Math 3070 § 1.	**Railway Example:**	Name: Example
Treibergs	**Power of One Sample t-Test**	May 25, 2011

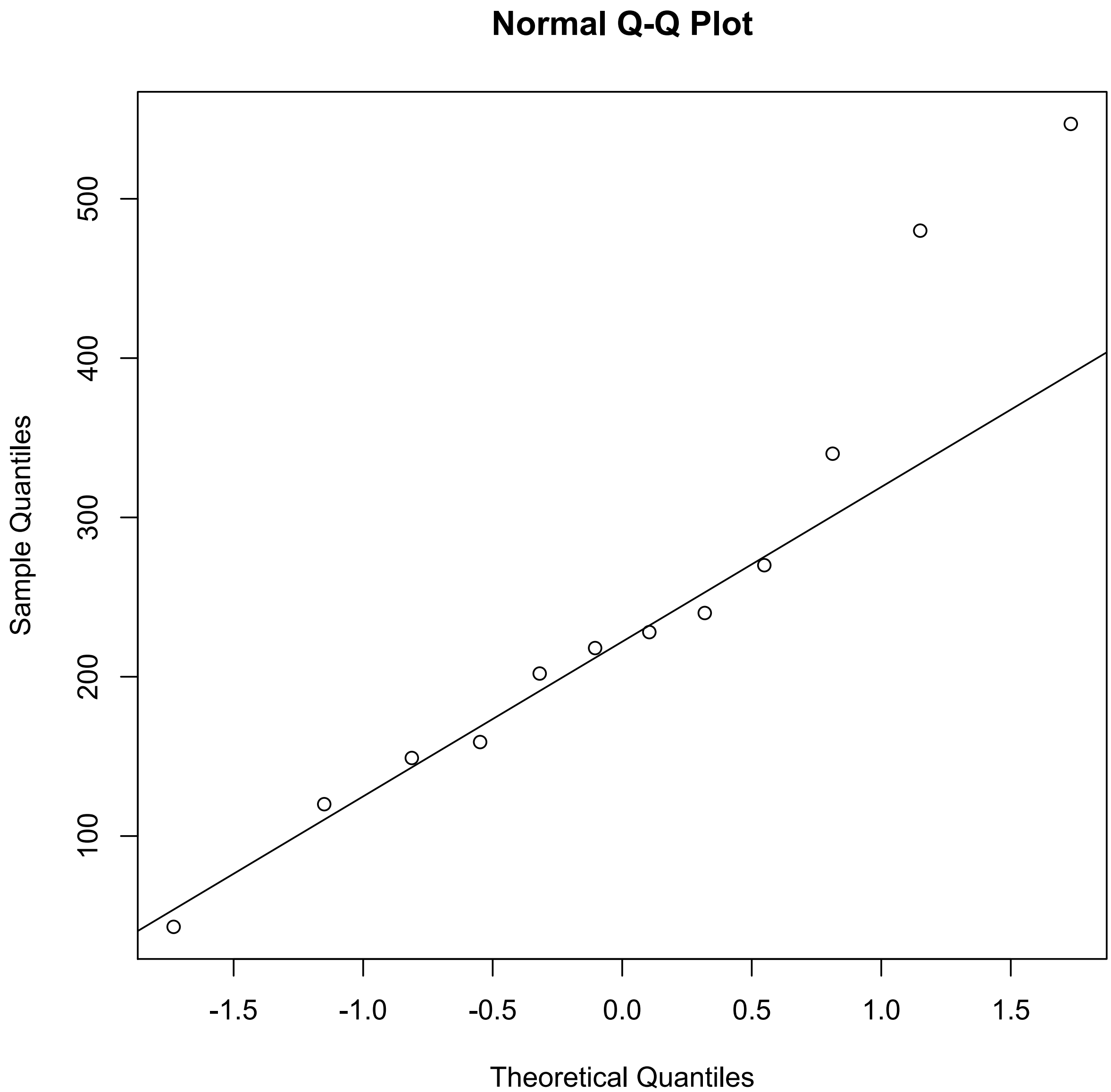
The power of a t-test is described by the operation characteristic curves of Table A.17 in Devore. It is also canned in **R**. In this example, we shall use **R** to solve one of the textbook problems, as well as to illustrate the use of the OC-curves. Finally, we will plot our own OC curves.

R Session:

```
R version 2.11.1 (2010-05-31)
Copyright (C) 2010 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.34 (5589) i386-apple-darwin9.8.0]

[Workspace restored from /home/1004/ma/treibergs/.RData]

> ################### COMPUTING BETA FOR ONE-SAMPLE t-TEST ####################
>
> # In Example 8.10, a t-test is used to determine whether the true average voltage
> # drop is at most mu0 = 2.5. A sample of  n = 10  voltages are measured
> # to test H0: mu = mu0  vs.  Ha: mu > mu0 at significance level alpha = .05.
> # If sd = .100, how likely will H0 not be rejected when in fact mu=2.6?
> # We compute d=|2.5 -2.6|/.1 = 1.0. df = n - 1 = 9. On the df = 9 curve
> # bets is approx .1. let's see what R says it is using  beta = 1 - power.
>
> ptt <- power.t.test(delta=abs(2.5-2.6),sd=.1,n=9,type="one.sample",
+ alternative="one.sided"); ptt

     One-sample t test power calculation

              n = 9
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.8618137
    alternative = one.sided
```

```
> beta <- 1 - ptt$power; beta
[1] 0.1381863
>
> # The example continues to find  beta's  for upper-tailed tests alpha = .05, sd = .1
> # mu1 = 2.6,  mu0 = 2.5 and  n = 5, 10, 15.
>
> $ the variables associated to the power.test() list may be found by
> names(ptt)
[1] "n"      "delta"    "sd"    "sig.level"   "power"   "alternative"   "note"   "method"
>
> To access the power part of the list, we look at the ptt$power
>
> ptt <- power.t.test(delta = abs(mu1-mu0), sd=sd, n=c(5,10,15), type="one.sample",
+ alternative="one.sided"); ptt

     One-sample t test power calculation

              n = 5, 10, 15
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.5797374, 0.8975170, 0.9789162
    alternative = one.sided

> ptt$power
[1] 0.5797374 0.8975170 0.9789162

> # R can also find sample sizes for a given  power.  For power = .9:
>
>  power.t.test(delta=2.6-2.5,sd=.1,power=.9,sig.level=.05,type="one.sample",
+  alternative="one.sided")

     One-sample t test power calculation

              n = 10.08107
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.9
    alternative = one.sided

> # Hence we round up and use  n=11. The actual power for  n=11 is
>  power.t.test(delta=2.6-2.5,sd=.1,n=11,sig.level=.05,type="one.sample",alternative="one.sided")

     One-sample t test power calculation

              n = 11
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.924489
```

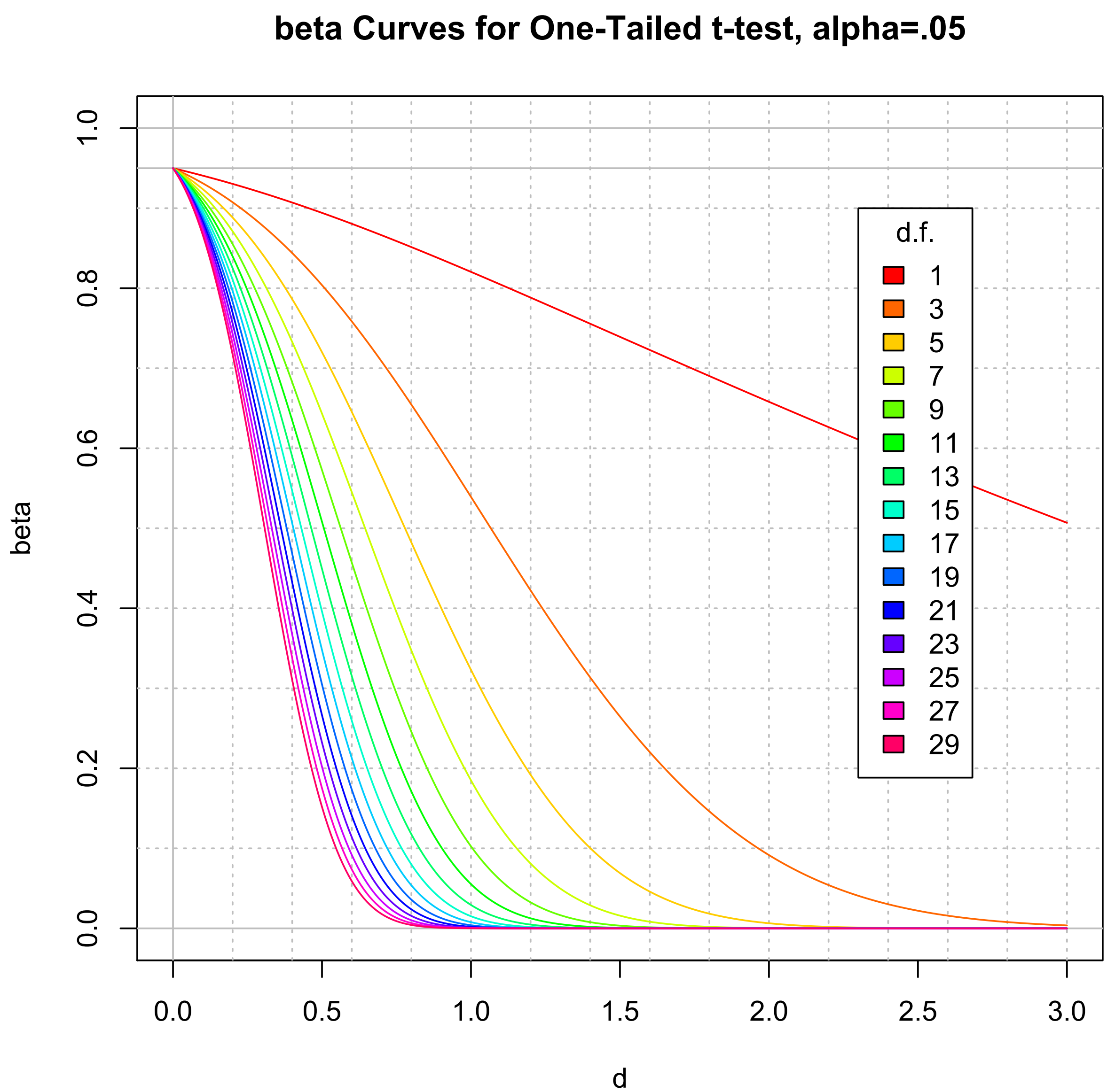
```
    alternative = one.sided

> # For power = .95,
>  power.t.test(delta=2.6-2.5,sd=.1,power=.95,sig.level=.05,type="one.sample",
+  alternative="one.sided")

     One-sample t test power calculation

              n = 12.32052
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.95
    alternative = one.sided

> # The actual power after rounding up to  n = 13:
>  power.t.test(delta=2.6-2.5,sd=.1,n=13,sig.level=.05,type="one.sample",
+  alternative="one.sided")

     One-sample t test power calculation

              n = 13
          delta = 0.1
             sd = 0.1
      sig.level = 0.05
          power = 0.9597032
    alternative = one.sided


> ############################# SOLVE DEVORE PROBLEM 9.29 ##########################

> # Repair time (in minutes) data for fixing a cracked railway is presented.
> # a.) Is normality plausible?
> # b.) Is there compelling evidence to conclude that repair time exceeds 200 min?
> #     (Test at alpha = .05.)
> # c.) Assuming sd = 150 and mu0 = 200, what is the probability of making a
> #     Type II error when the true repair time is  mu1=300 min?
>
> repair.time<-c(159,120,480,149,270,547,340,43,228,202,240,218)
> # Check:
> repair.time
 [1] 159 120 480 149 270 547 340  43 228 202 240 218

> qqnorm(repair.time);qqline(repair.time)
> # M3074PowerT5.pdf
```



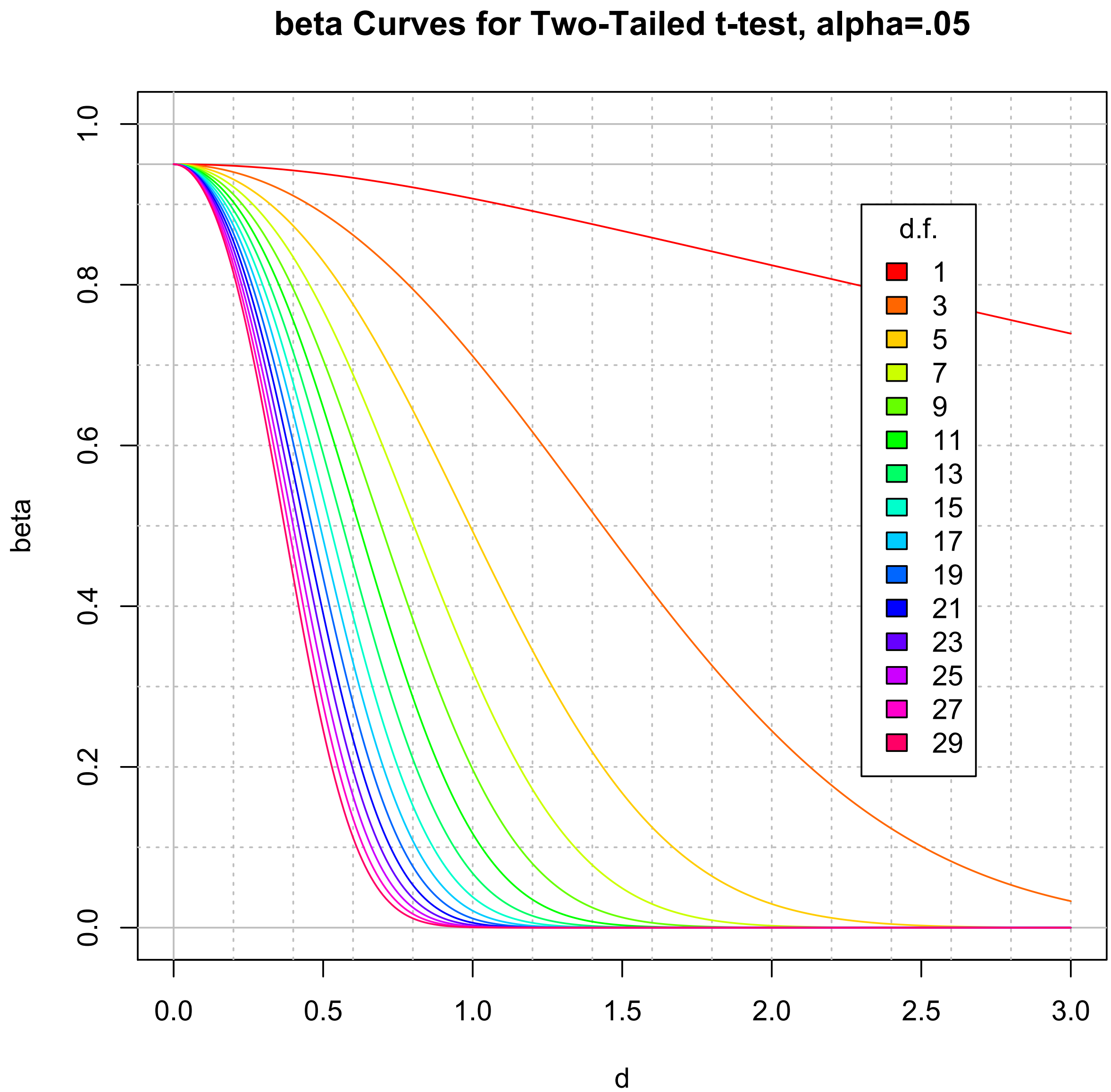

```
> # a.) For small n-size, even this QQ-plot is not deviating from normality
> #     very much. So normality is plausible.
>
>
> summary(repair.time)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   43.0   156.5   223.0   249.7   287.5   547.0
> rtsd <-sd(repair.time);rtsd
[1] 145.1490
> rtm <- mean(repair.time);rtm
[1] 249.6667
```

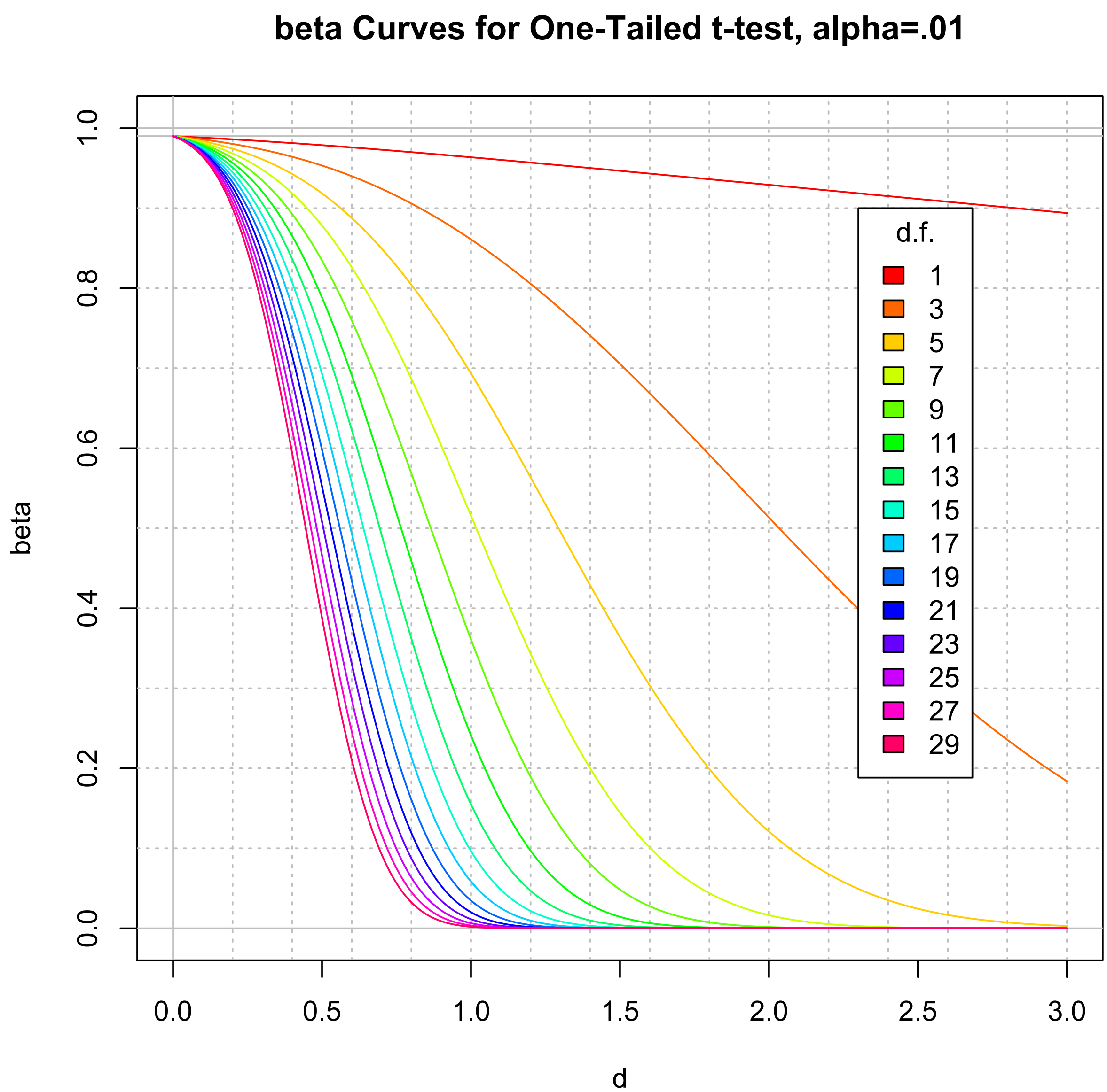
```
> # Is there evidence that rt > 200 min? Do the t-test.
> t.test(repair.time,alternative="greater",mu=200)

One Sample t-test

data:  repair.time
t = 1.1853, df = 11, p-value = 0.1304
alternative hypothesis: true mean is greater than 200
95 percent confidence interval:
 174.4174      Inf
sample estimates:
mean of x
 249.6667
>
> # Or do it "by hand."
>
> tval <- (rtm-200)/(rtsd/sqrt(12)); tval
[1] 1.185336
> pt(tval,11,lower.tail=FALSE)
[1] 0.1304300
>
> b.) The p-value exceeds .05 so not significant evidence that mu > 200.
>
> pttt <- power.t.test(delta=300-200, sd=150, n=12, sig.level=.05, type="one.sample",
+         alternative="one.sided"); pttt

     One-sample t test power calculation

              n = 12
          delta = 100
             sd = 150
      sig.level = 0.05
          power = 0.6981908
    alternative = one.sided

> # c.) Thus  beta = 1 - power = approx. .30
> 1-pttt$power
[1] 0.3018092
>


> ############## beta-CURVES (OPERATING CHARACTERISTIC CURVES) FOR t-TEST #############
>
> # The d values (difference of means). Same as d  if sd = 1.
> de <- seq(from=0,to=3,by=.01)
> ptt <- power.t.test(delta=de,sd=1,n=10,type="one.sample",alternative="one.sided")
>
> plot(de,1-ptt$power,type="n",col=2,xlim=c(0,3),ylim=0:1,
+ main="beta Curves for One-Tailed t-test, alpha=.05",xlab="d",ylab="beta")
> abline(h=c(0,.95,1),col="gray");abline(h=1:9/10,col="gray",lty=3)
> abline(v=0,col="gray");abline(v=1:15/5,col="gray",lty=3)
> for(j in 1:15)  lines(de,1-power.t.test(delta=de,sd=1,n=2*j,type="one.sample",
```

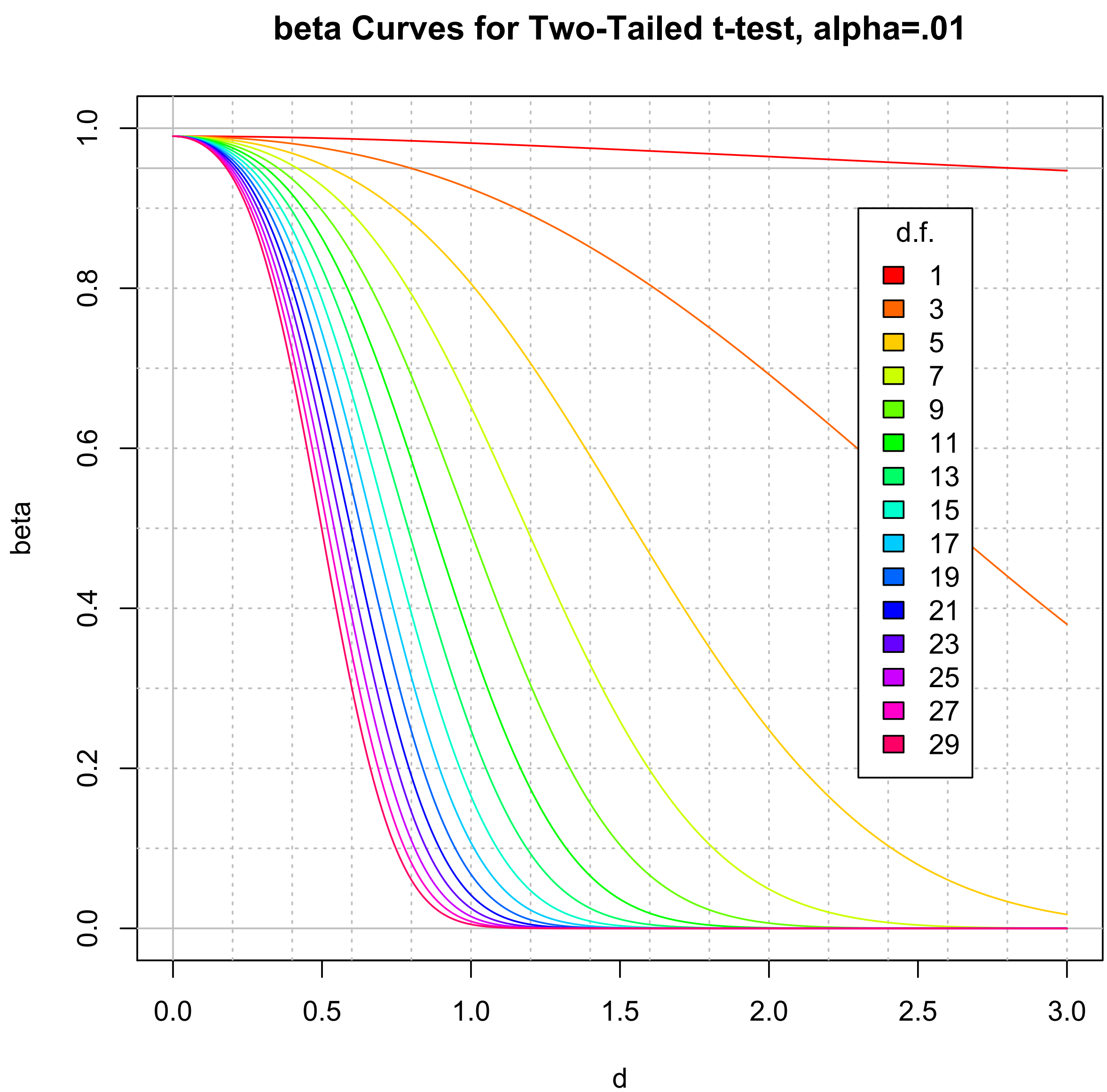
```
+ alternative="one.sided",sig.level=.05)$power,type="l",col=rainbow(15)[j])
> legend(2.3,.9,legend=1:15*2-1,fill=rainbow(15),title="d.f.",bg="white")
> # M3074PowerT1.pdf

>
>
>
> plot(de,1-ptt$power,type="n",col=2,xlim=c(0,3),ylim=0:1,
+ main="beta Curves for Two-Tailed t-test, alpha=.05",xlab="d",ylab="beta")
> abline(h=c(0,.95,1),col="gray");abline(h=1:9/10,col="gray",lty=3)
> abline(v=0,col="gray");abline(v=1:15/5,col="gray",lty=3)
> for(j in 1:15)  lines(de,1-power.t.test(delta=de,sd=1,n=2*j,type="one.sample",
+ alternative="two.sided",sig.level=.05,strict=T)$power,type="l",col=rainbow(15)[j])
> legend(2.3,.9,legend=1:15*2-1,fill=rainbow(15),title="d.f.",bg="white")
> # M3074PowerT2.pdf
>
>
>
> plot(de,1-ptt$power,type="n",col=2,xlim=c(0,3),ylim=0:1,
+ main="beta Curves for One-Tailed t-test, alpha=.01",xlab="d",ylab="beta")
> abline(h=c(0,.99,1),col="gray");abline(h=1:9/10,col="gray",lty=3)
> abline(v=0,col="gray");abline(v=1:15/5,col="gray",lty=3)
> for(j in 1:15)  lines(de,1-power.t.test(delta=de,sd=1,n=2*j,type="one.sample",
+ alternative="one.sided",sig.level=.01)$power,type="l",col=rainbow(15)[j])
> legend(2.3,.9,legend=1:15*2-1,fill=rainbow(15),title="d.f.",bg="white")
> # M3074PowerT3.pdf

>
> plot(de,1-ptt$power,type="n",col=2,xlim=c(0,3),ylim=0:1,
+ main="beta Curves for Two-Tailed t-test, alpha=.01",xlab="d",ylab="beta")
> abline(h=c(0,.99,1),col="gray");abline(h=1:9/10,col="gray",lty=3)
> abline(v=0,col="gray");abline(v=1:15/5,col="gray",lty=3)
> for(j in 1:15)  lines(de,1-power.t.test(delta=de,sd=1,n=2*j,type="one.sample",
+ alternative="two.sided",sig.level=.01,strict=T)$power,type="l",col=rainbow(15)[j])
> legend(2.3,.9,legend=1:15*2-1,fill=rainbow(15),title="d.f.",bg="white")
> # M3074PowerT4.pdf
>
>
```



beta Curves for One-Tailed t-test, alpha=.05
beta
0.0
0.2
0.4
0.6
0.8
1.0
d
0.0
0.5
1.0
1.5
2.0
2.5
3.0
d.f.
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29


beta Curves for Two-Tailed t-test, alpha=.05
1.0
0.8
0.6
0.4
0.2
0.0
beta
0.0
0.5
1.0
1.5
2.0
2.5
3.0
d
d.f.
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29


beta Curves for One-Tailed t-test, alpha=.01
beta
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
d
d.f.
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29


beta Curves for Two-Tailed t-test, alpha=.01
beta
1.0
0.8
0.6
0.4
0.2
0.0
d
0.0
0.5
1.0
1.5
2.0
2.5
3.0
d.f.
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29