Math 3070 § 1. **Hockey Example: Test Two Variances** Name: Example
Treibergs **with Unequal Degrees of Freedom** June 2, 2011

Data File Used in this Analysis:

```
# Math 3070 - 1      Hockey Stick Example       June 4, 2011
# Treibergs
#
# From Navidi, "Statistics for Engineers and Scientists, 2nd. ed.,"
# McGraw Hill, 2008
#
# The breaking strength of graphite-kevlar hockey sticks is recorded.
#
# Variables
#         Strength     (newtons)
#         Composite    A or B
Strength Compsite
487.3 A
444.5 A
467.7 A
456.3 A
449.7 A
459.2 A
478.9 A
461.5 A
477.2 A
488.5 B
501.2 B
475.3 B
467.2 B
462.5 B
499.7 B
470.0 B
469.5 B
481.5 B
485.2 B
509.3 B
479.3 B
478.3 B
491.5 B
488.5 B
```

R Session:

```
R version 2.10.1 (2009-12-14)
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ISBN 3-900051-07-0

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 Natural language support but running in an English locale
```

```
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Type 'q()' to quit R.

[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]
[Workspace restored from /Users/andrejstreibergs/.RData]

> tt <- read.table("M3074HockeyData.txt",header=TRUE)
> attach(tt)
> tt
   Strength Compsite
1     487.3        A
2     444.5        A
3     467.7        A
4     456.3        A
5     449.7        A
6     459.2        A
7     478.9        A
8     461.5        A
9     477.2        A
10    488.5        B
11    501.2        B
12    475.3        B
13    467.2        B
14    462.5        B
15    499.7        B
16    470.0        B
17    469.5        B
18    481.5        B
19    485.2        B
20    509.3        B
21    479.3        B
22    478.3        B
23    491.5        B
24    488.5        B
> ################# SUMMARIZE THE DATA ######################################
> tapply(Strength,Compsite,summary)
$A
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  444.5   456.3   461.5   464.7   477.2   487.3

$B
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  462.5   472.6   481.5   483.2   490.0   509.3

> tapply(Strength,Compsite,var)
       A        B
202.7175 182.6752
```
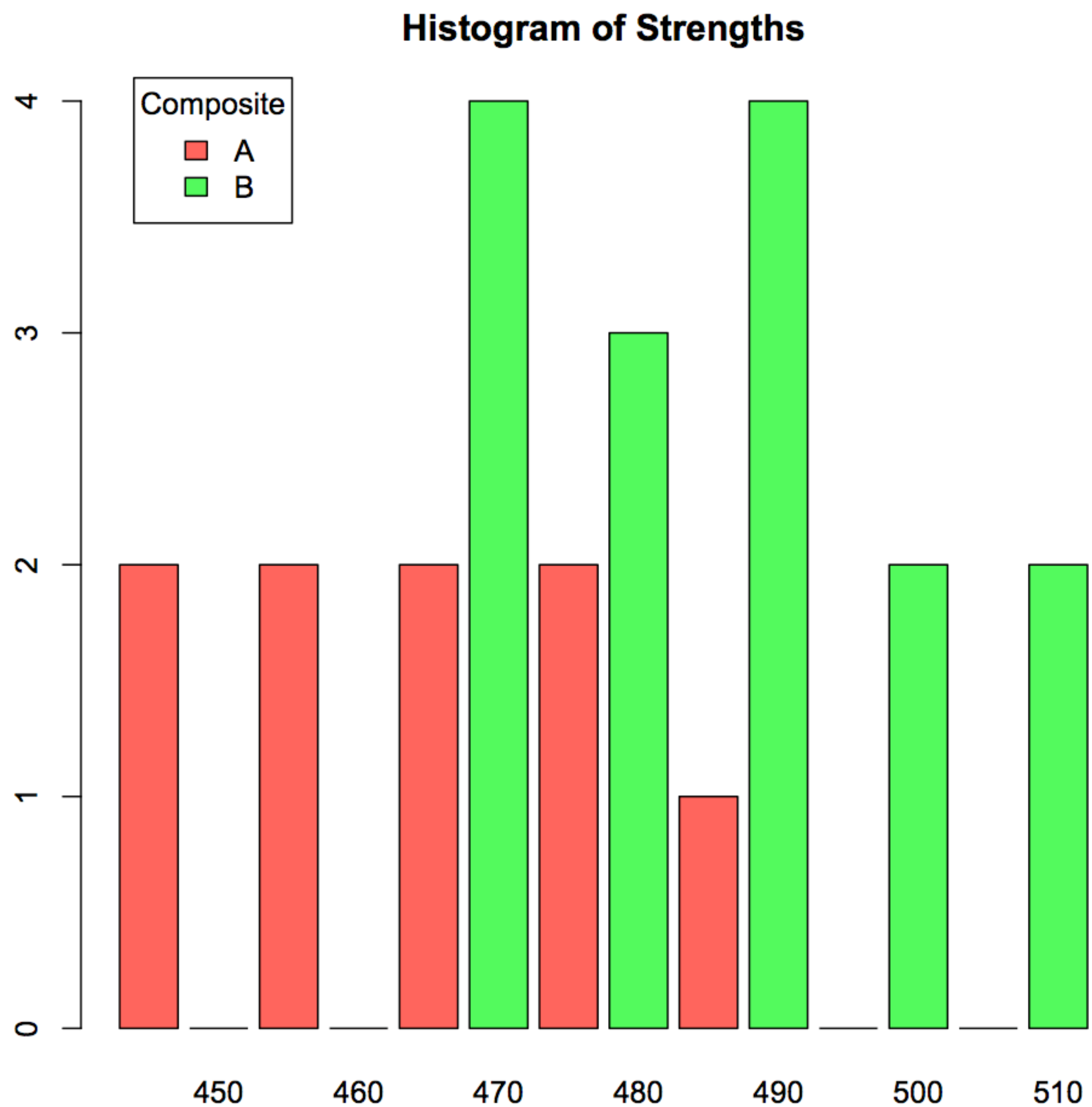
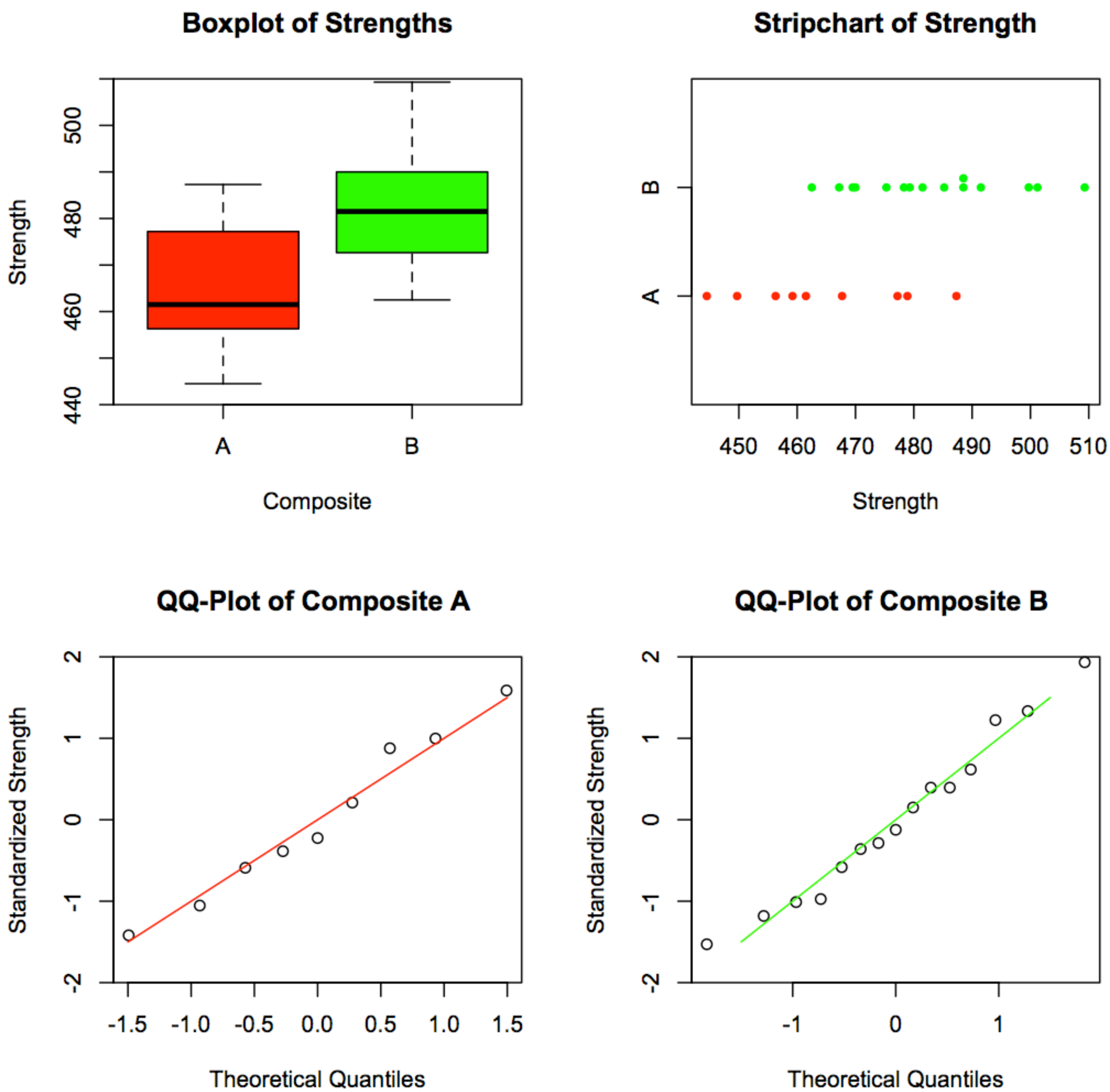

```
> # Pick off the two samples.
> y1 <- Strength[Compsite=="A"]
> y1
[1] 487.3 444.5 467.7 456.3 449.7 459.2 478.9 461.5 477.2
> y2 <- Strength[Compsite=="B"]
> y2
 [1] 488.5 501.2 475.3 467.2 462.5 499.7 470.0 469.5 481.5 485.2 509.3 479.3
[13] 478.3 491.5 488.5


> ###################### SIDE-BY-SIDE HISTOGRAM "BY HAND" ##################
> #  We shall use hist() to compute the bin counts. Then we'll use boxplot()
> # to draw the bars. Take a look at the data together.
> hist(Strength)
> # Take the same breakpoints.
> br <- seq(440,510,10)
> br
[1] 440 450 460 470 480 490 500 510

> # We can pick off number in each bin.
> hh <- hist(y1,breaks=br,ylim=c(0,6))

> # The class "histogram" is list with vectors used in plotting.
> hh
$breaks
[1] 440 450 460 470 480 490 500 510

$counts
[1] 2 2 2 2 1 0 0

$intensities
[1] 0.02222222 0.02222222 0.02222222 0.02222222 0.01111111 0.00000000 0.00000000

$density
[1] 0.02222222 0.02222222 0.02222222 0.02222222 0.01111111 0.00000000 0.00000000

$mids
[1] 445 455 465 475 485 495 505

$xname
[1] "y1"

$equidist
[1] TRUE

attr(,"class")
[1] "histogram"
```

```
> # We care about "counts"
>
> h1 <- hist(y1,breaks=br)
> h2 <- hist(y2,breaks=br)
> c1 <- h1$counts
> c2 <- h2$counts
> matr <- t(matrix(c(c1,c2),ncol=2))
> matr
     [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    2    2    2    2    1    0    0
[2,]    0    0    4    3    4    2    2

> nam <- rbind(rep("",7),br[2:8])
> colr <- rainbow(3,alpha=.7)[1:2]
> barplot(matr,beside=T,col=colr,space=c(0.2,0.2),names=nam)
> title("Histogram of Strengths")
> legend(.445,4.1,fill=colr,legend=c("A","B"),title="Composite")
> # M3074Hockey1.pdf
```



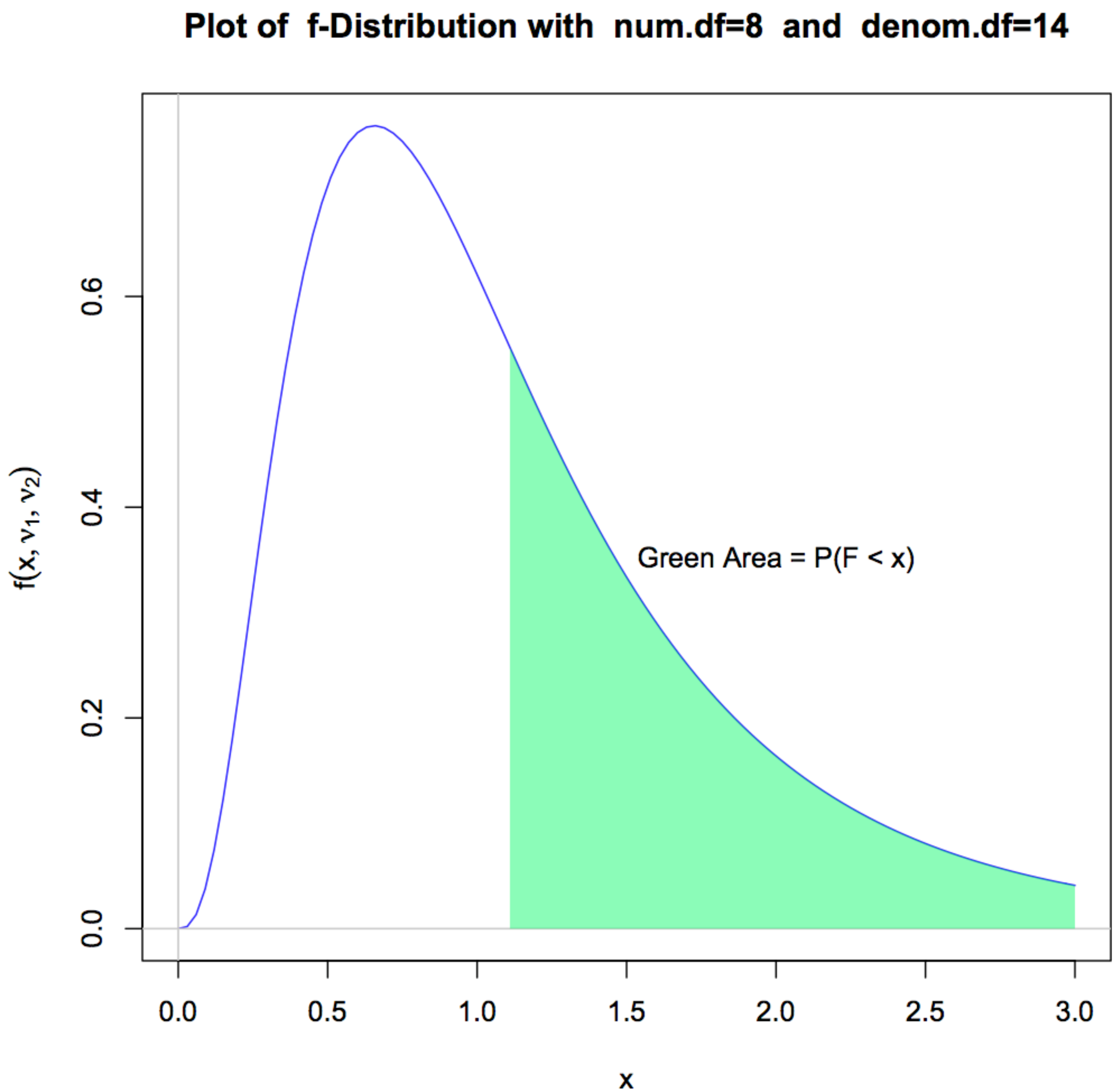
Histogram of Strengths
Composite
A
B
0
1
2
3
4
450
460
470
480
490
500
510

```
> ################## DIAGNOSTIC PLOTS ON STRENGTHS ############################
> layout(matrix(c(1,3,2,4),ncol=2)
>
> boxplot(Strength~Compsite,ylim=c(440,510),ylab="Strength",
+ xlab="Composite",main="Boxplot of Strengths",col=rainbow(4))
>
> stripchart(Strength~Compsite,col=rainbow(3),method="stack",
+ ylim=c(0,3), main="Stripchart of Strength",pch=16)
>
> qqnorm((y1-mean(y1))/sd(y1),ylab="Standardized Strength",
+ main="QQ-Plot of Composite A ",ylim=c(-2,2))
> curve(x-0,-1.5,1.5,col=rainbow(4)[1],add=T)
>
> qqnorm((y2-mean(y2))/sd(y2),ylab="Standardized Strength",
+ main="QQ-Plot of Composite B ",ylim=c(-2,2))
> curve(x-0,-1.5,1.5,col=rainbow(4)[2],add=T)
> # M3074Hockey2.pdf
>
> # The lengths of the boxes are about the same, indicating variances
> # should be nearly the same.
> # The normal QQ-Plots are linear indicating that both populations
> # are plausibly normal.
```


Boxplot of Strengths
Strength
440
460
480
500
A
B
Composite
Stripchart of Strength
B
A
450
460
470
480
490
500
510
Strength
QQ-Plot of Composite A
Standardized Strength
-2
-1
0
1
2
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Theoretical Quantiles
QQ-Plot of Composite B
Standardized Strength
-2
-1
0
1
2
-1
0
1
Theoretical Quantiles

```
>
> ##################### TEST FOR EQUALITY OF VARIANCES #####################
>
> var.test(y1,y2)

F test to compare two variances

data:  y1 and y2
F = 1.1097, num df = 8, denom df = 14, p-value = 0.8254
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.3377832 4.5827526
sample estimates:
ratio of variances
          1.109715

> # High p-value: we cannot reject null hypothesis that ratio of variances = 1

> #################### TEST FOR EQUALITY OF VARIANCE "BY HAND" ##############
> n1 <- length(y1); n1
[1] 9
> n2 <- length(y2); n2
[1] 15
> F <- var(y1)/var(y2); F
[1] 1.109715

> alpha <- .05

> fhighcrit <- qf(alpha/2,n1-1,n2-1,lower.tail=F); fhighcrit
[1] 0.2421504

> fhighcrit <- qf(alpha/2,n1-1,n2-1,lower.tail=FALSE); fhighcrit
[1] 3.285288

> # CI for sigma1^2 / sigma2^2
> c(F/fhighcrit,F/flowcrit)
[1] 0.3377832 4.5827526

> # With unequal df's, critical values are not reciprocal.
> 1/flowcrit
[1] 4.129665


> # For two-tailed test the p-value is smaller area multiplied by 2
> pv <- 2*pf(F,n1-1,n2-1,lower.tail=FALSE); pv
[1] 0.8253555
```

```
> #################### PLOT  f-DISTRIBUTION WITH AREA F<X ####################
> layout=(1)
> F <- 1.109715
> xmax <- 3
> curve(df(x,n1-1,n2-1),0,xmax,ylab=expression(f(x,nu[1],nu[2])),
+ main="Plot of  f-Distribution with  num.df=8  and  denom.df=14",col=4)
> abline(h=0,col="gray"); abline(v=0,col="gray")
> x <- seq(from=F, to=xmax, by=(xmax-F)/100)
> y <- df(x,n1-1,n2-1)
> polygon(c(F,x,x[101],F),c(0,y,0,0),col=rainbow(5,alpha=.5)[3],border=NA)
text(2,.35,"Green Area = P(F < x)")
> # M3074Hockey3.pdf
```



Plot of f-Distribution with num.df=8 and denom.df=14
Green Area = P(F < x)
f(x, ν1, ν2)
x
0.0
0.2
0.4
0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0