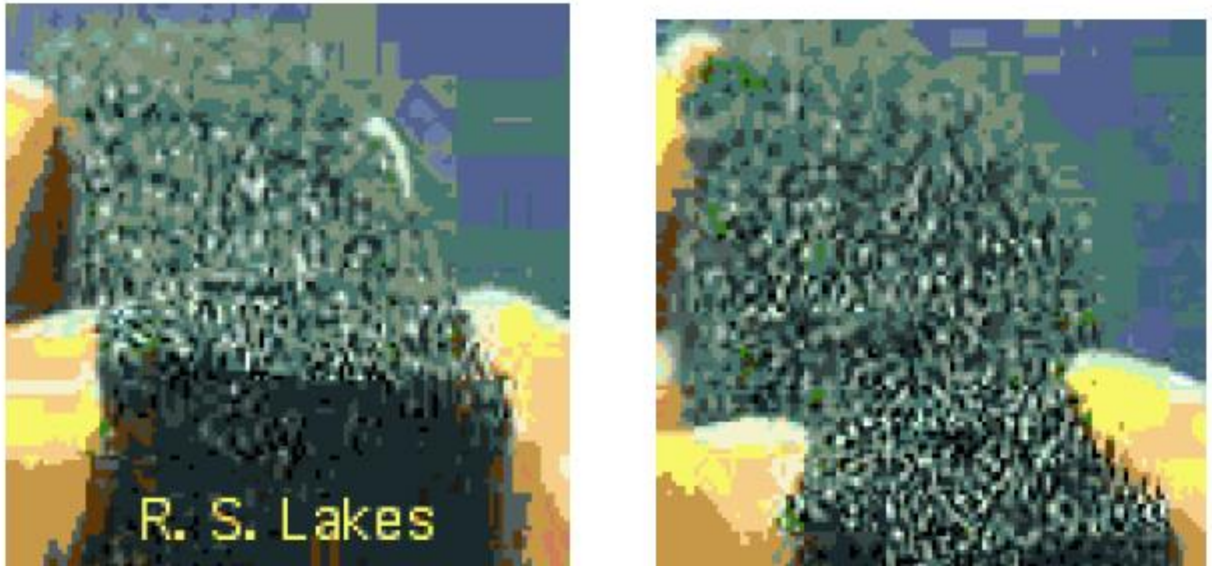


with Bückmann, Schittny, Thiel, Kadic, and Wegener (2014)

Experiment of R. Lakes (1987)

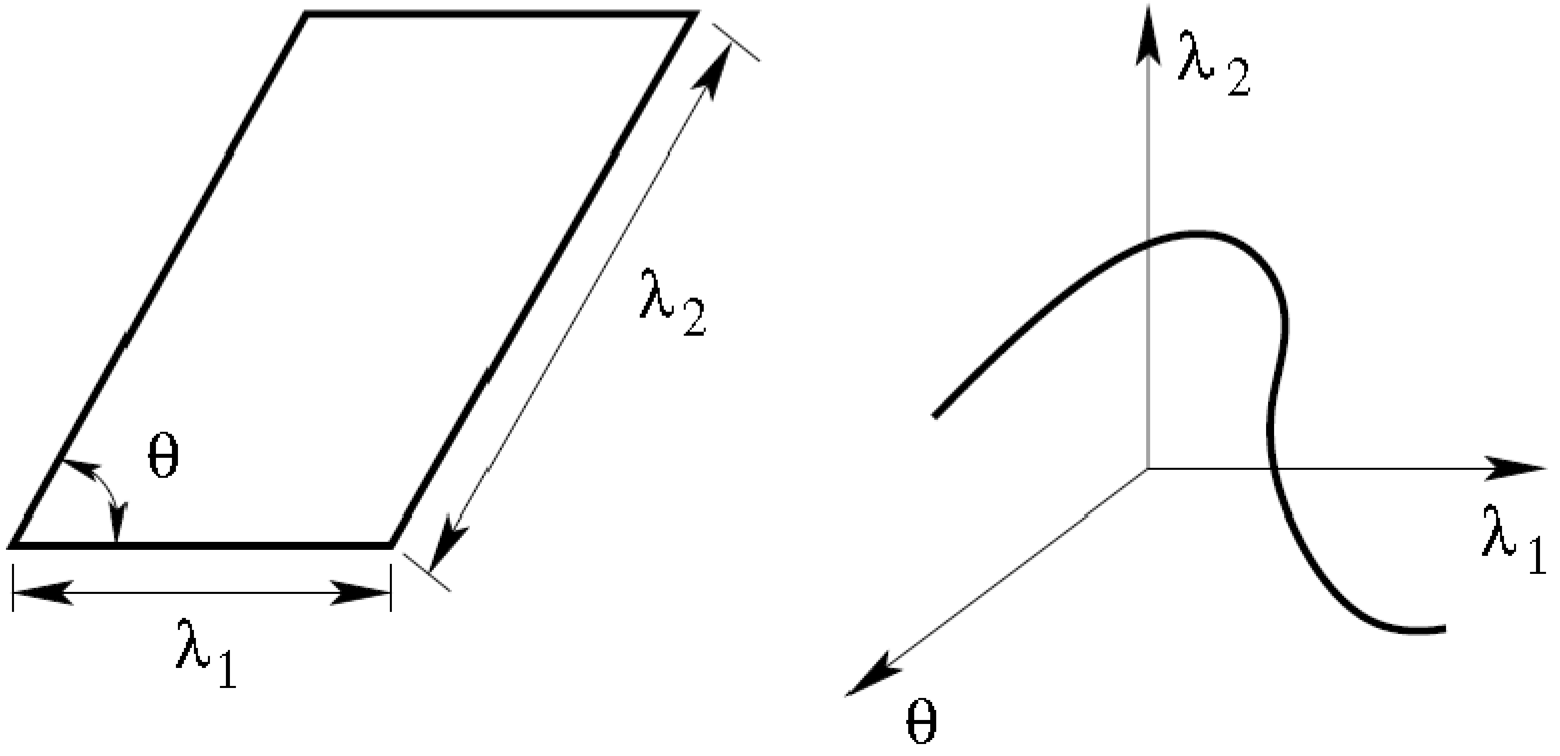


Normal Foam



These are ideal "Auxetic" materials

Unimode:

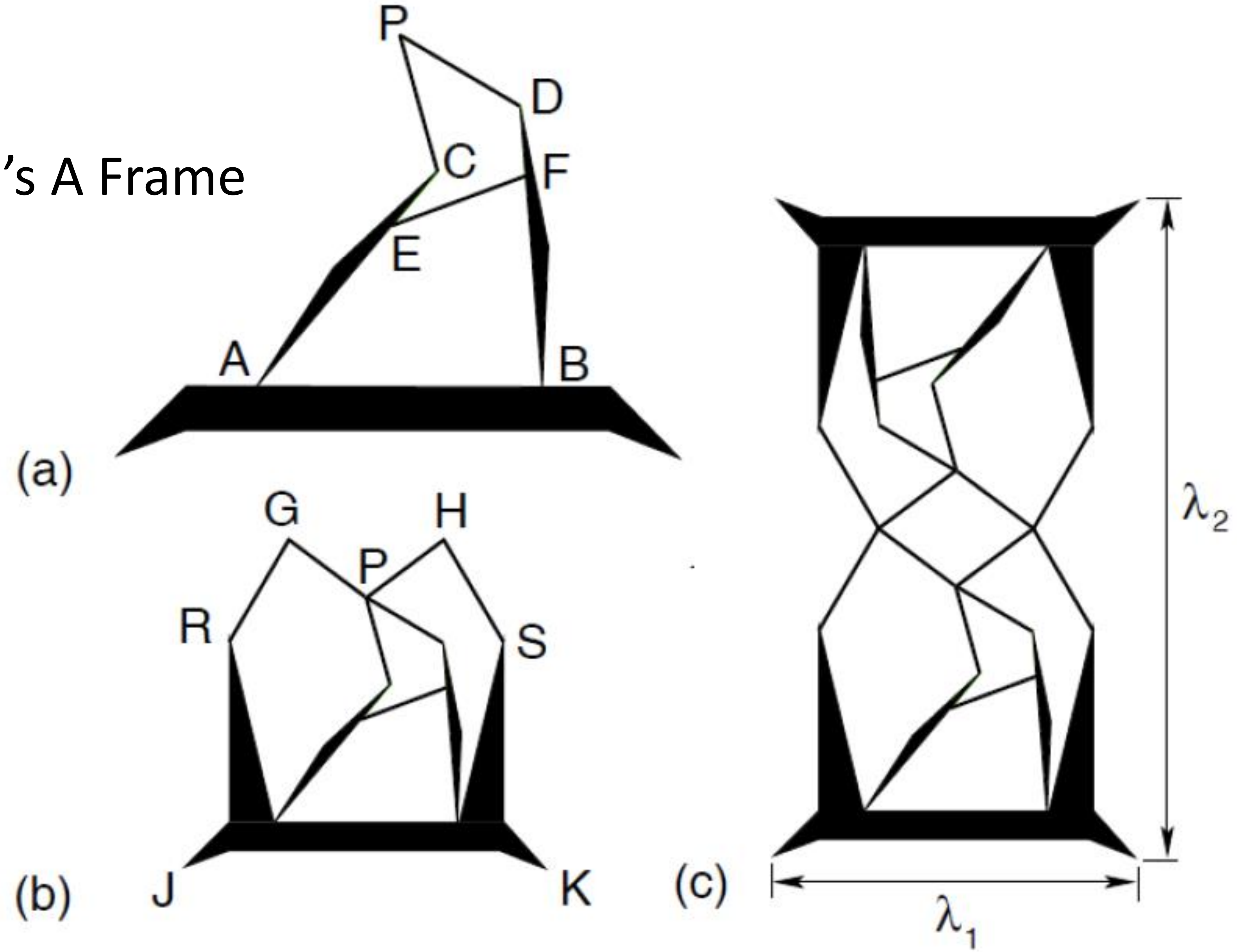


What trajectories $\lambda_1(t) = \lambda_2(t) = \theta(t)$ are realizable? (Answer: any trajectory!)

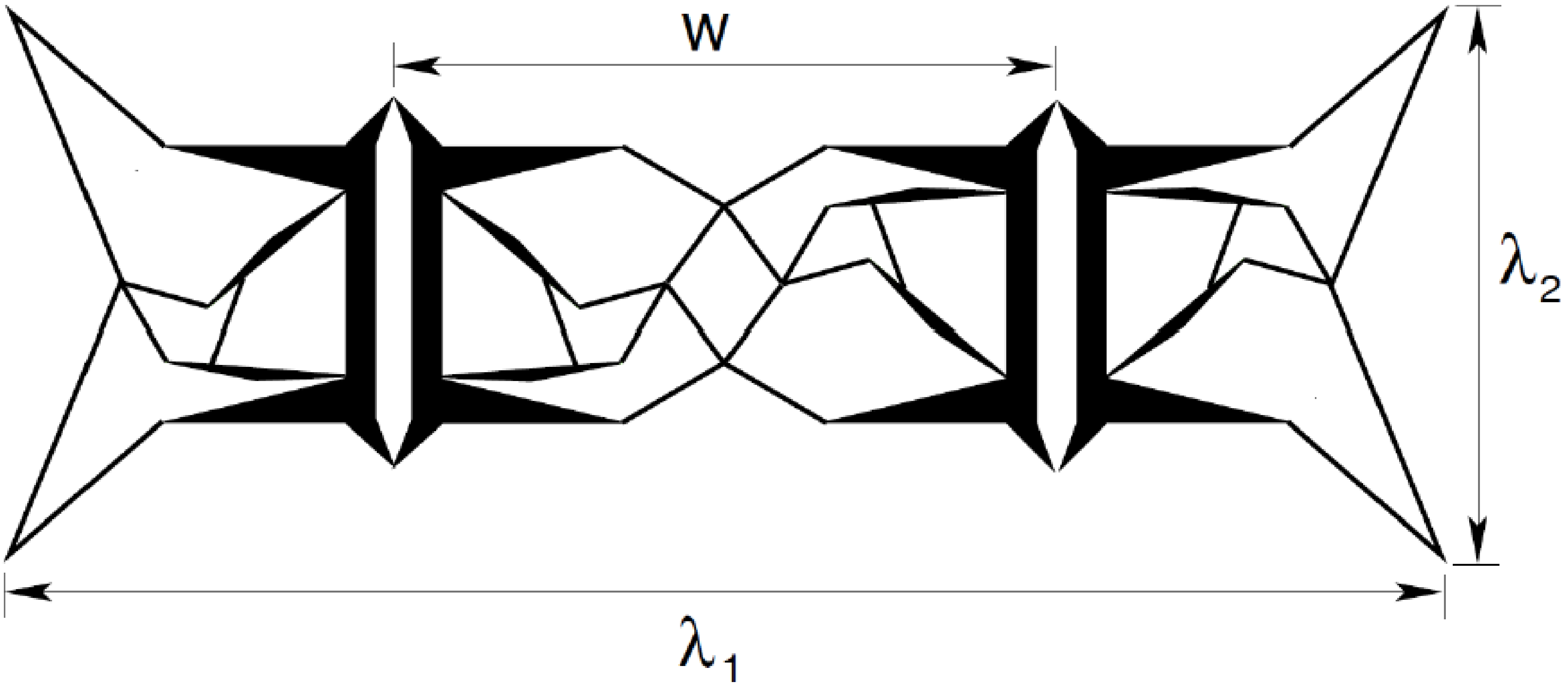
In a bimode material there is a surface of realizable motions.

Cell of the perfect expander: a unimode material

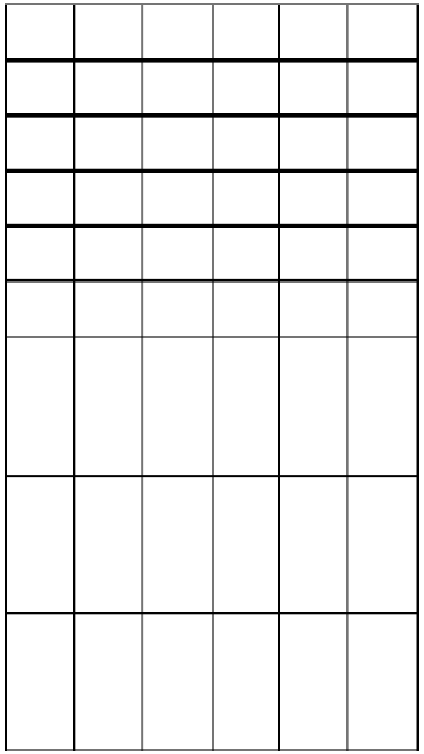
Hart’s A Frame

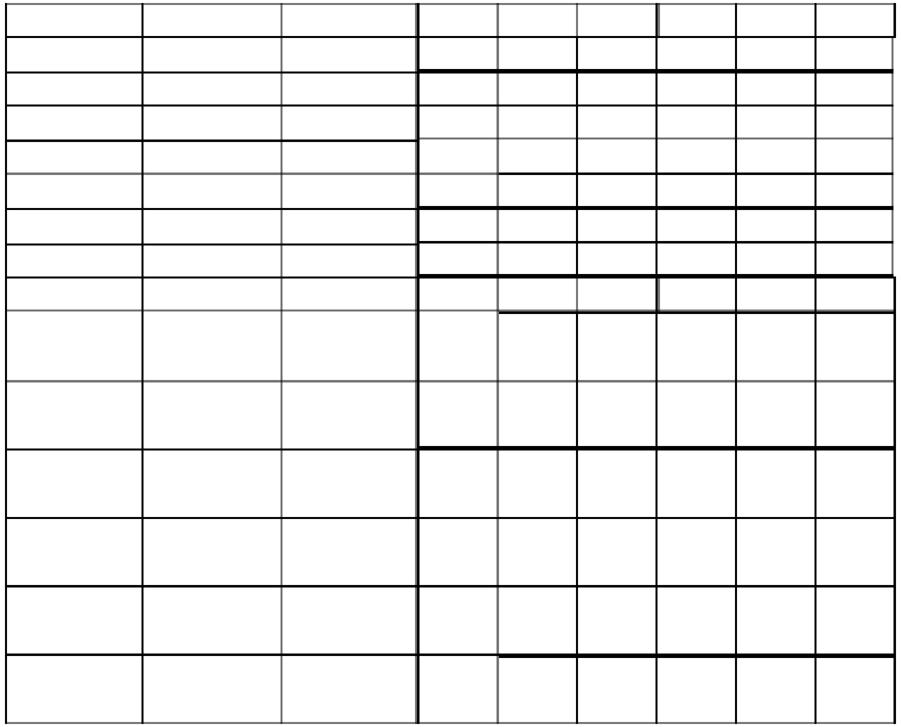


Cell of a bimode material



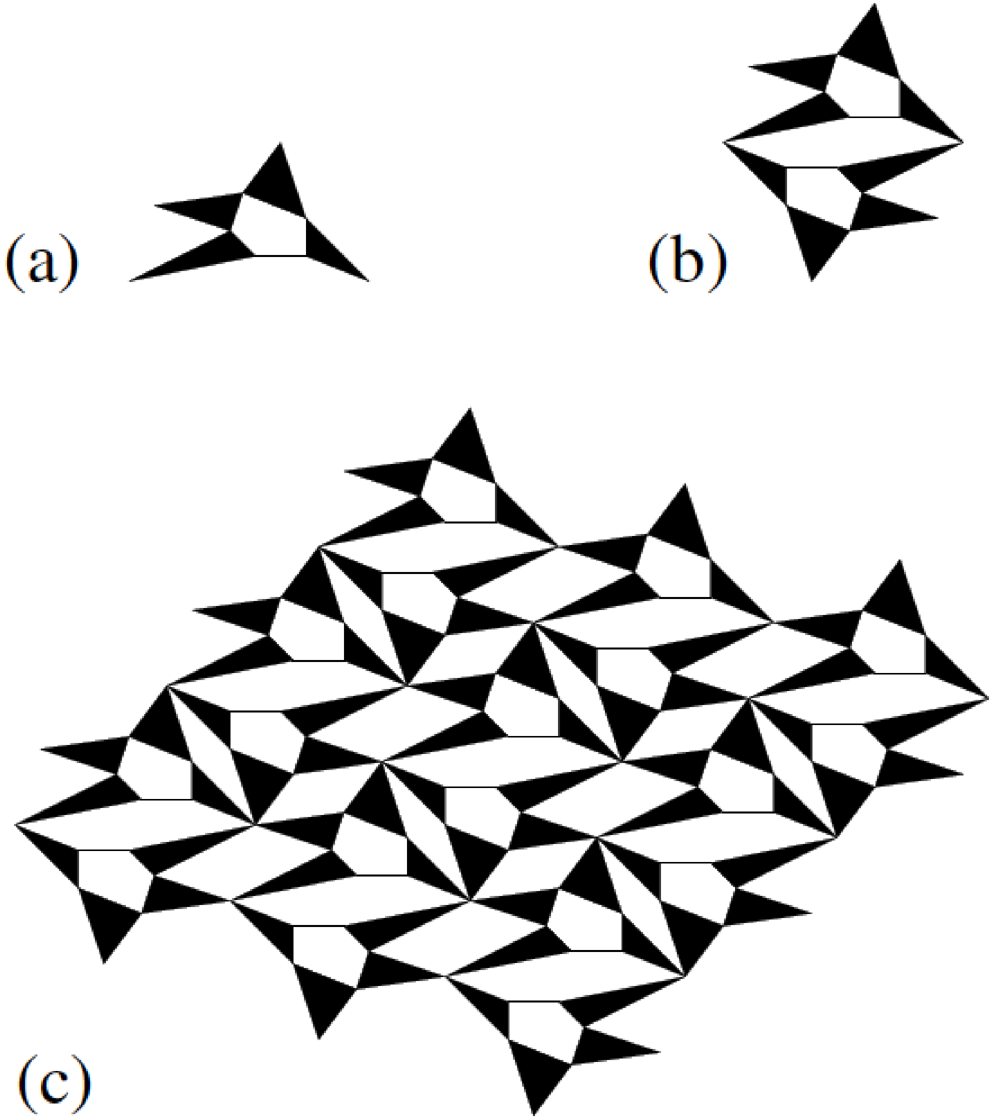
However neither are affine materials:


(a)


(b)

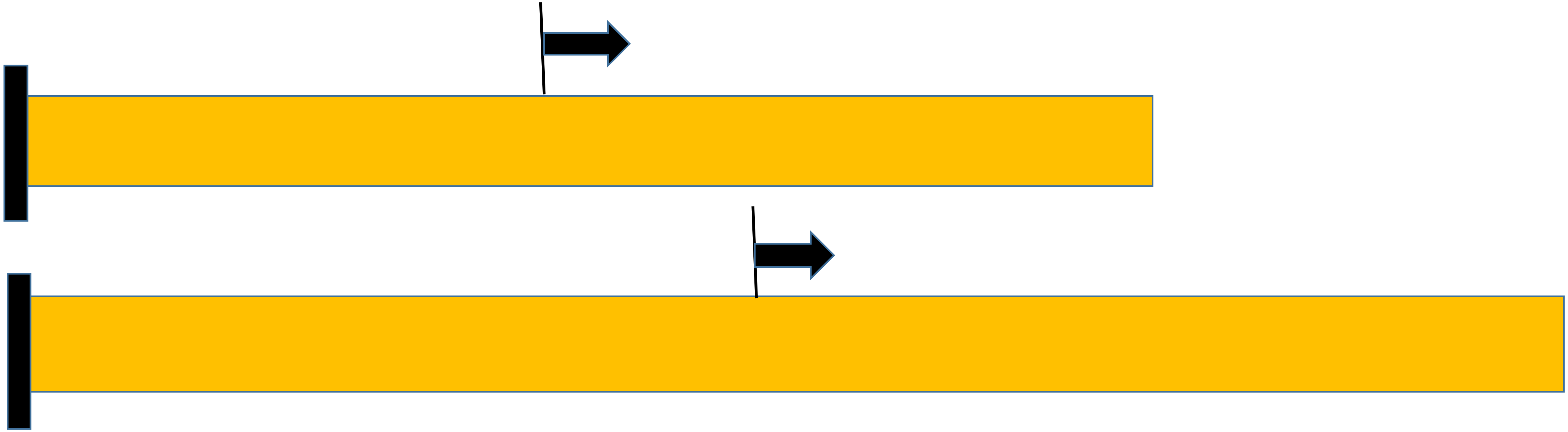
So can one get affine bimode materials?

Bimode material for which the only easy modes of deformations are affine ones

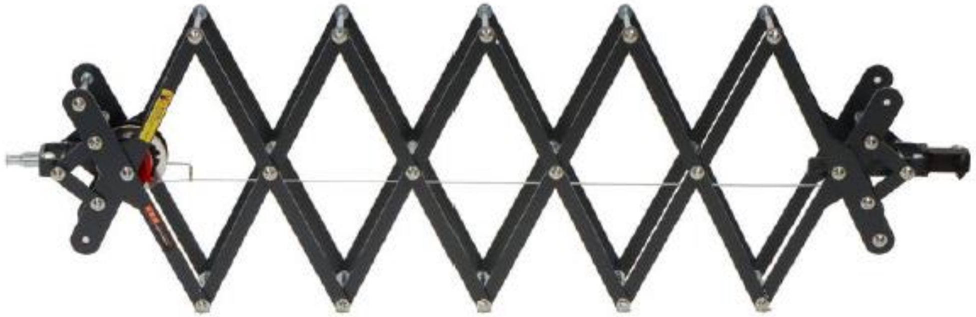


Characteristic Feature of Affine Materials:

They dislike strain gradients – elastic energy not just a function of $\nabla \mathbf{u}$



Example of Pierre Seppecher
Like a Pantograph:



Field Patterns: A new type of wave

With Ornella Mattei

Space-time microstructures

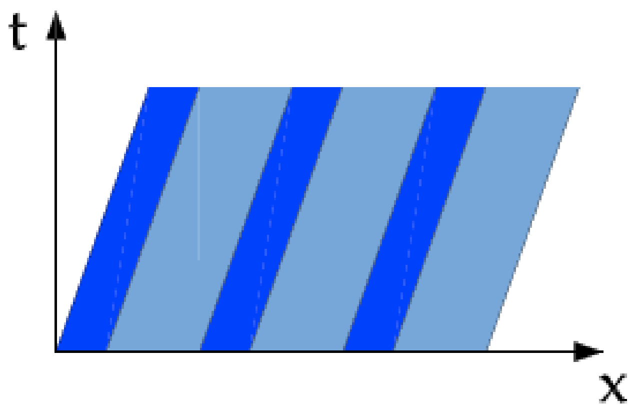
$$(a\,u_t)_t - (b\,u_x)_x = 0$$

Static materials: $a = a(x)$ and $b = b(x)$

Space-time microstructures: $a = a(x, t)$ and $b = b(x, t)$

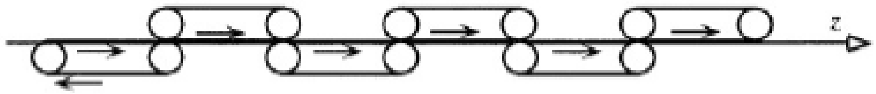
Activated materials:

The property pattern moves



Kinetic materials:

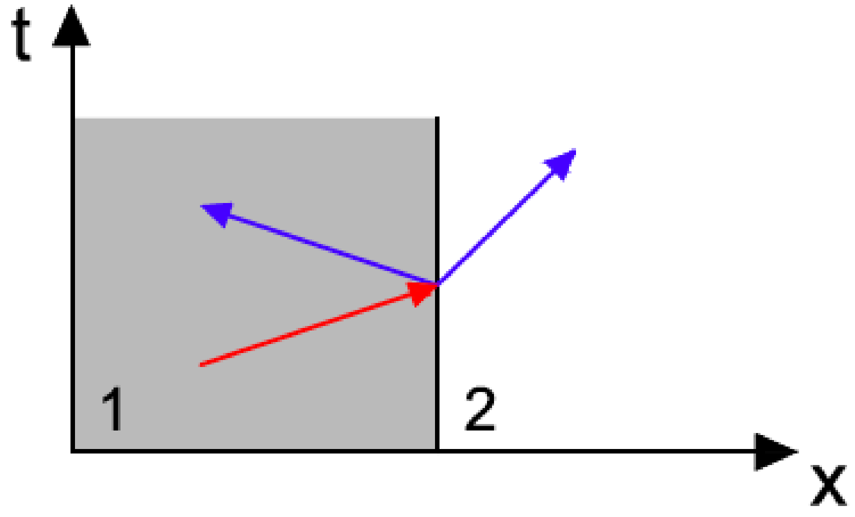
The material itself moves



[K.A. Lurie, An Introduction to the Mathematical Theory of Dynamic Materials (2007)]

Dynamic composites

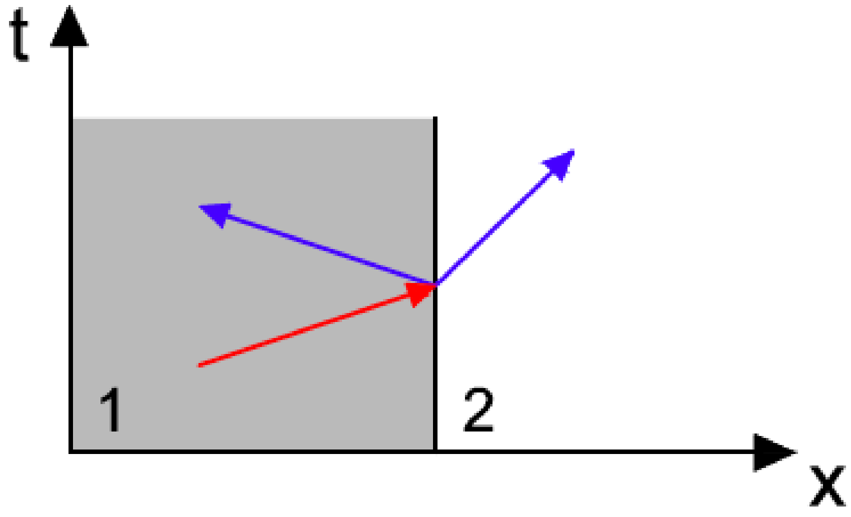
Pure space interface



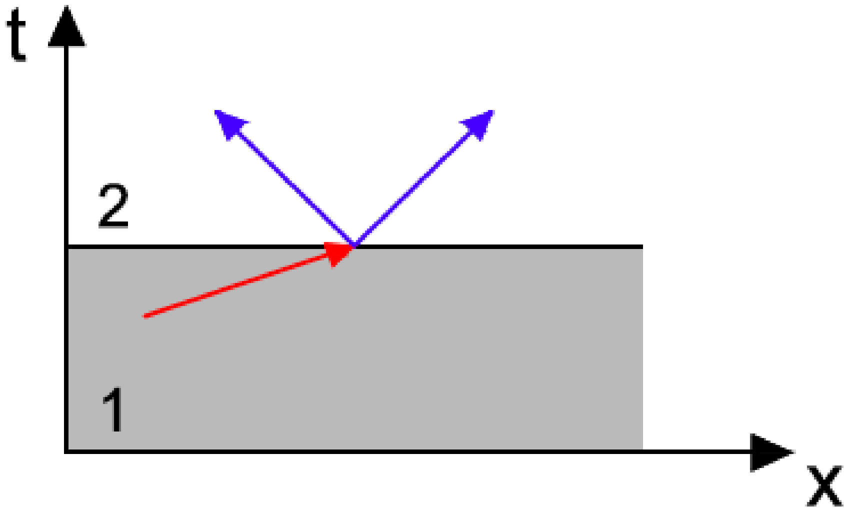
Pure time interface

Dynamic composites

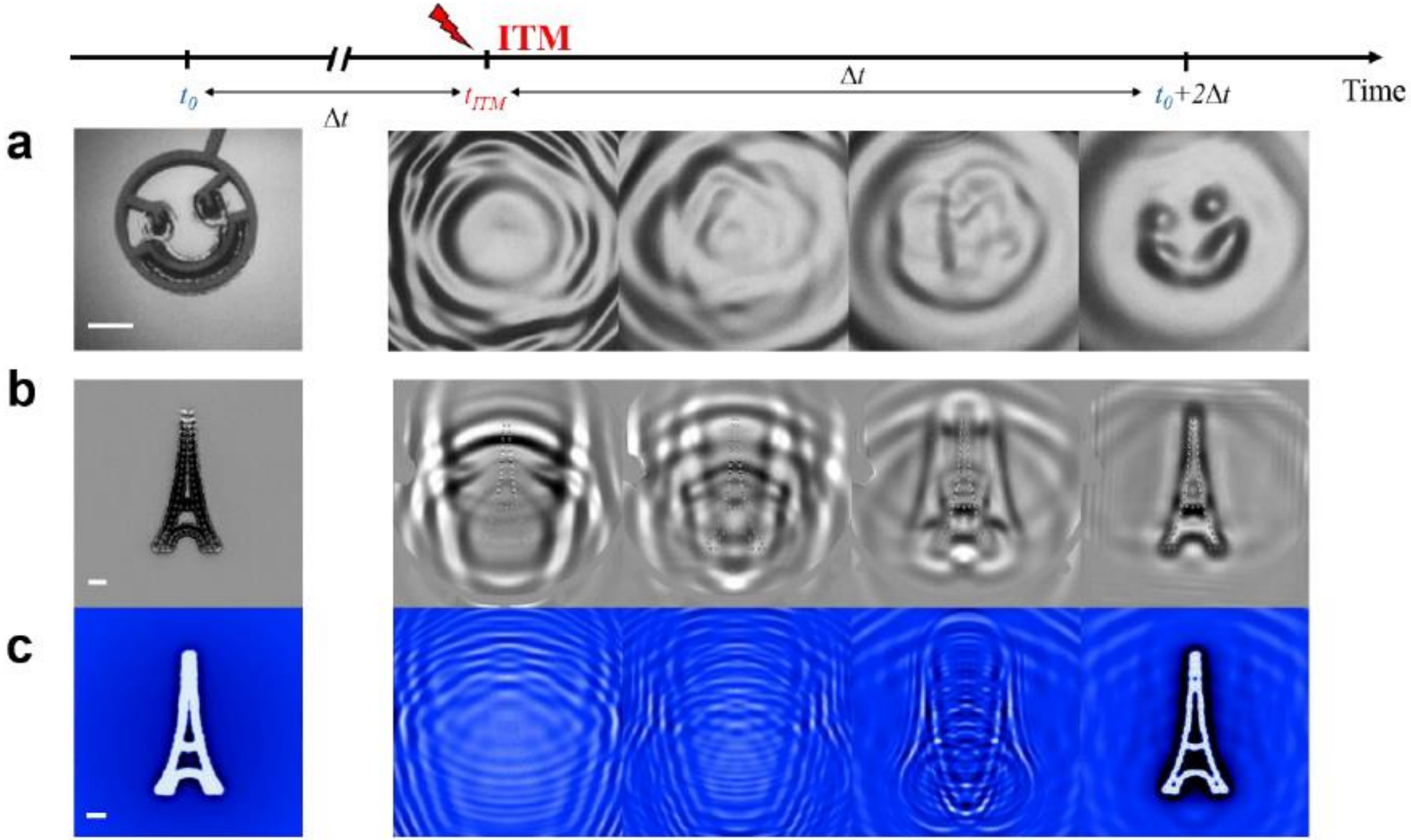
Pure space interface



Pure time interface

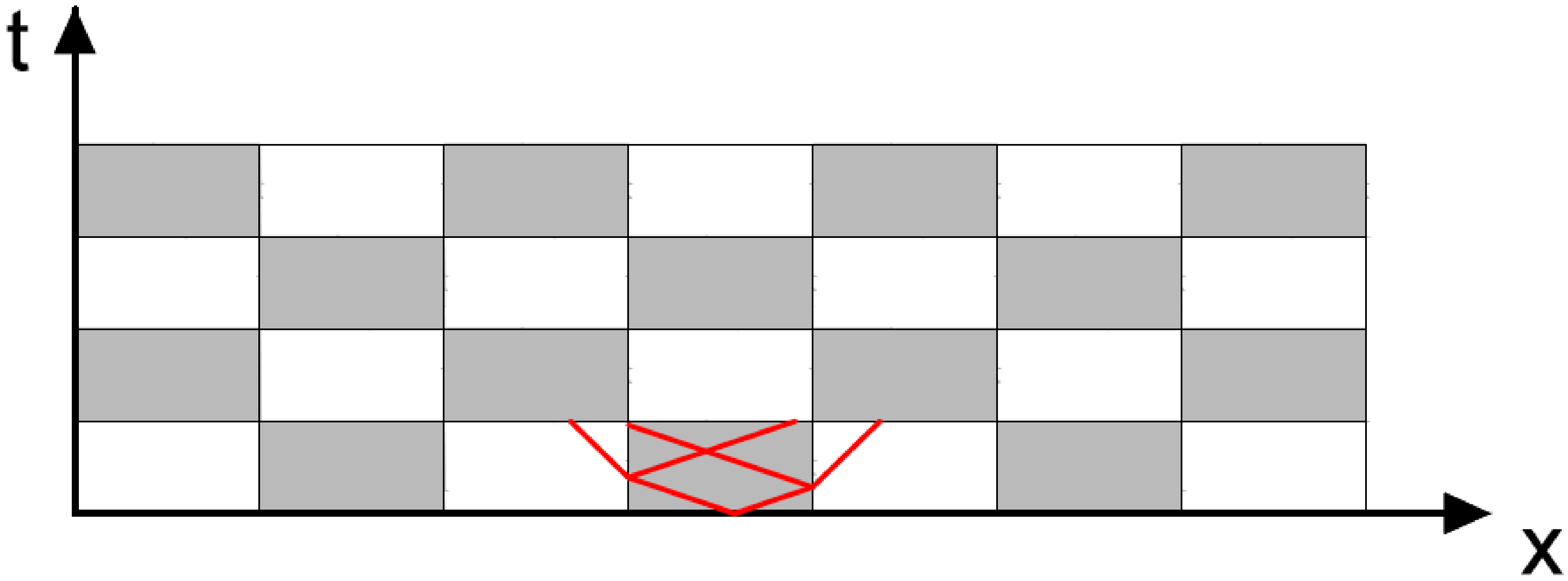


What happens at a time interface?



Bacot, Labousse, Eddi, Fink, and Fort, Nature 2016

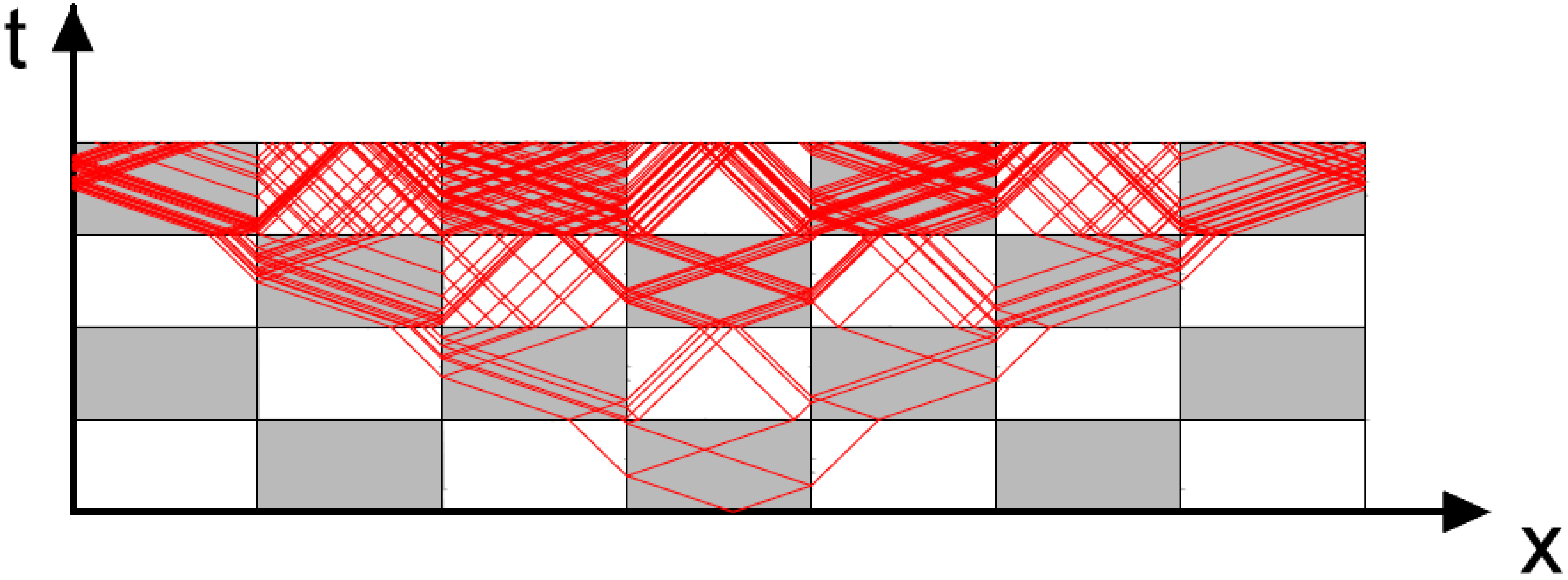
Evolution of a disturbance in a space-time checkerboard



Transmission conditions:

$$\begin{cases} V_1 = V_2 \\ \mathbf{n} \cdot \sigma_1 \nabla V_1 = \mathbf{n} \cdot \sigma_2 \nabla V_2 \end{cases}$$

Evolution of a disturbance in a space-time checkerboard

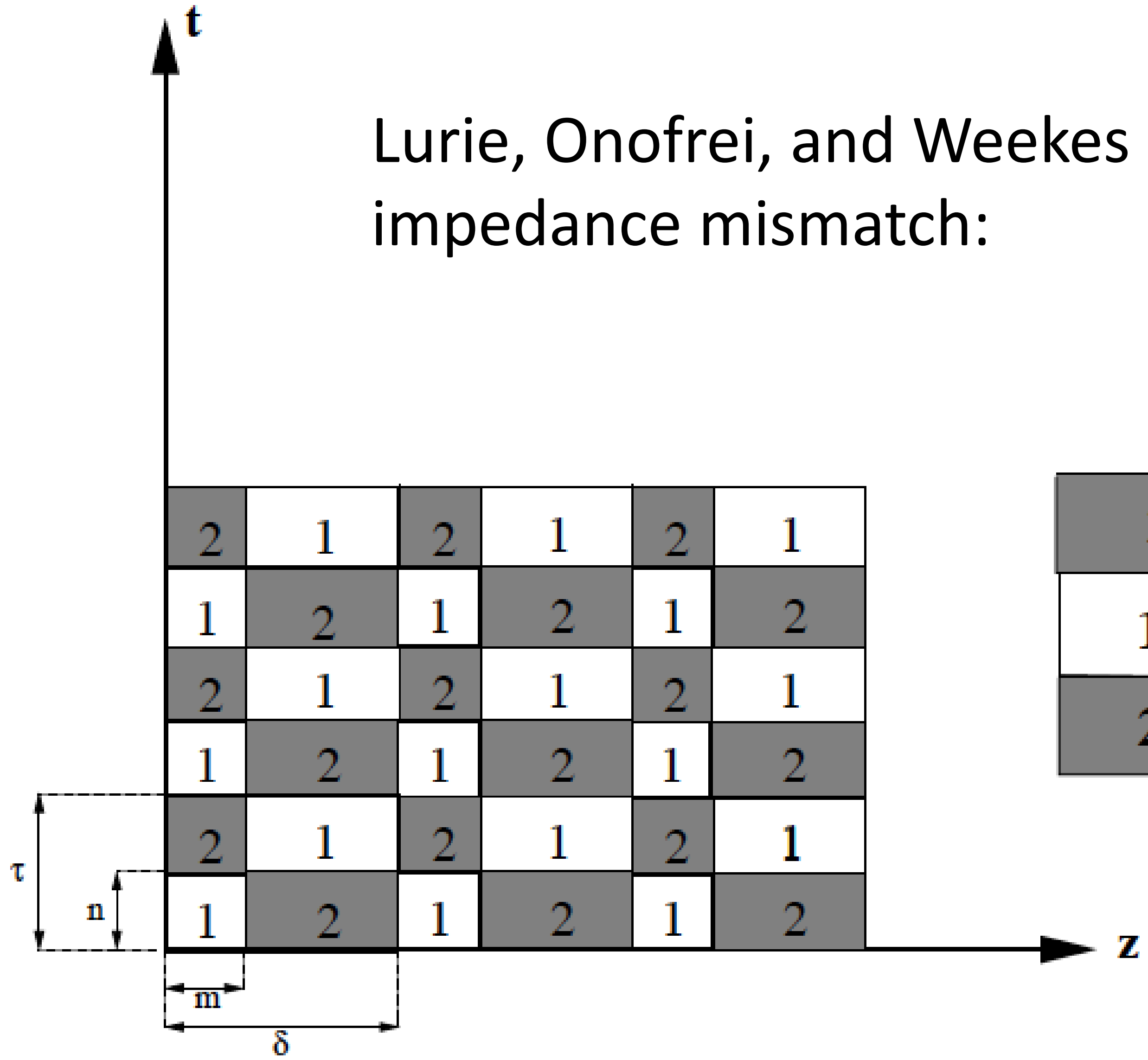


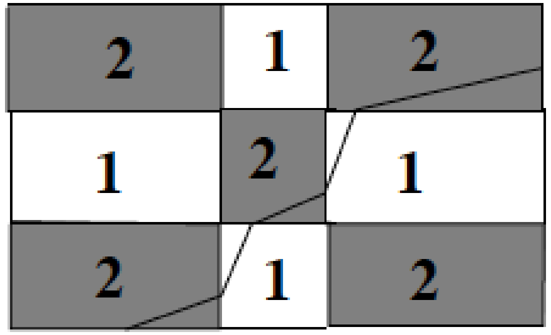
Transmission conditions:

$$\begin{cases} V_1 = V_2 \\ \mathbf{n} \cdot \boldsymbol{\sigma}_1 \nabla V_1 = \mathbf{n} \cdot \boldsymbol{\sigma}_2 \nabla V_2 \end{cases}$$

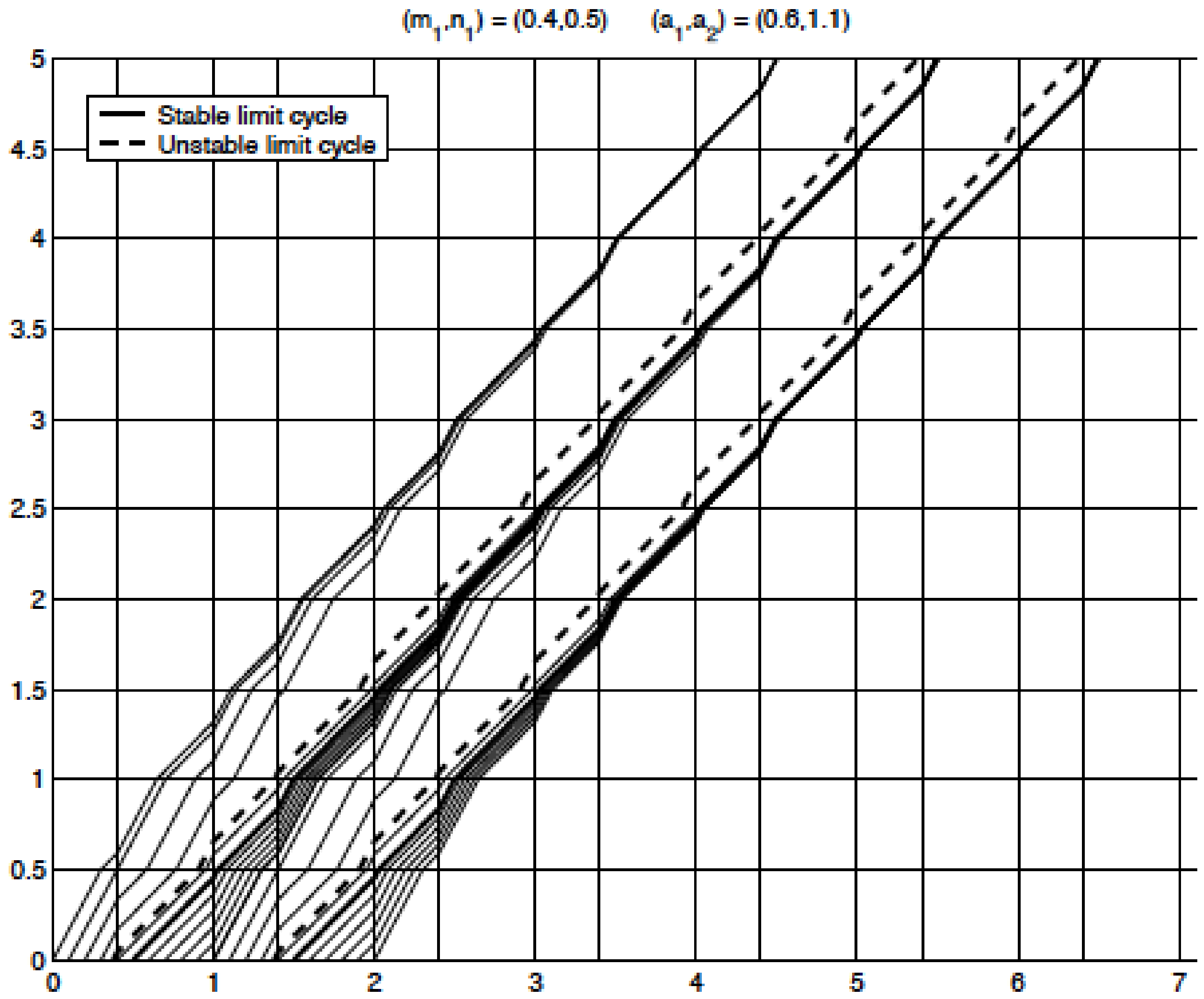
How to avoid this complicated cascade?

Lurie, Onofrei, and Weekes (2009) suggested having a zero impedance mismatch:





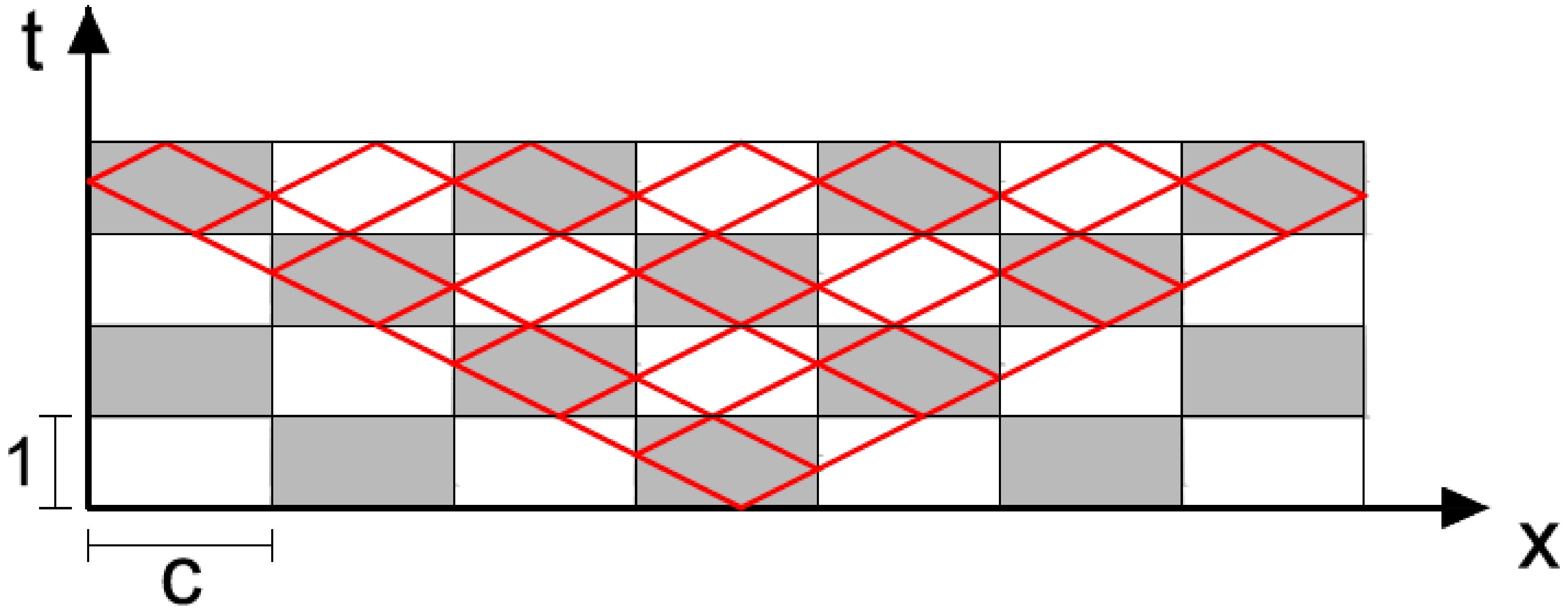
Curiously they found accumulations of the characteristic lines:



A bit like a shock but in a linear medium!

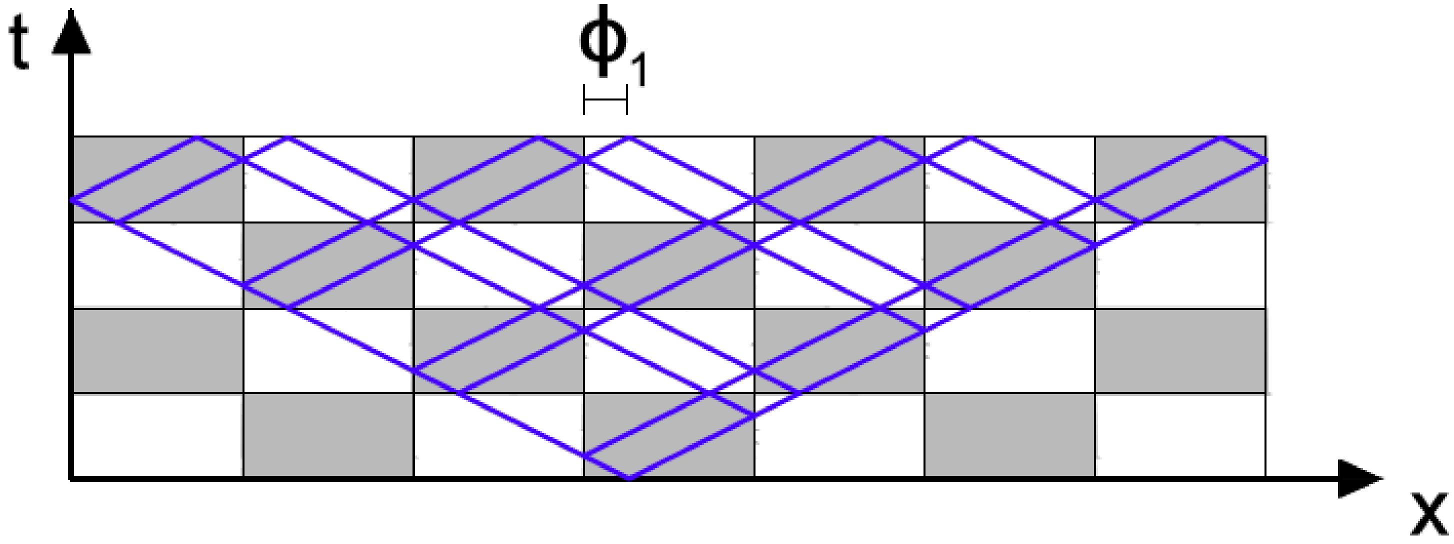
Field patterns in a space-time checkerboard

$$c_1 = c_2 = c \quad \Rightarrow \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$



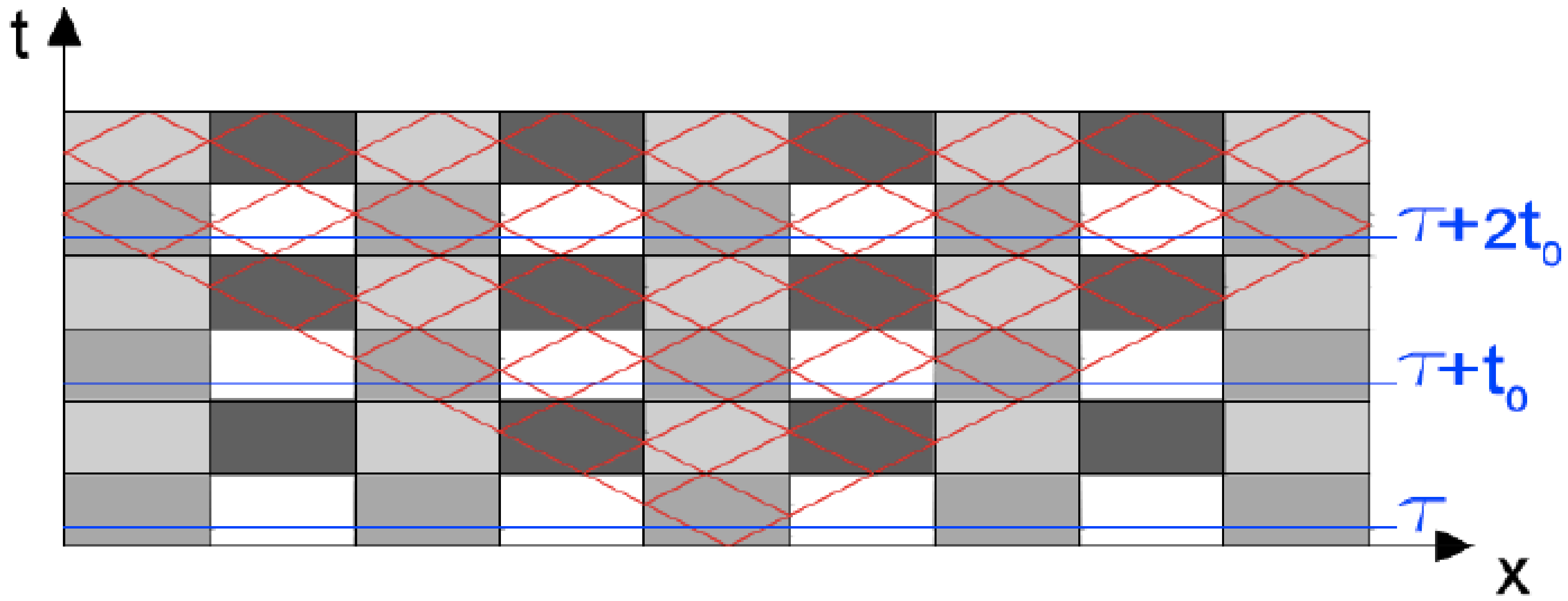
Families of field patterns

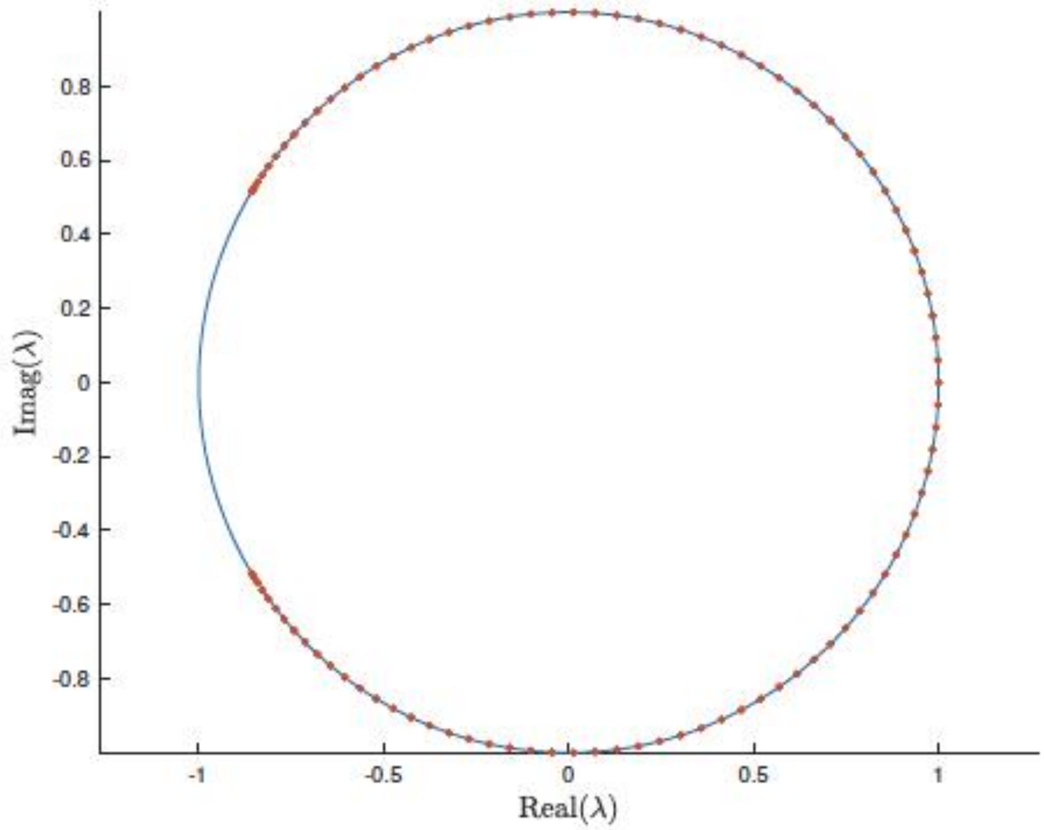
$$c_1 = c_2 = c \quad \Rightarrow \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$



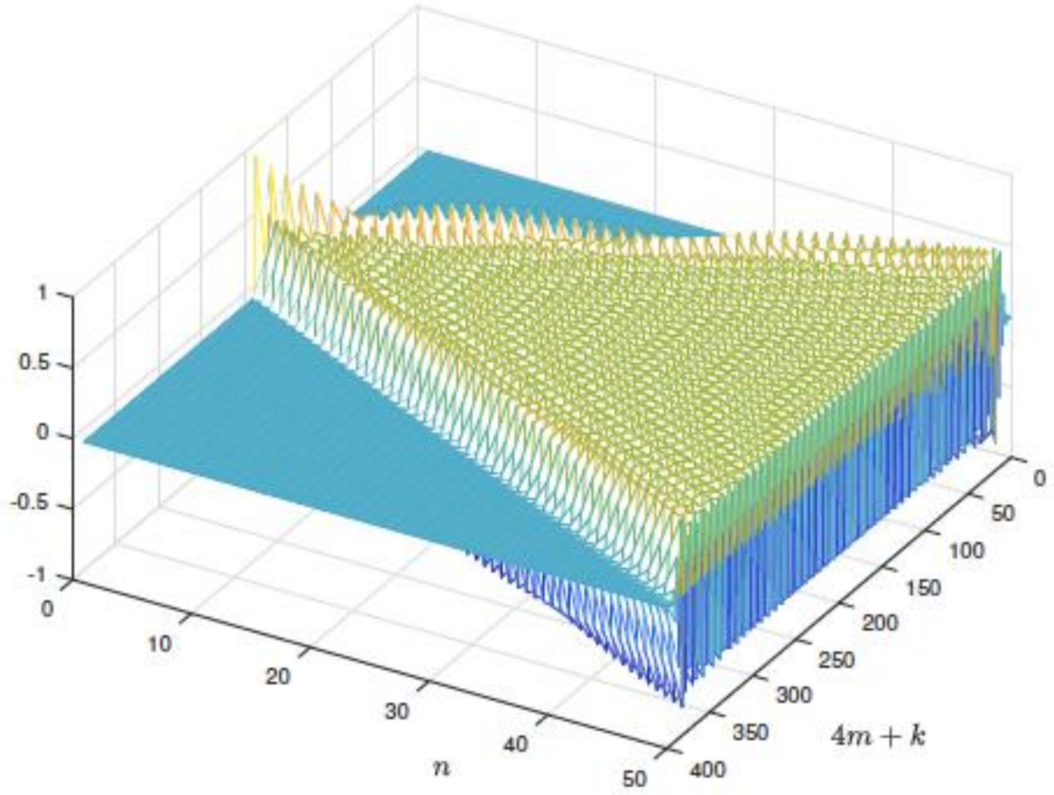
Field patterns are a new type of wave propagating along orderly patterns of characteristic lines which arise in specific space-time microstructures whose geometry in one spatial dimension plus time is somehow commensurate to the slope of the characteristic lines.

Checkerboard geometries where there is no blow up



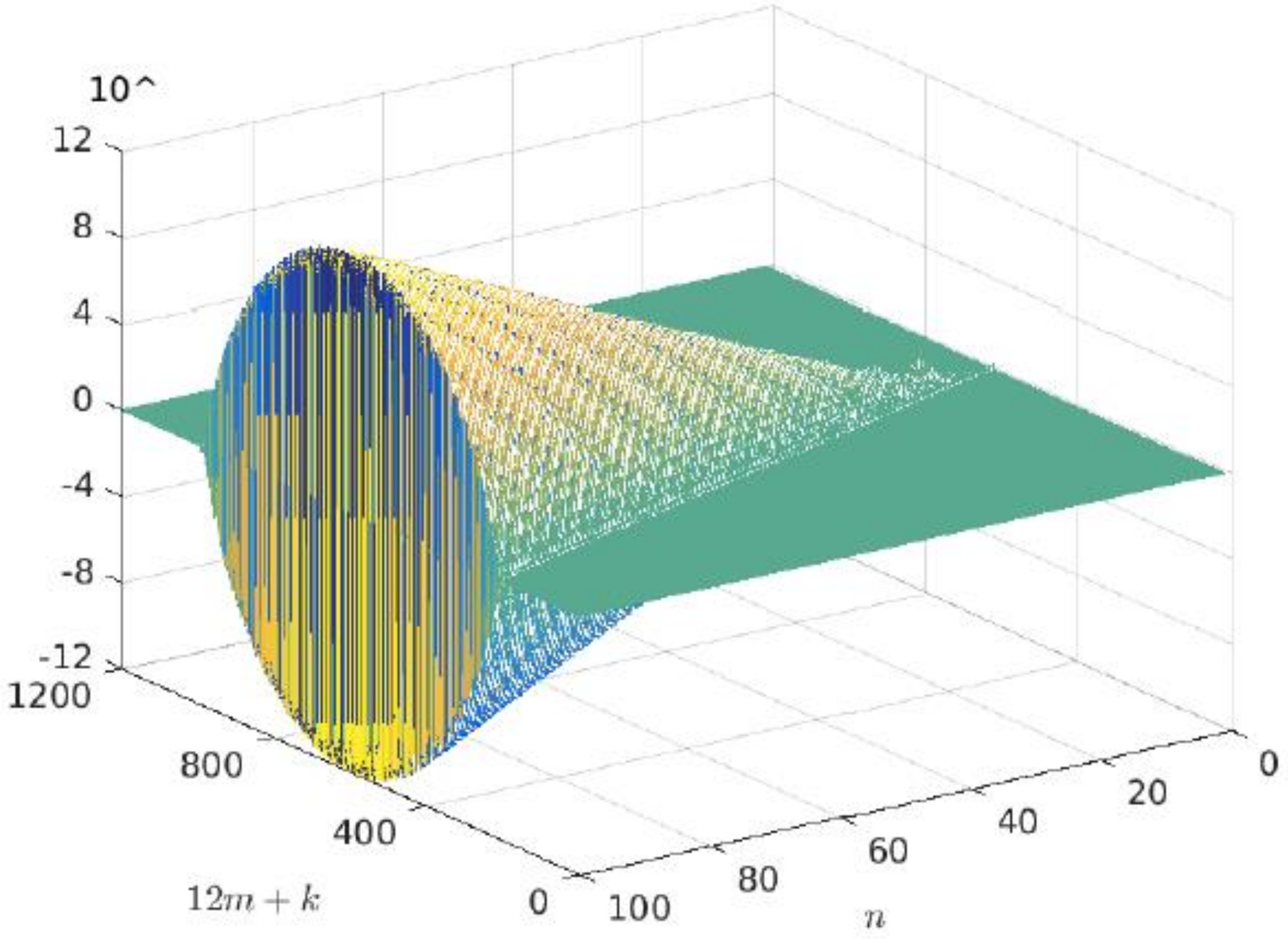


Related to PT-symmetry

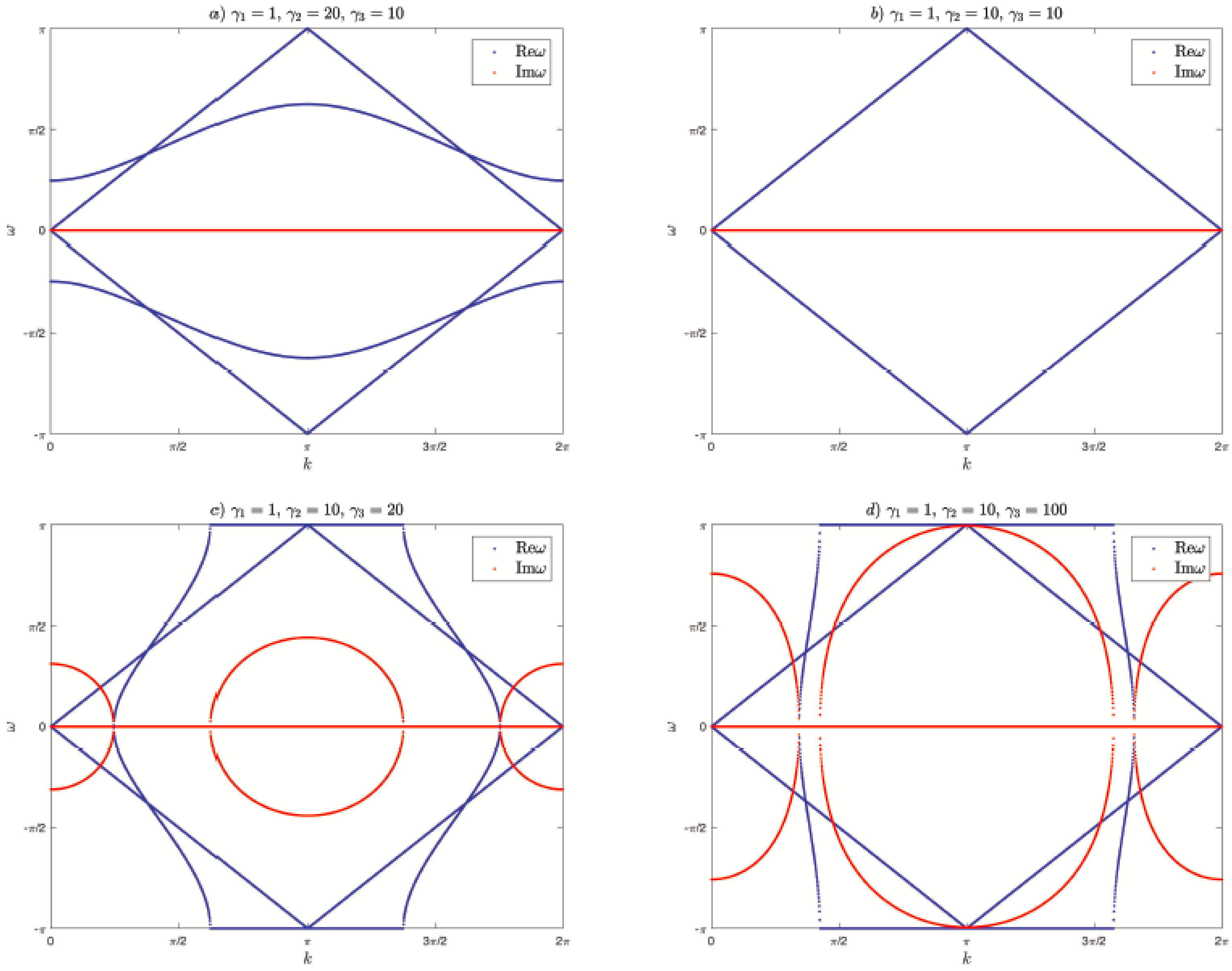


A New Wave

Blow up



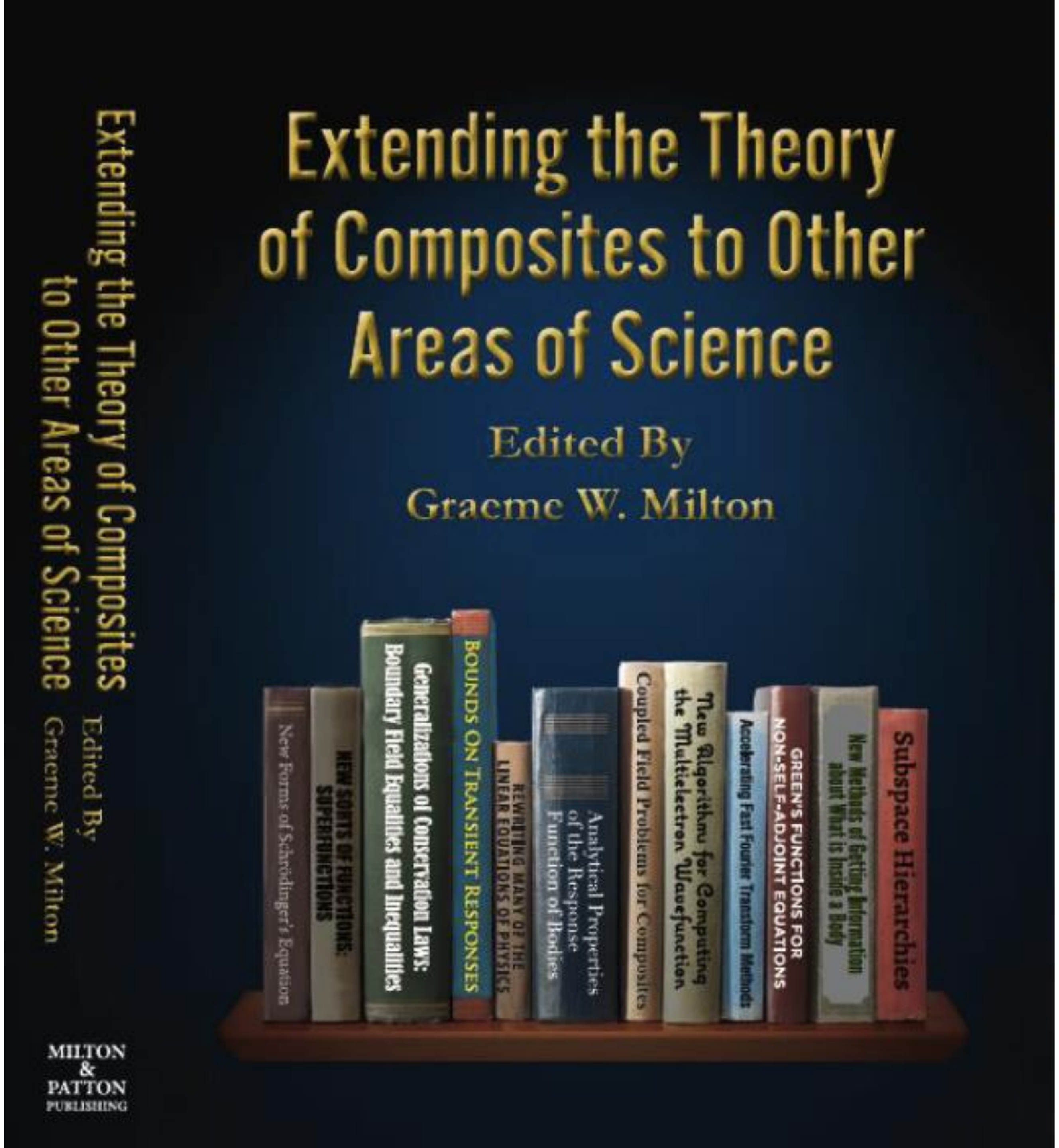
Dispersion diagrams for the three–phase checkerboard



Bloch Waves are:

Infinitely Degenerate!

Thank-you for listening



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