Math 3010 § 1.	**Final Exam**	Name:______________
Treibergs $\sigma\tau$		December 13, 2024

This is a closed book test except that you are a allowed "cheat sheet," an 8.5" x 11" piece of paper with notes on both sides. No other notes, books, papers, calculators, tablets, phones or messaging devices are permitted. Define terms, give complete solutions and explain your logic. There are [100] total points. **Do SEVEN of eight problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded. Do **BOTH PARTS** of problems 3,4,6,7.

1.______	/15
2.______	/14
3.______	/14
4.______	/14
5.______	/15
6.______	/14
7.______	/14
8.______	/14
Total______	/100

1. [15] Identify the mathematician associated to each formula. Describe in a few words the meaning of the formula.

Formula	Mathematician	Its Meaning
$\frac{1}{2^m}\sum_{k=0}^{s-1}\binom{m}{k}$		
$\sum_{k=0}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$		
$\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}\,dx$		
$\sum_{k=0}^{\infty}\frac{a}{4^k} = \frac{4a}{3}$		
$\sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_p \frac{1}{1-\frac{1}{p^s}}$		

2. For each quotation, identify the author and describe its mathematical importance.

a. [2] *A man has one pair of rabbits at a certain place entirely surrounded by a wall. We wish to know how many pairs can be bred from it in one year, if the nature of rabbits is that they breed every month one pair and begin to breed in the second month after their birth. . .*

`Mathematician:` `Importance:`

b. [2] *1.A point is that which has no part. 2.A line is a breadthless length. 3.The extremities of a line are points, 4.A straight lineis a line which lies evenly with the points on itself. 5.A surface is that which has length and breadth only. 6.The extremities of a surface are lines. 7.A plane surface is a surface which lies evenly with the straight lines on itself.*

`Mathematician:` `Importance:`

c. [2] *To divide a given square number into two squares. Given square number* 16. x^2 *one of the required squares. Therefore* $16 - x^2$ *must be equal to a square. Take a square of the form* $(mx - 4)^2$, m *being any integer and* 4 *being the number which is the square root of* 16, *e.g., take* $(2x - 4)^2$ *and equate it to* $16 - x^2$. *Therefore* $4x^2 - 16x + 16 = 16 - x^2$, *or* $5x^2 = 16x$ *and* $x = 16/5$. *The required squares are therefore* $256/25$ *and* $144/25$.

`Mathematician:` `Importance:`

d. [2] (Scribbled on the previous.) *On the other hand it is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or generally any power except a square into two powers with the same exponent. I have discovered a truly marvelous proof of this, which however the margin is not large enough to contain.*

`Mathematician:` `Importance:`

e. [2] *In each of 7 houses are 7 cats; each cat kills 7 mice; each mouse would have eaten 7 ears of spelt; each ear of spelt would have produced 7 hekat of grain. Query: how much grain is saved by the 7 houses' cats?*

`Mathematician:` `Importance:`

f. [2] *On an expedition to seize his enemy's elephants, a king marched two* yojanas *the first day. Say, intelligent calculator, with what increasing rate of daily march did he proceed, since he reached his foe's city, a distance of eighty* yojanas, *in a week?*

`Mathematician:` `Importance:`

g. [2] *At Königsberg in Prussia there is an island called "der Kneiphof," and the river surrounding it is divided into two branches. Over the branches of this river are seven bridges. Now the question is whether one can plan a walk so as to cross each bridge once and not more than once. I was told that some deny the possibility, others are doubtful but that nobody affirms it. Wherefrom, I formulated the following problem, framed in a very general way for myself: whatever the shape of the river or its division into branches may be, and whatever the number of bridges, to find out whether it is possible or not to cross each bridge exactly once.*

`Mathematician:` `Importance:`

3. Questions about Isaac Newton 1642–1727.

   (a) [7] Using Newton's method of fluxions, compute the slope of the tangent line to the witch of Agnesi curve at the point $(2, 1)$. Other methods will receve no credit.
   $xy^2 = 4(2 - y)$

   (b) [7] Using Newton's binomial series, compute the power series for
   $f(x) = \int_0^x \frac{dx}{(1 - x^2)^{3/2}}$

4. More questions about Isaac Newton.

   (a) [6] Using Newton's version of Newton's Method, starting from $x_1 = 2$, find the next two approximations to the root of $f(x) = x^2 - 5$. (Your score will be [3] for using the modern version.)

   (b) [4] Name at least two contribution Newton made to astronomy and our understanding of the solar system.

   (c) [2] Did Newton invent calculus? Was he first to do so?

   (d) [2] Name two other non-mathematical discoveries of Newton.

5. Determine whether the following statements are true or false.

   (a) [2] With the substitution $x = a - b$, Cardano reduced the solution of the cubic equation $x^3 + px = q$ to the solution of a quadratic equation for a^3.

TRUE: ◯	FALSE: ◯

   (b) [2] For $x, y > 0$ the Napierian logarithm $\text{Nap.}\log(10^{-7}xy) = \text{Nap.}\log(x) + \text{Nap.}\log(y)$.

TRUE: ◯	FALSE: ◯

   (c) [2] A chain suspended from two points takes the shape of a parabola.

TRUE: ◯	FALSE: ◯

   (d) [2] There are infinitely many pairs of rational numbers (x, y) that satisfy $x^2 + y^2 = 1$.

TRUE: ◯	FALSE: ◯

   [7] Give a detailed explainantion of ONE of your answers (a)–(d) above.

6. (a) [7] Find all integers x that simultaneously satisfy the congruences.

$$x \equiv 3 \mod 4$$
$$x \equiv 1 \mod 5$$
$$x \equiv 2 \mod 9$$

(b) [7] Find an integer solution of $x^2 = 1 + 32y^2$.
[Hint: Start by noting that $(x, y, k) = (6, 1, 4)$ is a solution of $x^2 = 4 + 32y^2$.]

7. (a) [7] Recall the Fibonacci Sequence $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5, \ldots$. What is the recursion formula for generating the Fibonacci sequence? Prove that no two consecutive Fibonacci numbers F_n and F_{n+1} have a factor $d > 1$ in common.

(b) [7] Prove the following consequence Pascal found for binomial coefficients.

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-2}{r-1} + \binom{n-3}{r-1} + \cdots + \binom{r-1}{r-1}$$

8. [14] Find the slope of the tangent line to $y = f(x) = \sqrt{x}$ at $x = a > 0$ using one of three early methods: Descartes method of finding the normal, Fermat's method of ad-equality or Lagrange's algebraic method. [Other methods will receive no credit.]