Math 3070 § 1. Treibergs | **Approximating the Binomial by Poisson Distribution Example.** | Name: Example June 13, 2011

The binomial distribution describes the probability that there will be x successes in a sample of size n, chosen with replacement from a population where the probability of success is p. In **R**, this is given by

$$P(X = x) = \mathrm{b}(x; n, p) = \mathtt{dbinom}(\mathtt{x}, \mathtt{n}, \mathtt{p}.$$

If $n \to \infty$, $p \to 0$ and $np \to \lambda$ with $\lambda > 0$, then $b(x, n, p) \to p(x; \lambda)$, where the Poisson pmf is given in **R** by

$$P(X = x) = p(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} = \mathtt{dpois}(x, \lambda)$$

Devore's rule of thumb, is that it's O.K. to approximate a binomial pmf by Poisson pmf if n is large and p is small.

> In a binomial experiment where each trial results in S or F and the sampling is with replacement from a population where the probability of success is p, if the sample size $n > 50$ and $np < 5$, then the experiment may be analyzed as if it were a Poisson experiment with $\lambda = np$.

We print and graph the binomial pmf and its Poisson approximation. We see how the errors compare as n increases and p decreases for several np's

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R Session:

```
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[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

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```

```
> ############## SUBPROGRAM TO LIST BINOMIAL VS POISSON PMF'S ###############
> # We use fix(lis)
> lis<- function(n,p,a,b){}
> fix(lis)
> lis <- function(n,p,a,b)
+               {
+                lambda <- n*p
+                x<- matrix(numeric((b-a+1)*3),ncol=3,
+                dimnames = list(a:b,c("Binomial","Poisson","Difference")))
+                x[,1]<-dbinom(a:b,n,p)
+                x[,2]<-dpois(a:b,lambda)
+                x[,3]<-x[,1]-x[,2]
+                error <- max(abs(x[,3]))
+                cat("Poisson Approx to Binomial\n n =", n, "\n p =", p,
+                "\n lambda =", lambda, "\n")
+                print (x)
+                cat("\n Error (Maximum Absolute Difference) = ",error,
+                "\n\n")
+               }

> lis(10,.3,0,10)
Poisson Approx to Binomial
 n = 10
 p = 0.3
 lambda = 3
      Binomial      Poisson    Difference
0  0.0282475249 0.0497870684 -0.0215395435
1  0.1210608210 0.1493612051 -0.0283003841
2  0.2334744405 0.2240418077  0.0094326328
3  0.2668279320 0.2240418077  0.0427861243
4  0.2001209490 0.1680313557  0.0320895933
5  0.1029193452 0.1008188134  0.0021005318
6  0.0367569090 0.0504094067 -0.0136524977
7  0.0090016920 0.0216040315 -0.0126023395
8  0.0014467005 0.0081015118 -0.0066548113
9  0.0001377810 0.0027005039 -0.0025627229
10 0.0000059049 0.0008101512 -0.0008042463

 Error (Maximum Absolute Difference) =  0.04278612

> # Devore's rule of thumb does not hold. Error = .04 is large.
```

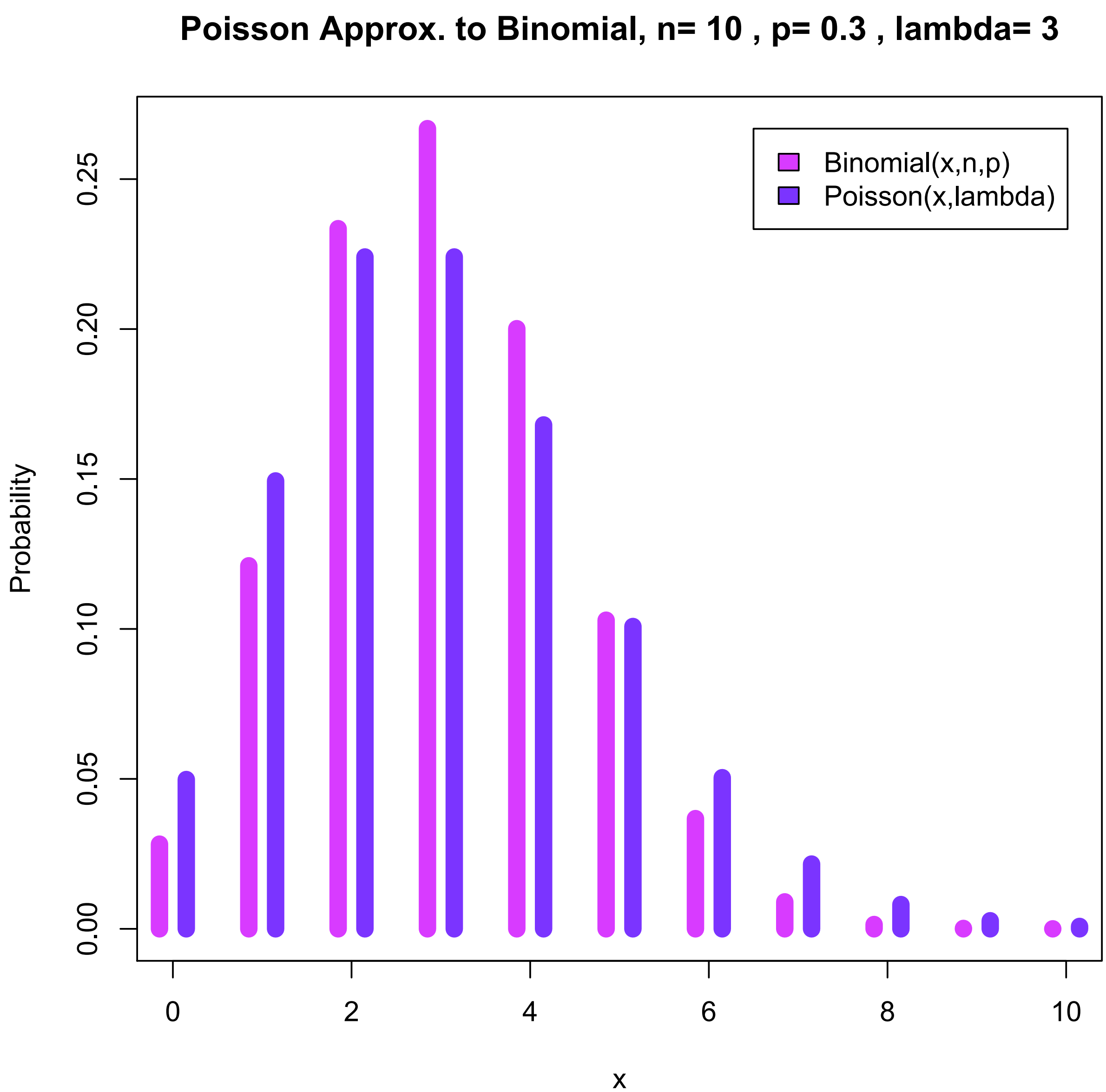
```
> lis(100,.03,0,10)
Poisson Approx to Binomial
 n = 100
 p = 0.03
 lambda = 3
       Binomial      Poisson    Difference
0  0.0475525079 0.0497870684 -0.0022345604
1  0.1470696121 0.1493612051 -0.0022915930
2  0.2251529629 0.2240418077  0.0011111553
3  0.2274741275 0.2240418077  0.0034323198
4  0.1706055956 0.1680313557  0.0025742399
5  0.1013080650 0.1008188134  0.0004892516
6  0.0496096195 0.0504094067 -0.0007997873
7  0.0206037006 0.0216040315 -0.0010003309
8  0.0074077738 0.0081015118 -0.0006937380
9  0.0023419766 0.0027005039 -0.0003585273
10 0.0006591336 0.0008101512 -0.0001510176

 Error (Maximum Absolute Difference) =  0.00343232


> # Devore's rule of thumb does hold. Error = .003 is smaller.


> lis(1000,.003,0,10)
Poisson Approx to Binomial
 n = 1000
 p = 0.003
 lambda = 3
       Binomial      Poisson    Difference
0  0.0495630828 0.0497870684 -2.239855e-04
1  0.1491366584 0.1493612051 -2.245467e-04
2  0.2241537439 0.2240418077  1.119363e-04
3  0.2243785721 0.2240418077  3.367645e-04
4  0.1682839291 0.1680313557  2.525734e-04
5  0.1008690833 0.1008188134  5.026984e-05
6  0.0503333690 0.0504094067 -7.603768e-05
7  0.0215065348 0.0216040315 -9.749664e-05
8  0.0080325937 0.0081015118 -6.891811e-05
9  0.0026641033 0.0027005039 -3.640064e-05
10 0.0007944212 0.0008101512 -1.573001e-05

 Error (Maximum Absolute Difference) =  0.0003367645

 > # Devore's rule of thumb does hold. Error = .0003 is small.
```

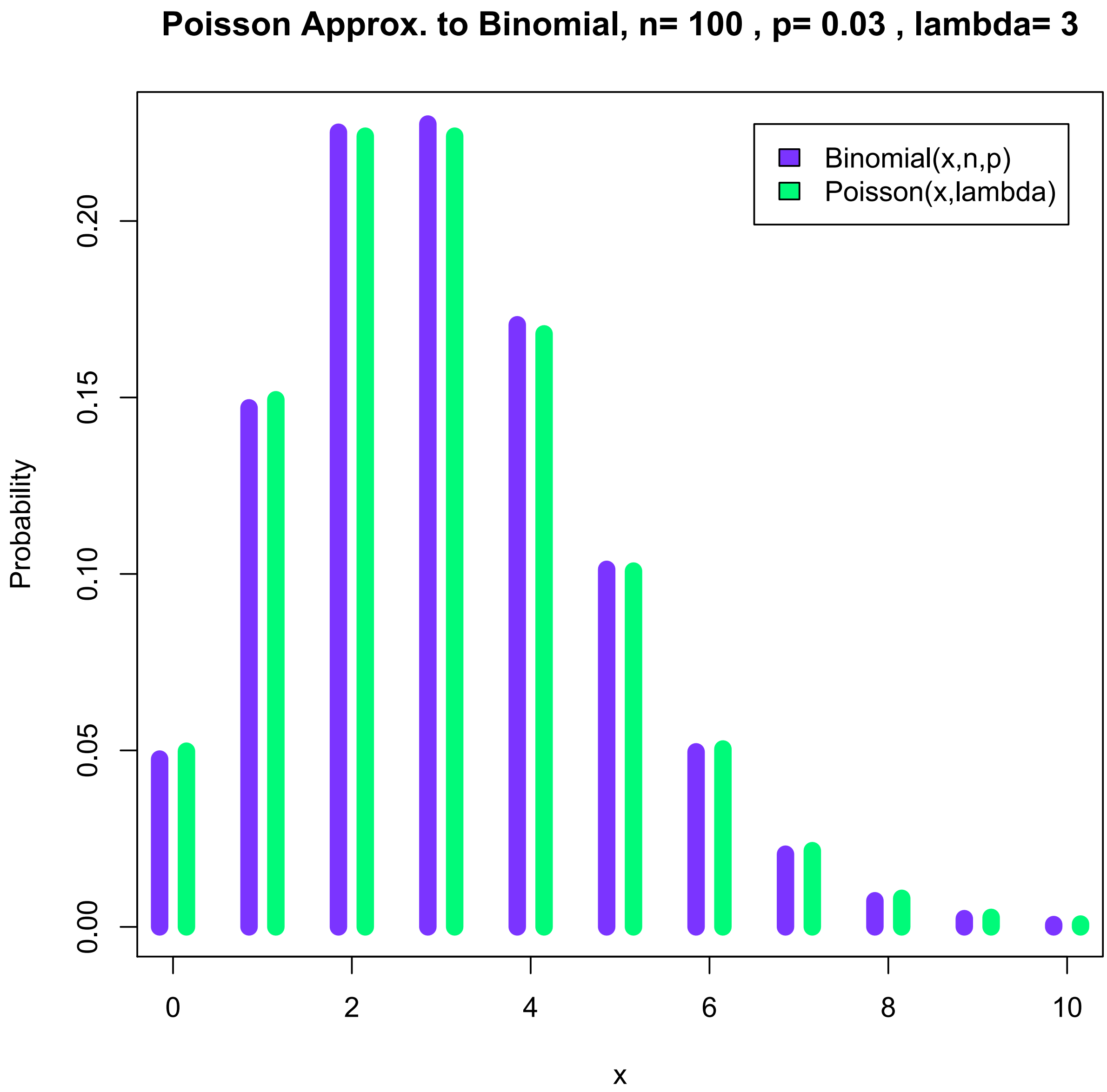
```
> lis(10000,.0003,0,10)
Poisson Approx to Binomial
 n = 10000
 p = 3e-04
 lambda = 3
      Binomial      Poisson    Difference
0  0.049764665 0.0497870684 -2.240362e-05
1  0.149338796 0.1493612051 -2.240922e-05
2  0.224053009 0.2240418077  1.120125e-05
3  0.224075421 0.2240418077  3.361327e-05
4  0.168056566 0.1680313557  2.520996e-05
5  0.100823853 0.1008188134  5.039553e-06
6  0.050401841 0.0504094067 -7.565634e-06
7  0.021594307 0.0216040315 -9.724596e-06
8  0.008094625 0.0081015118 -6.886842e-06
9  0.002696859 0.0027005039 -3.645123e-06
10 0.000808572 0.0008101512 -1.579117e-06

 Error (Maximum Absolute Difference) =  3.361327e-05


 > # Devore's rule of thumb does hold. Error = .00003 is small.
```

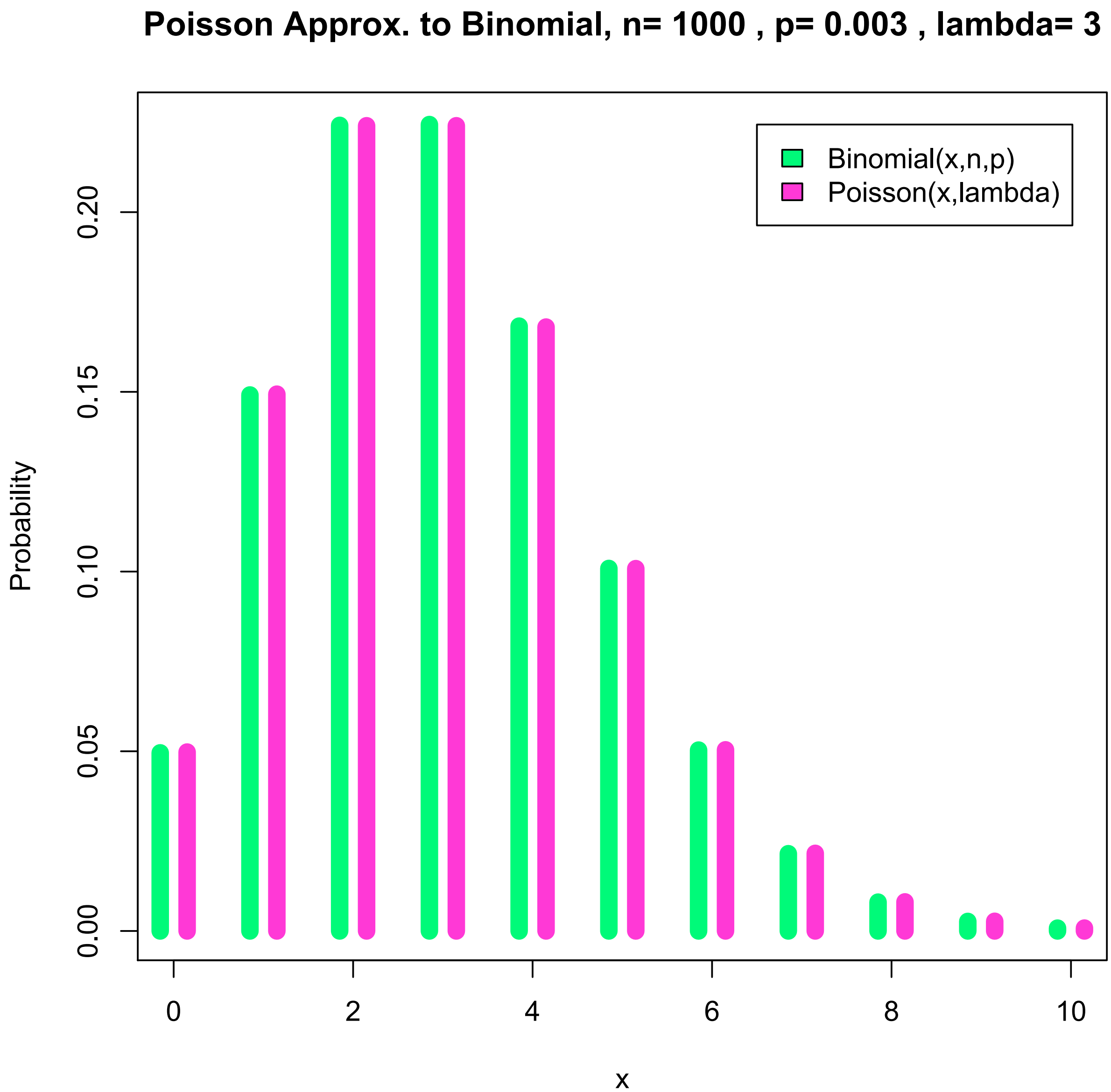
```
> lis(100000,.00003,0,10)
Poisson Approx to Binomial
 n = 1e+05
 p = 3e-05
 lambda = 3
       Binomial      Poisson    Difference
0  0.0497848280 0.0497870684 -2.240412e-06
1  0.1493589646 0.1493612051 -2.240468e-06
2  0.2240429279 0.2240418077  1.120201e-06
3  0.2240451684 0.2240418077  3.360697e-06
4  0.1680338763 0.1680313557  2.520523e-06
5  0.1008193175 0.1008188134  5.040802e-07
6  0.0504086505 0.0504094067 -7.561833e-07
7  0.0216030592 0.0216040315 -9.722092e-07
8  0.0081008232 0.0081015118 -6.886341e-07
9  0.0027001394 0.0027005039 -3.645625e-07
10 0.0008099932 0.0008101512 -1.579727e-07

 Error (Maximum Absolute Difference) =  3.360697e-06


> # Devore's rule of thumb does hold. Error = .000003 is small.
```

```
> ################### ERROR FOR VARIOUS n AND p ############################
> ps <- c(.00001,.0001,.001,.0025,.005,.01,.02,.04,.1,.2,.4)
> ns <- c(10,100,1000,10000)
> ps
 [1] 0.00001 0.00010 0.00100 0.00250 0.00500 0.01000 0.02000 0.04000 0.10000 0.20000 0.40000
> ns
[1]    10   100  1000 10000
> z <- matrix(1:88,ncol=8)
> for (i in 1:4)
+               {
+                for(j in 1:11)
+                              {
+                               z[j,2*i-1]<- round(mx(ns[i],ps[j]),10)
+                               if((ns[i] > 50) &  (ns[i]*ps[j]< 5))
+                                                                     {
+                                                                      z[j,2*i]<-"<-RoT OK   "
+                                                                     }
+                               else
+                                                                     {
+                                                                      z[j,2*i]<-"   "
+                                                                     }
+                              }
+                }
> print(z,quote=FALSE)
        10                 100                        1000                       10000
0.00001 0.000000001        0.00000001   <-RoT OK    0.0000000985 <-RoT OK    0.0000008596 <-RoT OK
0.0001  0.0000000999       0.0000009852 <-RoT OK    0.0000085968 <-RoT OK    0.000018395  <-RoT OK
0.001   0.0000098608       0.0000860427 <-RoT OK    0.0001840471 <-RoT OK    0.0000626009
0.0025  0.0000603445       0.0004268818 <-RoT OK    0.0002944837 <-RoT OK    0.0000995888
0.005   0.0002330077       0.0011419247 <-RoT OK    0.0004402467             0.0001413404
0.01    0.0008679829       0.0018501965 <-RoT OK    0.0006301754             0.000200809
0.02    0.0030034018       0.0027433451 <-RoT OK    0.0009017519             0.0002862738
0.04    0.0088855799       0.0040216823 <-RoT OK    0.0012977906             0.0004112312
0.1     0.0195410478       0.0067553111             0.0021557941             0.0006823442
0.2     0.0313193215       0.0104648974             0.0033276335             0.0010528718
0.4     0.0554558412       0.0182721056             0.00580186               0.0018354604
>
> # "RoT OK" = Rule of Thumb holds.
```

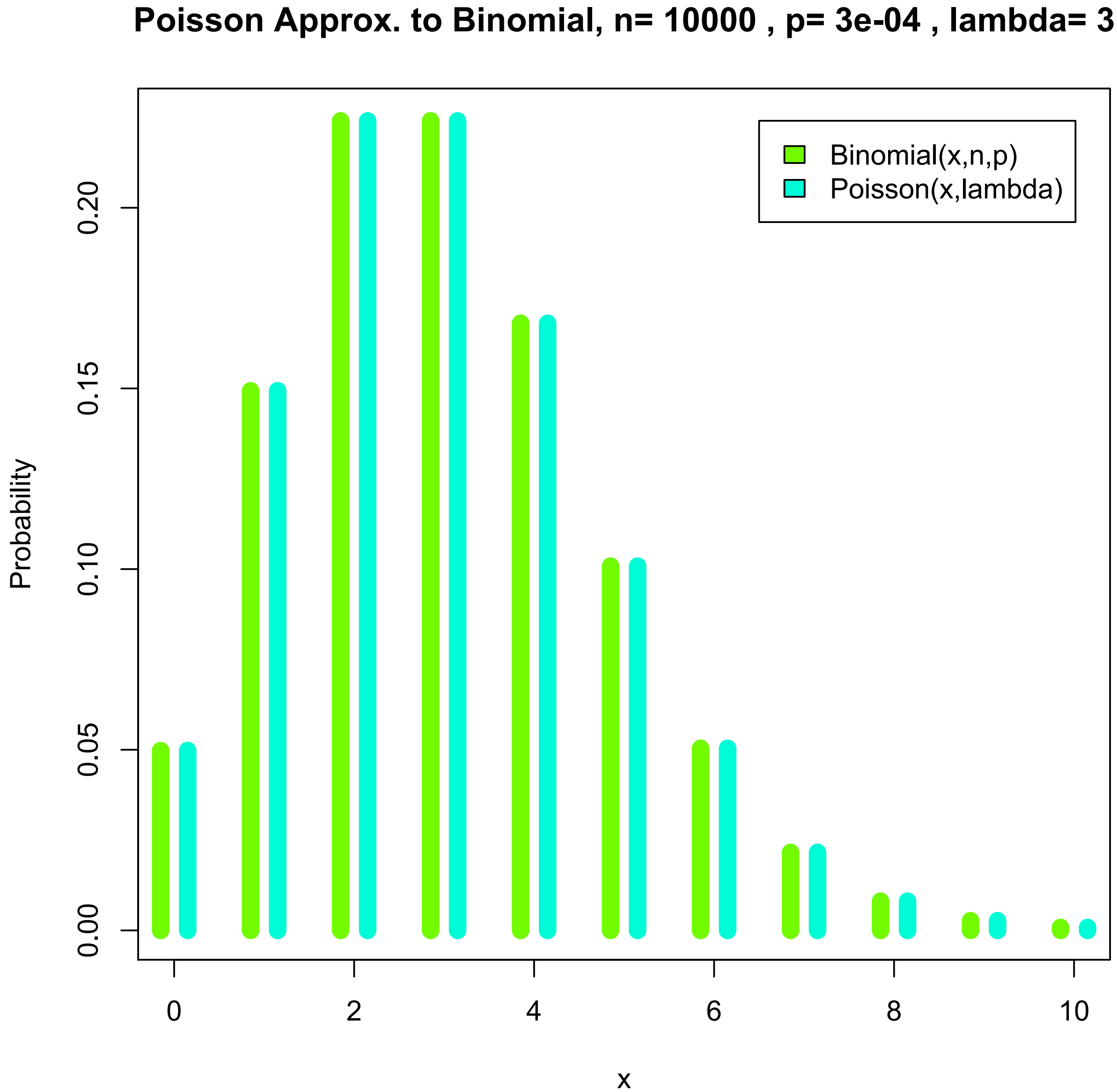
```
> ################## PLOT POISSON AND BINOMIAL PMF'S ##############
> # Write a subprogram to do plot of Binomial vs. Poisson
> pl <- function(n,p,a,b)
+                {
+                 clr<-rainbow(15)[ceiling(runif(2,4,15))]
+                 lambda <- n*p
+                 mx <- max(dbinom(a:b,n,p))
+                 plot(c(a:b,a:b),c(dbinom(a:b,n,p), dpois(a:b,lambda)),
+                 type="n", main = paste("Poisson Approx. to Binomial, n=", n,
+                 ", p=", p, ", lambda=",lambda), ylab = "Probability", xlab="x")
+                 points((a:b)-.15,dbinom(a:b,n,p), type = "h",
+                 col = clr[1], lwd = 10)
+                 points((a:b)+.15,dpois(a:b,lambda), type="h",
+                 col = clr[2], lwd=10)
+                 legend(b-3.5, mx, legend=c("Binomial(x,n,p)",
+                 "Poisson(x,lambda)"), fill = clr, bg="white")
+                }
> pl(10,.3,0,10)
> # M3074PoisApprox1.pdf
>
> pl(100,.03,0,10)
> # M3074PoisApprox2.pdf
>
> pl(1000,.003,0,10)
> # M3074PoisApprox3.pdf
>
> pl(10000,.0003,0,10)
> # M3074PoisApprox4.pdf
```


Poisson Approx. to Binomial, n= 10 , p= 0.3 , lambda= 3
Binomial(x,n,p)
Poisson(x,lambda)
Probability
0.00
0.05
0.10
0.15
0.20
0.25
0
2
4
6
8
10
x


Poisson Approx. to Binomial, n= 100 , p= 0.03 , lambda= 3
Binomial(x,n,p)
Poisson(x,lambda)
Probability
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
x



Poisson Approx. to Binomial, n= 1000 , p= 0.003 , lambda= 3
Binomial(x,n,p)
Poisson(x,lambda)
Probability
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
x


Poisson Approx. to Binomial, n= 10000 , p= 3e-04 , lambda= 3
Binomial(x,n,p)
Poisson(x,lambda)
Probability
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
x