Homework for Math 2200 §2, Spring 2011

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Our text is Kenneth H. Rosen, *Discrete Mathematics and its Applications, 6th ed.* Mc Graw Hill, New York, 2007. Please read the relevant sections in the text as well as any cited reference. Each assignment is due the following Thursday, or on Apr. 28, whichever comes first. **Hand in only the even-numbered exercises.** It is recommended that you do the odd-numbered ones too. Please copy or paraphrase each question. Answer the question using complete sentences. Assignments taken from the text `page[exercises]`.

1. [Due Jan. 13.] **Propositional Logic.**

   Problems from the text

   ```
    18[18bcd, 19bcd, 22b, 23abc, 29ac, 30de]
    29[25, 30, 41]
   ```

2. [Due Jan. 20.] **Quantifiers.**

   Problems from the text

   ```
    47[14bc, 15bc, 23ac, 24bd, 33be, 37bc]
    60[14df, 15ef, 24b, 25b, 28cf, 33d, 40bc]
    72[8, 9ab, 10ad, 13b, 16ac]
   ```

3. [Due Jan. 27.] **Proofs.**

   Problems from the text

   ```
    85[8, 11, 15, 18a, 24, 25, 26, 39]
   102[2, 4, 5, 7, 23, 29, 30, 32]
   119[1, 5, 6, 16, 21, 28cd]
   ```

4. [Due Feb. 10.] **Sets & Functions.**

   Problems from the text

   ```
   130[3, 14, 18e, 19, 20, 30, 35, 37, 39, 41]
   146[12, 13, 17, 18, 21, 34, 35, 38, 77]
   ```

5. [Due Feb. 17.] **Summation. Countability.**

   Problems from the text

   ```
   162[12, 16, 24, 32, 36, 37, 41]
   ```

6. [Due Feb. 24.] **Modular Arithmetic.**

   Problems from the text

   ```
   209[9bc, 16, 22, 23, 24, 31]
   217[3ef, 5, 8, 12ab]
   ```

7. [Due Mar. 3.] **Equivalence Relations.**

   Problems from the text

   ```
   217[14, 17, 18, 19, 21, 24, 31]
   562[1, 2bc, 9, 16]
   ```

   Please hand in these additional problems.

   A2. Let the relation $\sim$ on the real numbers be given by

   $$x \sim y \text{ if and only if } x - y \text{ is a rational number.}$$

   Prove that $\sim$ is an equivalence relation by checking the axioms:

   i. (reflexivity): $x \sim x$ for every x;

   ii. (symmetry): If $x \sim y$ then $y \sim x$ for every x, y;

   iii. (transitivity): If $x \sim y$ and $y \sim z$ then $x \sim z$ for every x, y, z.

   A4. Let another relation $\mathcal{R}$ on the real numbers be given by

   $$x\mathcal{R}y \text{ if and only if } x - y \text{ is zero or is irrational.}$$

   Is $\mathcal{R}$ is an equivalence relation? Prove that it is or identify the axions that do not hold.

8. [Due Mar. 17.] **Applications of Number Theory.**

   Problems from the text

   ```
   230[17, 24, 29, 30]
   244[2ceg, 8, 9, 10, 12]
   ```

   Please hand in these additional problems.

   A2. Find the base 8, or octal, expansion of $(54321)_{10}$.

   A4. Convert $(1357246)_8$ to its binary expansion and $(1101101001)_2$ to its hexadecimal expansion.

   A6. Prove that if p is a prime number, then the only solutions of $x^2 \equiv 1 \pmod{p}$ are integers x such that $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$.

   A8. Prove that $(a+1)^n \equiv 1 \pmod{a}$ for any positive integers a, n such that $a \geq 2$.

9. [Due Mar. 31.] **Mathematical Induction.**

   Problem from the text

   ```
   281[40]
   ```

   Please hand in these additional problems.

   Let n be a positive integer. For each problem, use mathematical induction to prove the statement.

   A2. $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

   A4. If $n \geq 2$ then $n! < n^n$.

   A6. For all $n \geq 2$, $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} < 2 - \frac{1}{n}$.

   A8. $3 \mid (n^3 + 2n)$.

   A10. $133 \mid \left(11^{n+1} + 12^{2n-1}\right)$.

10. [Due Apr. 7.] **Recursive Definitions.**

   Problem from the text

   ```
   292[5, 8, 9, 12, 36]
   308[3, 8, 9, 13]
   330[11, 18, 23]
   ```

   Please hand in these additional problems.

   Define a sequence recursively by

   $$\ell_0 = 2, \qquad \ell_1 = 1, \qquad \ell_n = \ell_{n-1} + \ell_{n-2} \text{ for } n = 2, 3, 4, \ldots.$$

   A2. Let f_n be the Fibonacci numbers. Show that for positive integers n,

   $$\ell_n = f_{n-1} + f_{n+1}.$$

   A4. Show that for all nonnegative integers n,

   $$\sum_{k=0}^{n} {\ell_k}^2 = 2 + \ell_n \ell_{n+1}.$$

11. [Due Apr. 14.] **Counting.**

   Problem from the text

   ```
   344[4, 7, 8, 16, 24, 33, 39]
   353[4, 7, 9, 15, 30]
   456[1, 3, 8, 15]
   471[4a, 41]
   ```

12. [Due Apr. 21.] **Combinations and Permutations.**

   Problem from the text

   ```
   361[12, 18, 22]
   369[8, 12, 13, 22, 29]
   398[14, 21, 23, 24, 39]
   504[2, 8, 10]
   ```