Math 3010 § 1. **Final Exam** Name:_____
Treibergs $\alpha\pi$ Decenber 8, 2025

This is a closed book test except that you are allowed three "cheat sheets," 8.5" x 11" pieces of paper with notes on both sides. No other notes, books, papers, calculators, tablets, phones or messaging devices are permitted. Define terms, give complete solutions and explain your logic. There are [100] total points. **Do SEVEN of eight problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded. Do **BOTH PARTS** of problems 3,4,6,7.

1.	/15
2.	/15
3.	/14
4.	/14
5.	/15
6.	/14
7.	/14
8.	/14
9.	/14
Total	/100

1. [15] **Algebra and Analysis.** Here a list of areas where mathematics developed. For each region identify two mathematicians, one who contributed to algebra and one who contributed to analysis.

Region	Two Mathematicians	Their Contributions in Algebra and Analysis
Greece		
Italy		
England		
France		
Germany		

2. [15] Identify the mathematician associated to each formula. Describe in a few words the meaning of the formula.

Formula	Mathematician	Its Meaning
$\chi = V - E + F$		
$f(x) = \sum_{k=1}^{\infty} a^k \cos(b^k x)$ $0 < a < 1, \quad ab > 1 + \frac{3}{2}\pi$		
$P = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-\frac{x^2}{2\sigma^2}}\, dx$		
$AC \cdot BD =$ $AB \cdot CD + BC \cdot DA$		
$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$		

3. Questions about Isaac Newton 1642–1727.

   (a) [7] Using Newton's method of fluxions, compute the slope of the tangent line to the Witch of Agnesi curve $yx^2 = a^2(a - y)$ at the point (x, y) on the curve where $a > 0$ is constant. Other methods will receive no credit.

   (b) [7] Describe the relate of the work of Kepler and Newton about the solar system. Include a discussion of both the force of gravity and the orbits of planets.

4. More questions about Isaac Newton.

   (a) [7] Using Newton's method, approximate root of $f(x) = x^2 - 5$. Do at least two iterations. You may use the modern approach instead of Newton's .

   (b) [7] Compare and contrast calculus of Newton and Leibnitz.

5. Determine whether the following statements are true or false.

(a) [2] Both the Greeks and the Chinese used polygons to approximate the value of π.

TRUE: ◯	FALSE: ◯

(b) [2] Pythagoras knew of infinitely many integer solutions of $x^2 + y^2 = z^2$ that are not multiples of each other.

TRUE: ◯	FALSE: ◯

(c) [2] If p is a prime number, then p^3 is perfect.

TRUE: ◯	FALSE: ◯

(d) [2] Euler found the sum $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$.

TRUE: ◯	FALSE: ◯

[7] Give a detailed explainantion of ONE of your answers (a)–(d) above.

6. (a) [7] Find all integers x that simultaneously satisfy the congruences.

$$\begin{aligned} x &\equiv 1 \mod 3 \\ x &\equiv 2 \mod 7 \\ x &\equiv 3 \mod 10 \end{aligned}$$

(b) [7] Use Brahmagupts's method to find an integer solution of $x^2 = 32y^2 + 1$. Other methods will receive no points. [HINT: Start by noting that $(x, y, k) = (6, 1, 4)$ is a solution of $x^2 = 32y^2 + k$.]

7. (a) [7] If a fair die is tossed five times, what is the probability that at least three sixes are thrown?

(b) [7] Prove al-Hayatham's formula $\displaystyle\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

8. Questions about the set theory of Georg Cantor.

   (a) [2] What does it mean for a set S to be *countable?*.

   (b) [6] Show that the rational numbers $\mathbb{Q}$ are countable.

   (c) [6] Show that the real numbers numbers $\mathbb{R}$ are not countable.

9. Questions about Euclid's Fifth Postulte.

   (a) [4] State Euclid's Fifth Postulate..

   (b) [5] Saccheri proposed to study geometries in which the fifth postulate is replaced by another postulate. Explain what is his *hypothesis of the acute angle*.

   (c) [5] Lobachevski and Bolyai and others constructed geometries that satisfy Saccheri's hypothesis of the acute angle. Describe one of these non-Euclidean geometries. Indicate why the fifth postulate doesn't hold in these geometries.