Homework for Math 3010 §1, Spring 2024

A. Treibergs, Instructor

April 12, 2024

Our text is by Victor J. Katz, *A History of Mathematics, 3rd. ed.*, Pearson India 2019. ISBN: 978-9353433000. Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 19, whichever comes first.

Your written work reflects your professionalism. Make answers complete, self contained and written in good English. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Monday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 before 4:00 pm Friday afternoon will be considered to be on time.

Please hand in on paper problems A1 – A3 on Friday, January 12.

A1. Exercises from from Katz's *A History of Mathematics.*

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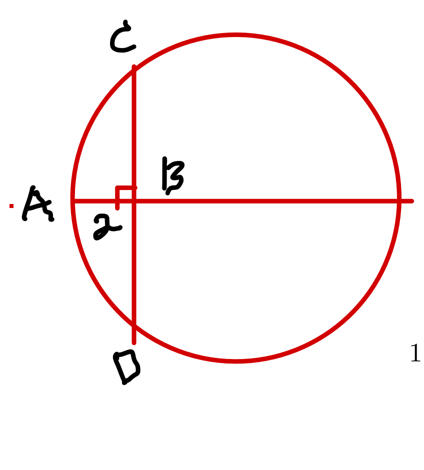
28[17, 18, 20, 21, 22]
```

A2. Compute the sum using Babylonian arithmetic. Convert the summands and your answer to decimals and check that your addition is correct.

$$(5, 51, 12, 49) + (13, 45, 19) = ?$$

A3. From Bunt *et. al.* 63[8].

Solve the following problem that occurs on a Babylonian tablet. Given that the circumference of a circle is 60 and the length of the saggita $\overline{AB}$ is 2, calculate the length of the chord $\overline{CD}$ in the figure.



Please hand in problems B1 on Friday, January 19.

B1. Exercises from from Katz's *A History of Mathematics.*

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28[2, 4, 24, 25, 28, 30]
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Please hand in problems C1–C3 on Friday, Jan. 26.

C1. Exercises from from Katz's *A History of Mathematics.*

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28[10]
47[3, 7, 8, 10, 12]
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C2. From Bunt *et. al.* 40[4]

1. Use the ancient Egyptian procedure to find the area of a circle with diameter 12.
2. What would the area be if the modern value of π were used instead?
3. What percentage error made by the ancient Egyptians?

C3. From Bunt *et. al.* 40[3]. The Egyptian calculation for the area of an arbitrary quadrilateral is to multiply half the sum of opposite sides by half the sum of the other two sides. Show that the Egyptian procedure for finding the area of a quadrilateral gives the correct result if the quadrilateral is a rectangle and gives too large a number if the figure is a nonrectangular parallelogram or a trapezoid. Is the procedure ever correct for a quadrilateral that is not a rectangle. (Can you prove it?)

Please hand in problems D1–D5 on Friday, Feb. 2.

D1. Exercises from from Katz's *A History of Mathematics.*

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48[21]
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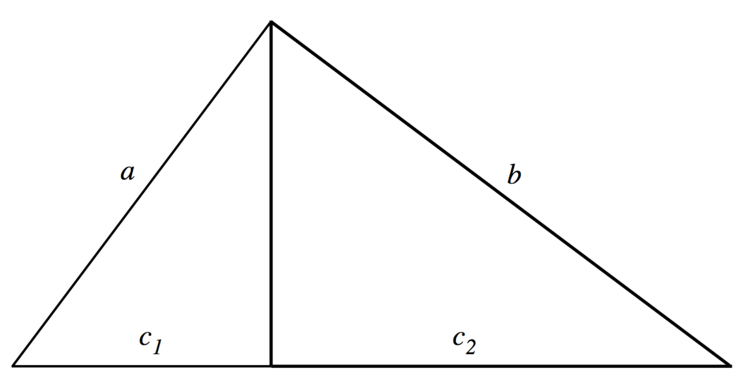
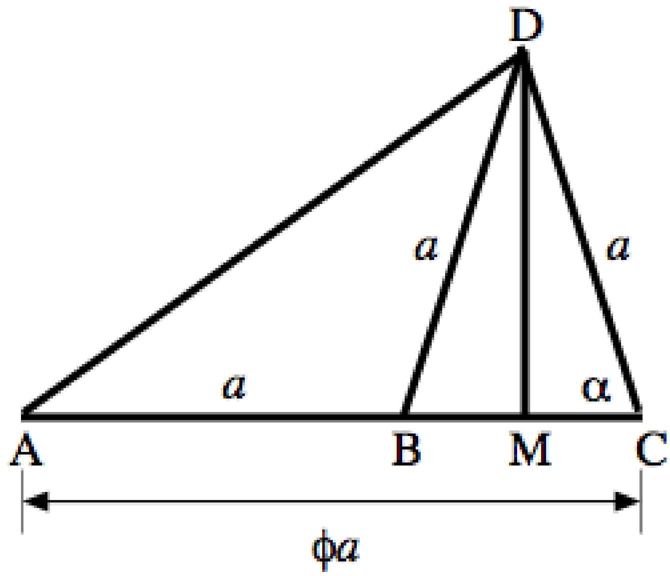
D2. In Book VI of *Elements*, Euclid gives the following argument for the Pythagorean Theorem based on similar triangles. Show that the three triangles in the figure are similar, and hence prove the Pythagorean theorem by equating ratios of corresponding sides. [Stillwell *Mathematics and its History.* p. 10]



D3. Show that the Golden ratio ϕ is irrational.

D4. Check Euclid's construction of the regular pentagon. Suppose that collinear points A, B, C have distances $a = AB$, $\phi a = AC$. Construct the isosceles triangle $\triangle(BDC)$ with $a = BD = CD$. Let M be the bisector between B and C. Let the angle $\alpha = \angle(ACD)$.



a. Find the length of DM using the triangle $\triangle(BMD)$

b. Show that $AD = AC$ using the triangle $\triangle(AMD)$.

c. Express the angles of triangle $\triangle(ABD)$ in terms of α and show that $\alpha = 72°$. Thus it is the central angle of a sector of a regular pentagon. HINT: The sum of the interior angles of any triangle is $180°$.

D5. On a separate piece of paper, write your Essay on Mathematics of Antiquity proposal. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what the Greeks thought about π. Better say that you will describe how Archimedes showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Katz).

Please hand in your essay E1 on Friday, Feb. 9.

E1. Essay on the Mathematics of Antiquity. Write an essay about a specific mathematical discovery/ theorem/ method that occurred before Christ.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.
- You must draw on a bare minimum of three book and journal sources. It is good if you include "primary" sources, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.
- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)
- Please attach your essay proposal from last week to your paper.

Please hand in problems F1–F4 on Friday, Feb. 16.

F1. Katz's *A History of Mathematics.*

```
90[7 Show area BEFG equals area ABML any way you can, 10, 19]
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F2. Show that for any integers a and b, there are integers m and n so that $\gcd(a, b) = ma + nb$. [Stillwell 3.3.2]

F3. Deduce from problem 3.3.2 that the equation $ax + by = c$ with integer coefficients has an integer solution x, y if $\gcd(a, b)$ divides c. [Stillwell 3.3.3]

F4 The equation $12x + 15y = 1$ has no integer solution. Why? [Stllwell 4.4.4]

Please hand in problems G1–G4 on Friday, February 23.

G1. Exercises from from Katz's *A History of Mathematics.*

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128[1, 11]
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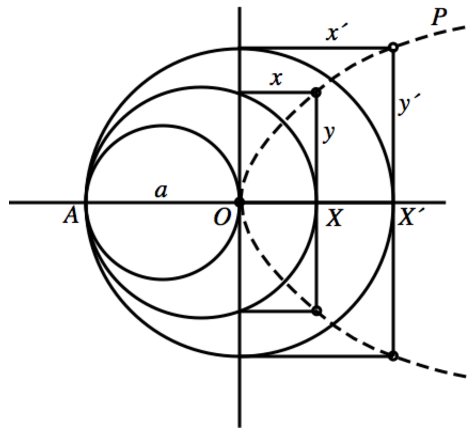
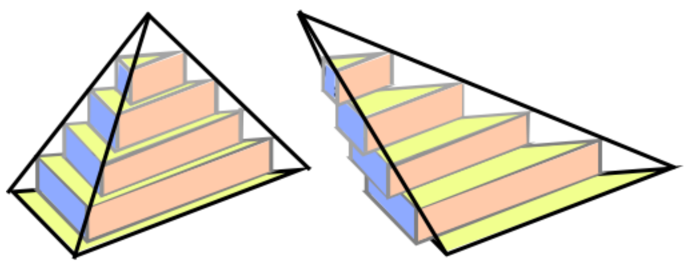
G2. Show that if $a \mid bc$ and $\gcd(a, b) = 1$ then $a \mid c$.

G3. Here is how Menaechmus constructed the parabola as the locus of points $\mathcal{P} = \{(x, y)\}$. Given points A, O and X on line such that a is the distance AO and x is the distance OX. Let L be a perpendicular line through O. For each x, construct a circle whose diameter is AX. Then y is the distance from O to the intersection point of L with the circle. Show that Menaechmus' consruction yields the parabola. Show also that the parabola passes through the point (a, a).



G4. Show that any two tetrahedra with the same base and height can be approximated arbitrarily closely by the same prisms, differently stacked, as in the Figure. Deduce that two tetrahedra with the same base and height have equal volume. [Stillwell *Mathematics and its History.* p. 61]



[HINT.] If you have not had Math 3210, you may assume that the apex of the tetrahedron is directly over the base, which yields to a simpler analysis. Be careful! You have to take into account how skew your tetrahedron is. If T and T' are the tetrahedra and S_n and S'_n are the approximating stacks of prisms with n levels, we have volumes $V(S_n) = V(S'_n)$ but we need to establish for Eudoxus's exhaustion method, that for any positive number $\epsilon > 0$, the stacks and tetrahedra are close enough so that both $|V(T) - V(S_n)| < \frac{\epsilon}{2}$ and $|V(T') - V(S'_n)| < \frac{\epsilon}{2}$ for some n large enough, depending on ϵ. You will need to compare inner and outer approximations for this. A problem occurs if the stacks spill outside the tetrahedron, as in the second diagram. We conclude from the triangle inequality, that for any $\epsilon > 0$ there is an n such that

$$\begin{aligned}
|V(T) - V(T')| &= |V(T) - V(S_n) + V(S_n) - V(S'_n) + V(S'_n) - V(T')| \\
&\le |V(T) - V(S_n)| + |V(S_n) - V(S'_n)| + |V(S'_n) - V(T')| \\
&< \frac{\epsilon}{2} + 0 + \frac{\epsilon}{2} = \epsilon.
\end{aligned}$$

Since ϵ may be any positive number no matter how small, $V(T) = V(T')$.

Please hand in problems H1 on Friday, Mar 1.

H1. Exercises from from Katz's *A History of Mathematics.*

```
128[19, 26]
168[1, 4, 22]
```

Please hand in problems I1–I2 on Friday, Mar. 15.

I1. Exercises from from Katz's *A History of Mathematics.*

```
226[2, 3, 6, 8, 16, 18, 19]
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I2. On a separate piece of paper, write your Essay on Renaissance Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay in two weeks. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what Omar Khayyam thought about Euclid. Better say you will describe Khayyam's suggestions to replace the parallel postulate which anticipate the modern development of non-Euclidean geometry. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Katz). One of your sources must be a primary source, quoting directly from the mathematician being discussed.

Please hand in your essay J1 on Friday, March 22.

J1. Essay on the Renaissance Mathematics. Write an essay about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.
- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.

- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)
- Please attach your essay proposal from last week to your paper.

Please hand in problems K1–K3 on Friday, March 29.

K1. Exercises from from Katz's *A History of Mathematics.*

```
263[23, 24, 25]
318[1, 11]
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K2. Find a solution using Bhaskara II's method.

$$83x^2 + 1 = y^2$$

K3. Use Brahmagupta's quadratic interpolation formula to find the Indian value for Sin(1000′), where $R = 3438$.

Minutes	Sine	Sine Difference
0	0	* * *
225	225	225
450	449	224
675	671	222
900	890	219
1125	1105	215
1350	1315	210

Please hand in problems L1 on Friday, Apr. 5.

L1. Exercises from from Katz's *A History of Mathematics.*

```
318[9, 16]
359[2, 31, 40]
418[33]
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Please hand in problems M1-M6 on Friday, Apr. 12

M1. Exercises from from Katz's *A History of Mathematics.*

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579[9, 24, 25]
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M2. Recall Napier's logarithm $\mathrm{Nlog}(x) = m$ if $10^7(1-10^{-7})^m = x$. Show that

$$\mathrm{Nlog}(x) + \mathrm{Nlog}(y) = \mathrm{Nlog}(xy) + \mathrm{Nlog}(1)$$

M3. Show that the binomial series gives

$$\frac{1}{\sqrt{1-t^2}} = 1 + \frac{1}{2}t^2 + \frac{1\cdot 3}{2\cdot 4}t^4 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}t^6 + \cdots$$

Then use

$$\sin^{-1} x = \int_0^x \frac{dt}{\sqrt{1-t^2}}$$

to derive Newton's series for $\sin^{-1} x$.

M4. Use Fermat's method of ad-equation to find the slope of the curve $f(x) = x^2 - \sqrt{x}$ at $x > 0$.

M5. Use Newton's version of Newton's method to approximate the root of $x^2 - 2 = 0$ to an accuracy of eight decimal places.

M6. On a separate piece of paper, write your Essay on Modern Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred after 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what the Henri Poincaré thought about the fundamental group. Better to describe not only why Poincaré invented the fundamental group, but say what it is and prove some things about it. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Katz). One of your sources must be a primary source, quoting directly from the mathematician being discussed.

Please hand in problems N1 on Mon. Apr. 22.

N1. Essay on the Modern Mathematics. Write an essay about a specific mathematical discovery/ theorem/ method that occurred after 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- Your writing should show that you have learned and understood something about your topic. There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate. Present mathematics that you understand so that your mathematical arguments flow in a logical order and are not just an unsubstantiated and disconnected sequence of statements.
- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, *quoting directly from the mathematician you're discussing*, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.
- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)
- Please attach your essay proposal from last week to your paper.

The Final Exam is Wed., May 1, 1:00 - 3:00 PM in the usual room JTB 120.