Homework for Math 3010 §1, Spring 2024

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Our text is by David Burton, *History of Mathematics: An Introduction 7th ed.*, McGraw Hill 2010, ISBN-10: 0073383155, ISBN-13: 978-0073383156. It is available via Inclusive Access (Bookshelf tab in Canvas). Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 19, whichever comes first.

Your written work reflects your professionalism. Make answers complete, self contained and written in good English. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer. You may discuss homework problems but you are expected to write up solutions on your own. If you use ideas from other textbooks or the internet, you are expected to cite such sources. Representing work of others as your own is plagiarism and is contrary to the student code.

Homework from Wednesday to Monday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 before 4:00 pm Friday afternoon will be considered to be on time. Notation: `page[problems]`.

Please hand problems A on Friday, Aug. 23.

A. Exercises from Burton's *The History of Mathematics.*

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18[3]
51[1, 2, 7, 13]
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Please hand in problems B1 – B3 on Friday, Aug. 30.

B1. Exercises from Burton's *The History of Mathematics.*

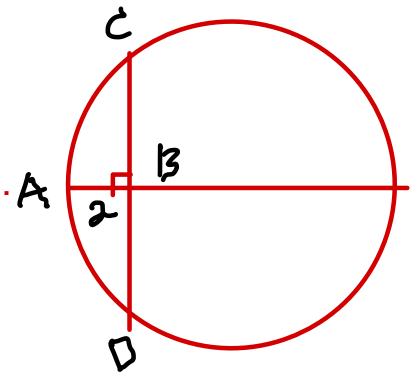
```
28[2, 3, 4]
61[1, 2, 6]
71[3]
```

B2. Compute the sum using Babylonian arithmetic. Convert the summands and your answer to decimals and check that your addition is correct.

$$(5, 51, 12, 49) + (13, 45, 19) =?$$

B3. From Bunt *et. al.* 63[8].

Solve the following problem that occurs on a Babylonian tablet. Given that the circumference of a circle is 60 and the length of the saggita $\overline{AB}$ is 2, calculate the length of the chord $\overline{CD}$ in the figure.

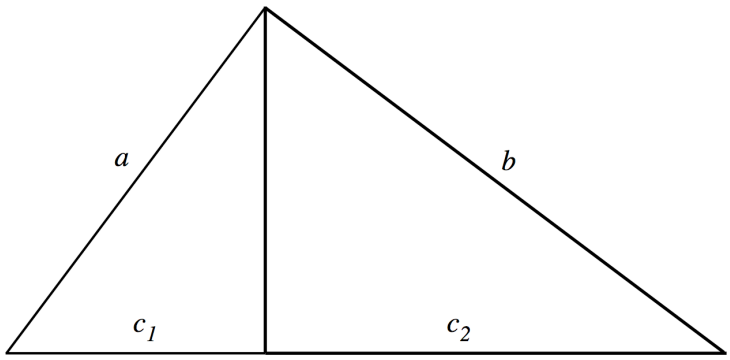


Please hand in problems C1–C3 on Friday, Sept. 6.

C1. Exercises from Burton's *The History of Mathematics.*

18[7]
103[2, 11b, 14]

C2. In Book VI of *Elements*, Euclid gives the following argument for the Pythagorean Theorem based on similar triangles. Show that the three triangles in the figure are similar, and hence prove the Pythagorean theorem by equating ratios of corresponding sides. [Stillwell *Mathematics and its History.* p. 10]



C3. Show that the Golden ratio ϕ is irrational.

Please hand in problems D1–D2 on Friday, Sept. 13.

D1. Exercises from Burton's *The History of Mathematics.*

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118[5, 7]
127[1, 2, 4]
```

D2. On a separate piece of paper, write your Essay on Mathematics of Antiquity proposal. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what the Greeks thought about π. Better say that you will describe how Archimedes showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Burton).

Please hand in your essay E1 on Friday, Sept. 20.

E1. Essay on the Mathematics of Antiquity. Write an essay about a specific mathematical discovery/ theorem/ method that occurred before Christ.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.
- You must draw on a bare minimum of three book and journal sources. It is good if you include "primary" sources, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the author cited in a Wikipedia article.
- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)
- Please attach your approved essay proposal from last week to your paper.

Please hand in problems F on Friday, Sept. 27.

F. Exercises from Burton's *The History of Mathematics.*

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168[4, 8]
182[12, 16, 24]
192[3]
```

Please hand in problems G on Friday, Oct. 4.

G. Exercises from Burton's *The History of Mathematics.*

```
208[1c, 3, 6]
231[1, 5, 12]
```

Please hand in problems H on Friday, Oct. 18.

H. Exercises from Burton's *The History of Mathematics.*

```
263[2, 4, 5, 6, 7]
```

Please hand in problems I1–I5 on Friday, Oct. 25.

I1. Exercizes from Burton's *The History of Mathematics.*

```
232[21, 23]
263[17]
```

I2. Solve the following problem from *Lilavati* of Bhaskara II. "A person gave three *drammas* on the first day, and continued to distribute alms increasing by two [a day]; and he thus bestowed on the priests three hundred and sixty *drammas*: say quickly in how many days?" [Calinger, *Classics of Mathematics,* p. 224.]

I3. Assume $u_1^2 = Dv_1^2 + k_1$ and $u_2^2 = Dv_2^2 + k_2$. Prove that

$$(u_1u_2 + Dv_1v_2)^2 = D(u_1v_2 + u_2v_1)^2 + k_1k_2.$$

[Katz, *A History of Mathematics,* p. 262.]

I4. Find an integral solution of $x^2 = 83y^2 + 1$ using Brahmagupta's method.

[Katz, *A History of Mathematics,* p. 262.]

I5. On a separate piece of paper, write your Essay on Renaissance Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay in two weeks. Be sure to include in your proposal

- Working Title

- Short but specific description of what your essay is about. Don't just say you will discuss what Omar Khayyam thought about Euclid. Better say you will describe Khayyam's suggestions to replace the parallel postulate which anticipate the modern development of non-Euclidean geometry. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Burton). One of your sources MUST BE a primary source, quoting directly from the mathematician being discussed.

Please hand in your essay J1 on Friday, Nov. 1.

J1. Essay on the Renaissance Mathematics. Write an essay about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.
- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.
- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)
- Please attach your essay proposal from last week to your paper.

Please hand problems K1 – K2 on Friday, Nov. 8.

K1. Exercises from Burton's *The History of Mathematics.*

285[2, 4, 7, 15]
293[2, 4]

K2. Problems from Alcuin's *Problems for the Quickening of the Mind of the Young.*

a. A dog chasing a rabbit, which has a start of 150 feet jumps 9 feet every time the rabbit jumps 7 feet. In how many leaps does the dog overtake the rabbit? [Struik, *A Concise History of Mathematics,* p. 85.]

b. A man must ferry a wolf, a goat and a head of cabbage across a river. The available boat, however, can carry only the man and one other thing. The goat cannot be left alone with the cabbage, nor the wolf with the goat. How should the man ferry the three items across the river? [Katz, *A History of Mathematics,* p. 359.]

Please hand problems L1 on Friday, Nov. 15.

L1. Exercises from Burton's *The History of Mathematics.*

326[2, 3ace]
334[1b]
361[6, 7]

Please hand in problems M1 – M4 on Friday, Nov. 22.

M1. Exercises from Burton's *The History of Mathematics.*

380[7, 10b]
467[10a]
493[2, 3]

M2. Use Descarte's method to find the normal vector to the curve $x^2 - y^2 = -9$ at the point $P_0 = (4, 5)$.

M3. Use Fermat's method of ad-equation to find the slope of the curve $f(x) = x^2 - \sqrt{x}$ at $x > 0$.

M4. On a separate piece of paper, write your Essay on Modern Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred after 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what the Henri Poincaré thought about the fundamental group. Better to describe not only why Poincaré invented the fundamental group, but say what it is and prove some things about it. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.

- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Burton). One of your sources must be a primary source, quoting directly from the mathematician being discussed.

Please hand in problems N1 on Wed., Dec. 4.

N1. Essay on the Modern Mathematics. Write an essay about a specific mathematical discovery/ theorem/ method that occurred after 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics and mathematical explanation in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate. Your mathematical arguments should be careful and thorough enough to convince the reader that you understand the mathematics.
- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, *quoting directly from the mathematician you're discussing*, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.
- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide.
- Please attach your essay proposal from last week to your paper.

The FINAL EXAM is Fri., Dec. 13 at 10:30 AM - 12:30 PM in the usual LCB 225.

Do not hand in problems O. These are Questions for Final Exam Study

O1. Use Newton's version of Newton's method to approximate the root of $x^2 - 5 = 0$ to an accuracy of eight decimal places.

O2. Show that the binomial series gives

$$\frac{1}{\sqrt{1-t^2}} = 1 + \frac{1}{2}t^2 + \frac{1\cdot 3}{2\cdot 4}t^4 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}t^6 + \cdots$$

Then use

$$\sin^{-1} x = \int_0^x \frac{dt}{\sqrt{1-t^2}}$$

to derive Newton's series for $\sin^{-1} x$.