Answers to Final Review Extra Problems

math 2210

Pg 646-647 #1) $y = 2(x-1) - \frac{1}{x-1}$, 3

2) $x = -(y-1)^2 - 1$, $\frac{1}{2\sin 1}$

3) $y = e^{-x^2}$, $-2e^{-1}$

20) $\sin 2\theta = 1 \Rightarrow \theta = \pi/4$

21) $r = 2\cos\theta \sec 2\theta$

Pg 714-716 #1) $12\hat{i} + 19\hat{j}$

2) $\langle -8, 18 \rangle$

3) -8

4) 51

5) $\sqrt{29} - \sqrt{17}$

6) $2\sqrt{10}$

7) $\arctan(5/2)$

8) $\langle 4/\sqrt{17}, -1/\sqrt{17} \rangle$

9) $\frac{1}{\sqrt{29}}(5\hat{i} - 2\hat{j})$

10) $\frac{-3}{\sqrt{29(17)}}$

11) (a) $\sqrt{38}$ (b) $(2, -7/2, 5/2)$

(c) $x^2 + y^2 + z^2 + 2x + 8y - 6z + 10 = 0$

(d) $y = -4$ (e) $x = 5 + 6t$, $y = -3 + t$, $z = 2 - t$

(f) $6x + y - z = 25$

12) $x = 0$

13) $6x - 15y + 5z = 30$

17) sphere ctr $(7, -3, 4)$ $r = 8$

19) plane; x-int $1/3$, y-int -2, z-int 5

21) elliptic cylinder, y-axis

23) hyperboloid of 1 sheet; x axis

25) hyperboloid of 2 sheets, z-axis

27) hyperbolic paraboloid

28) $\langle -15, -13, -8 \rangle$

29) 34

30) $\sqrt{42}$

31) $\sqrt{33} + \sqrt{37}$

32) $\langle 3/\sqrt{24}, -1/\sqrt{26}, -4/\sqrt{26} \rangle$

33) $3/\sqrt{26}, -1/\sqrt{26}, -4/\sqrt{26}$

34) $\frac{-27}{\sqrt{26(37)}}$

35) $\langle 22, -2, 17 \rangle$

36) 80

39) 26

40) $\vec{0}$

41) $\vec{0}$

42) $\langle 16, -16, 16 \rangle$

43) $\langle -4, -10, 4 \rangle$

44) $\pm \langle 6/\sqrt{41}, -2/\sqrt{41}, 1/\sqrt{41} \rangle$

45) (a) $1/\sqrt{6}, -2/\sqrt{6}, 1/\sqrt{6}$

(b) $\frac{1}{\sqrt{66}} \langle 1, 4, 7 \rangle$

(c) $x + 4y + 7z = 5$

(d) $x = 2 + 7t$, $y = -1 - 7t$, $z = 1 + 3t$

(e) 59 (f) $\cos^{-1}\left(\frac{59}{\sqrt{3745}}\right)$

(g) $\sqrt{66}$

51) $(2\sqrt{2}, -\pi/4, 1)$, $(3, -\pi/4, \arccos(1/3))$

52) cyl $(6, 3\pi/4, 6\sqrt{3})$ cart. $(-3\sqrt{2}, 3\sqrt{2}, 6\sqrt{3})$

53) $z = -\sqrt{\frac{x^2+y^2+z^2}{2}}$ bottom half of cone

54) $(x^2+y^2)^{3/2} = x^2 - y^2$

55) $x=1$ plane

56) $\frac{1}{3}(x^2+y^2+z^2) = \sqrt{x^2+y^2+z^2}$

57) $r=1$, $\rho^2 \sin^2\varphi = 1$

58) $z = \cos 2\theta$, $\rho = \cos\varphi \csc^2\varphi \sec 2\theta$

59) $r^2 + z^2 - 2z = 0$

$\rho = 2\cos\varphi$

60) $3z = 2r\cos\theta + r\sin\theta - 4$

$\rho = \frac{4}{2\sin\varphi\cos\theta + \sin\varphi\sin\theta - 3\cos\varphi}$

Pg 752-753

#1) $\vec{r}\,'(t) = 2t\hat{i} + (8t - 4t^3)\hat{j}$, $\vec{r}\,''(t) = 2\hat{i} + (8 - 12t^2)\hat{j}$

2) $\vec{v}(t) = (1-\cos t)\hat{i} + (\sin t)\hat{j}$, $\vec{a}(t) = \sin t(\hat{i}) + \cos t\,\hat{j}$, $|\vec{v}(t)| = \sqrt{2(1-\cos t)}$

5) $x = 4+6t$, $y = 4+4t$, $z = 1+t$

7) $-e^{-t}\hat{i} + 2\cos 2t\,\hat{j} + 2t^{3/2}\hat{k}$

Pg 818-819

#1) $\{(x,y): 4x^2 - 9y^2 \le 36\}$

3) $\{(x,y): z^2 > x^2 + y^2\}$

5) $f_x = 3x^2\cos y + 4$, $f_y = -x^3\sin y - 2y$

7) $f_x = \frac{2x}{y^2+z^2}$, $f_y = \frac{2y(z^2-x^2)}{(y^2+z^2)^2}$

$f_z = \frac{-2z(x^2+y^2)}{(y^2+z^2)^2}$

9) $f_x = 2xz\sqrt{2y+t}$ $f_z = x^2\sqrt{2y+t}$

$f_y = \frac{x^2 z}{\sqrt{2y+t}}$ $f_t = \frac{x^2 z}{2\sqrt{2y+t}}$

11) $f_{xx} = 6xy^2 + 12x^2$, $f_{yy} = 2x^3 - 18xy$

$f_{xy} = 6x^2 y - 9y^2$

15) $\Delta w = -1.13$ $dw = -1.1$

16) -0.17

17) $\frac{\partial s}{\partial x} = (2uv - w^3)(-y\sin x) + (u^2 + 2vw)e^{-y} + (v^2 - 3w^2 u)\left(\frac{y}{x}\right)$

$\frac{\partial s}{\partial y} = (2uv - w^3)\cos x + (u^2 + 2vw)(-xe^{-y}) + (v^2 - 3w^2 u)\ln x$

18) $\frac{\partial z}{\partial r} = \frac{2re^{4st} + 4r^3 t^2 e^{-2s}}{2\sqrt{r^2 e^{4st} + t^2 r^4 e^{-2s}}}$

$\frac{\partial z}{\partial s} = \frac{1}{\sqrt{x^2+y^2}}(2xrte^{2st} - yt r^2 e^{-s})$

19) $\frac{dw}{dt} = (\tan y)(3t^2) + (x\sec^2 y + \tan z)(-2e^{-2t}) + y\sec^2 z\,(-2/t^3)$

20) $-69/5$

21) $-14\sqrt{41}$

pg 818-819 (cont)

23) $-16(x+2)+4(y+1)-7(z-2)=0$

$x = -2-16t$
$y = -1+4t$
$z = 2-7t$

24) tangent plane has $\vec{n} = \left\langle \frac{2x}{a^2}, \frac{-2y}{b^2}, \frac{2z}{c^2} \right\rangle$ $\forall$ pts (x,y,z) on $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ $\Rightarrow$ tangent plane is $2\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = D$

but since all pts are on cone, then $D=0$ $\Rightarrow$ tangent plane

$\frac{x_0}{a^2}x - \frac{y_0}{b^2}y + \frac{z_0}{c^2}z = 0$
for a particular pt on plane (x_0, y_0, z_0)

27) min: $(0,-1,-2)$

28) $x=y = 2\sqrt[3]{\frac{V_0}{12}}$ $\quad z = 3\sqrt[3]{\frac{V_0}{12}}$

31) min at $\left(-\sqrt{8/3}, -\sqrt{2/3}, \sqrt{4/3}\right)$
$\left(-\sqrt{8/3}, \sqrt{2/3}, \sqrt{4/3}\right)$
$\left(\sqrt{8/3}, -\sqrt{2/3}, \sqrt{4/3}\right)$
$\left(\sqrt{8/3}, \sqrt{2/3}, -\sqrt{4/3}\right)$

max at $\left(\sqrt{8/3}, \sqrt{2/3}, \sqrt{4/3}\right)$
$\left(-\sqrt{8/3}, -\sqrt{2/3}, \sqrt{4/3}\right)$
$\left(-\sqrt{8/3}, \sqrt{2/3}, -\sqrt{4/3}\right)$
$\left(\sqrt{8/3}, -\sqrt{2/3}, -\sqrt{4/3}\right)$

32) $\left(\frac{30}{19}, \frac{5}{19}, \frac{21}{19}\right)$

pg 882-883

#1) $-5/84$

2) $\ln 2\left(\frac{1}{2}\ln 2 - 1\right)$

3) $63/4$

4) $-32/105$

5) $-107/210$

6) $\pi/32$

7) $\int_2^4 \int_{-\sqrt{x^2-4}}^{\sqrt{x^2-4}} f(x,y)\,dy\,dx$

8) $\int_0^4 \int_{-\sqrt{4+y^2}}^{\sqrt{4+y^2}} f(x,y)\,dx\,dy$

9) $\int_{-2}^{2} \int_{y^2-4}^{-y^2+y} f(x,y)\,dx\,dy$

10) $\int_{-1}^{1} \int_{3x^2}^{-x^2+4} f(x,y)\,dy\,dx$

11) R bndd by $x = e^y$, $x = y^3$, $y = -1$ + $y = 1$

12) R bndd by $y = x$ + $y = -x^2$ between $x = -1$ + $x = 0$

13) $\int_0^9 \int_0^{\sqrt{x}} y e^{-x^2}\,dy\,dx = (1 - e^{-81})/4$

14) $\int_0^1 \int_{y^2}^{y} e^{x/y}\,dx\,dy$; $\frac{e}{2} - 1$

19) $\pi a^3/6$

23) 13

24) $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\frac{1}{2}\sqrt{y-x^2}}^{\frac{1}{2}\sqrt{y-x^2}} f\,dz\,dx\,dy$; $\int_{-2}^{2} \int_{x^2}^{4} \int_{-\frac{1}{2}\sqrt{y-x^2}}^{\frac{1}{2}\sqrt{y-x^2}} f\,dz\,dy\,dx$

$\int_{-2}^{2} \int_{-\frac{1}{2}\sqrt{4-x^2}}^{\frac{1}{2}\sqrt{4-x^2}} \int_{x^2+4z^2}^{4} f\,dy\,dz\,dx$; $\int_{-1}^{1} \int_{-2\sqrt{1-z^2}}^{2\sqrt{1-z^2}} \int_{x^2+4z^2}^{4} f\,dy\,dx\,dz$

$\int_{-1}^{1} \int_{4z^2}^{4} \int_{-\sqrt{y-4z^2}}^{\sqrt{y-4z^2}} f\,dx\,dy\,dz$; $\int_{0}^{4} \int_{-\frac{1}{2}\sqrt{y}}^{\frac{1}{2}\sqrt{y}} \int_{-\sqrt{y-4z^2}}^{\sqrt{y-4z^2}} f\,dx\,dz\,dy$