Data File Used in this Analysis:

```
# Math 3080 - 1       Hardness Data       April 7, 2010
# Treibergs
#
# From Navidi, "Statistics for Engineers and Scientists," McGraw Hill 2006
#
# Taken from "...High Energy Electron Beam Case Hardening of Cast Iron,"
# (Surface Engineering, 2003.)
#
# Several factors are tested for an electron beam process that affects
# the hardness of metal.
# A = factor determined by travel speed in mm/sec
# B = factor corresponding to accelerating voltage in volts
# Hardness = resulting surface measured in Vickers Hardness
#
"A"   "B"   "Hardness"
10   10   875
10   10   896
10   10   921
10   10   686
10   10   642
10   10   613
10   20   712
10   20   719
10   20   698
10   20   621
10   20   632
10   20   645
10   30   568
10   30   546
10   30   559
10   30   757
10   30   723
10   30   734
20   10   876
20   10   835
20   10   868
20   10   812
20   10   796
20   10   772
20   20   889
20   20   876
20   20   849
20   20   768
20   20   706
20   20   615
20   30   756
20   30   732
```

```
20   30   723
20   30   681
20   30   723
20   30   712
30   10   901
30   10   926
30   10   893
30   10   856
30   10   832
30   10   841
30   20   789
30   20   801
30   20   776
30   20   845
30   20   827
30   20   831
30   30   792
30   30   786
30   30   775
30   30   706
30   30   675
30   30   568
```

R Session:

```
R version 2.10.1 (2009-12-14)
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  Natural language support but running in an English locale

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'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.31 (5537) powerpc-apple-darwin9.8.0]

> tt <- read.table("M3081dataHardness",header=TRUE)
Error in file(file, "rt") : cannot open the connection
In addition: Warning message:
In file(file, "rt") :
  cannot open file 'M3081dataHardness': No such file or directory
> tt <- read.table("M3081DataHardness.txt",header=TRUE)
```
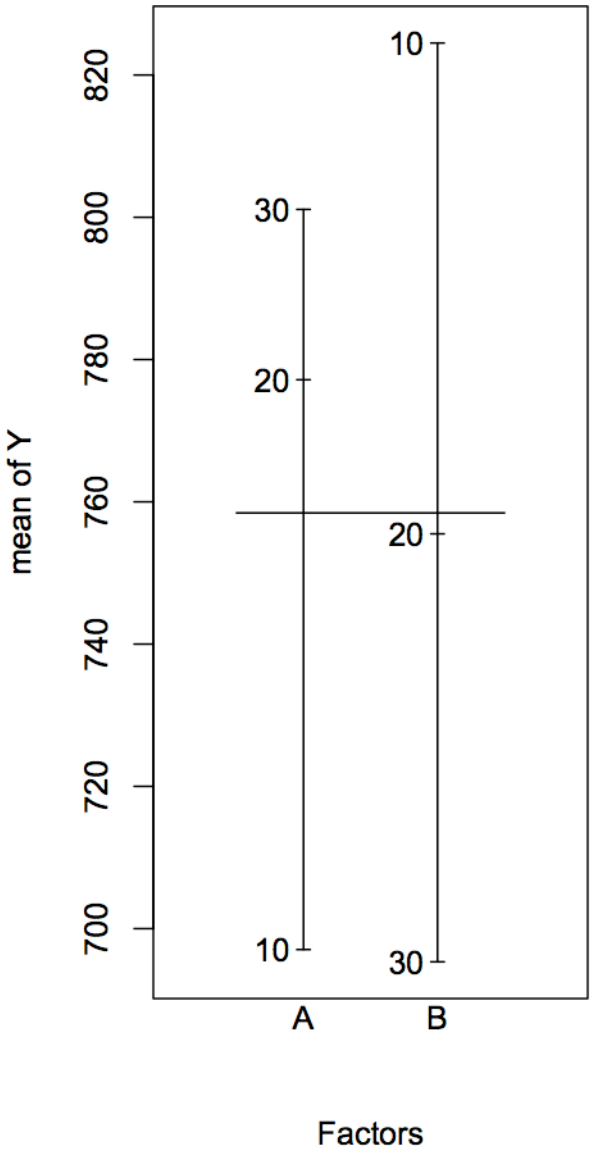

```
> tt
    A  B Hardness
1  10 10      875
2  10 10      896
3  10 10      921
4  10 10      686
5  10 10      642
6  10 10      613
7  10 20      712
8  10 20      719
9  10 20      698
10 10 20      621
11 10 20      632
12 10 20      645
13 10 30      568
14 10 30      546
15 10 30      559
16 10 30      757
17 10 30      723
18 10 30      734
19 20 10      876
20 20 10      835
21 20 10      868
22 20 10      812
23 20 10      796
24 20 10      772
25 20 20      889
26 20 20      876
27 20 20      849
28 20 20      768
29 20 20      706
30 20 20      615
31 20 30      756
32 20 30      732
33 20 30      723
34 20 30      681
35 20 30      723
36 20 30      712
37 30 10      901
38 30 10      926
39 30 10      893
40 30 10      856
41 30 10      832
42 30 10      841
43 30 20      789
44 30 20      801
45 30 20      776
46 30 20      845
47 30 20      827
48 30 20      831
49 30 30      792
50 30 30      786
```

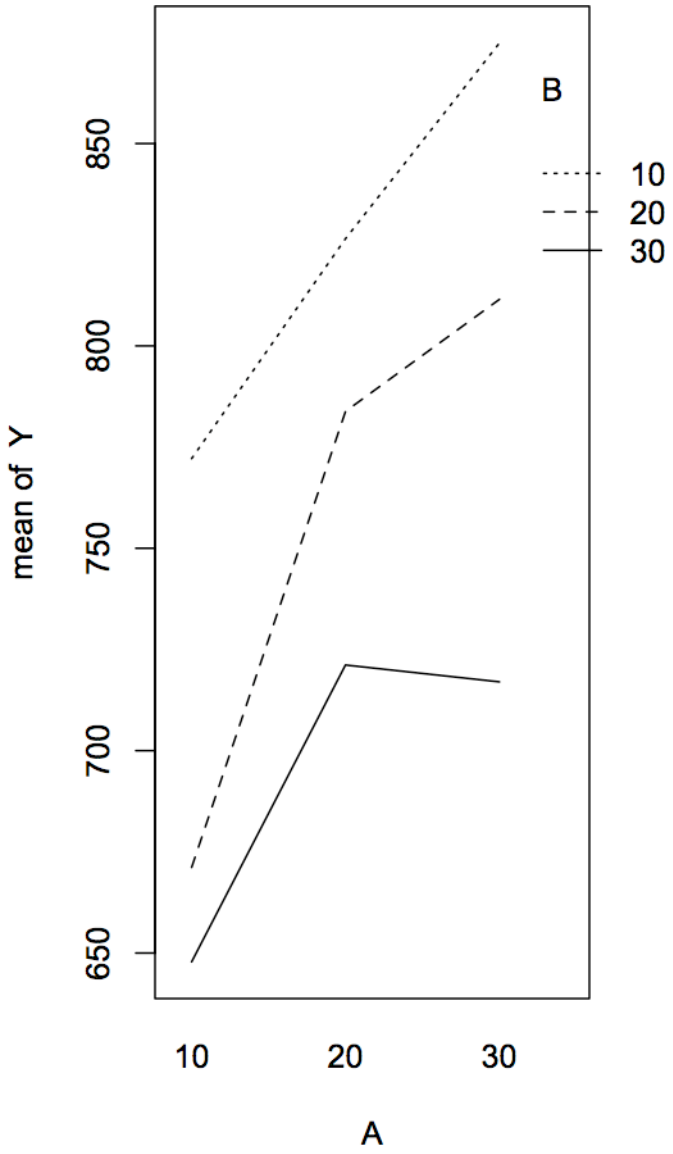
```
51 30 30      775
52 30 30      706
53 30 30      675
54 30 30      568
> attach(tt)
> Y <- Hardness
> A <- factor(A)
> B <- factor(B)

>#===================================PLOT DESIGN AND INTERACTION====================
> layout(matrix(1:2,ncol=2))
> plot.design(data.frame(A,B,Y))
> interaction.plot(A,B,Y)
>#===================================RUN TWO FACTOR ANOVA==========================
> f1 <- aov(Y~A*B);anova(f1)
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
A          2 106912   53456  8.7401  0.000621 ***
B          2 150390   75195 12.2945 5.497e-05 ***
A:B        4  11409    2852  0.4663  0.760062
Residuals 45 275228    6116
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

>#===================================THE INTERACTION TERM IS NEGLIGIBLE=============
>#==================TUKEY HSD TO SEE IF SIGNIFICANT DIFFERENCES IN EFFECTS=========

> sort(tapply(Y,A,mean))
      10       20       30
697.0556 777.1667 801.1111

> TukeyHSD(f1,which="A",ordered=TRUE)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = Y ~ A * B)

$A
           diff       lwr       upr     p adj
20-10  80.11111  16.93074 143.29148 0.0098593
30-10 104.05556  40.87518 167.23593 0.0006889
30-20  23.94444 -39.23593  87.12482 0.6315525
```

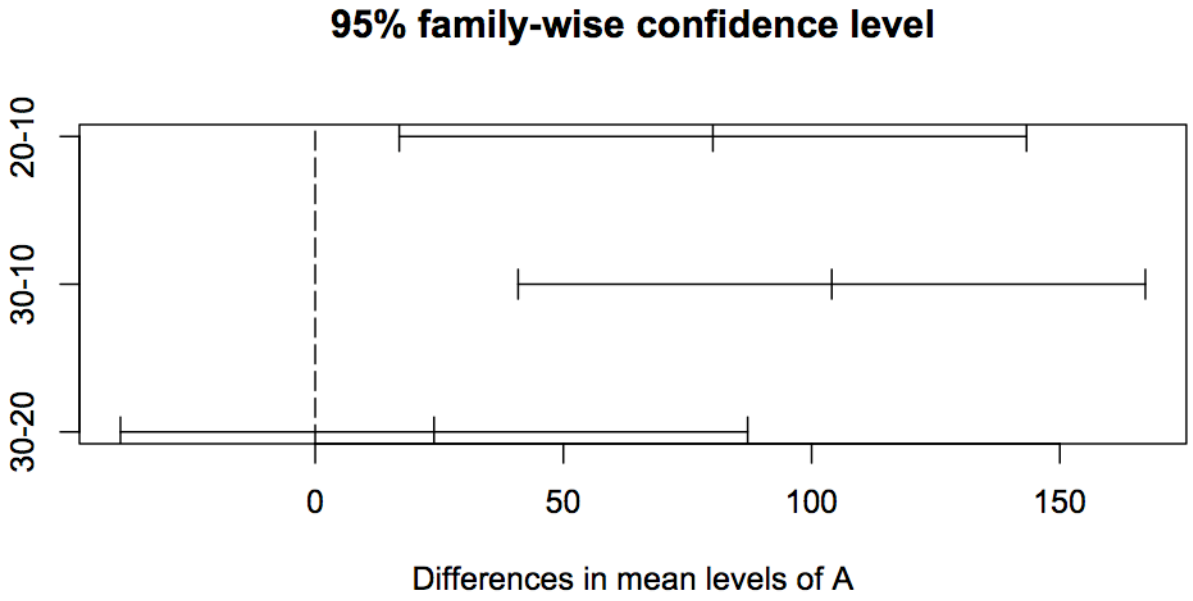
```
> sort(tapply(Y,B,mean))
      30       20       10
695.3333 755.5000 824.5000
> TukeyHSD(f1,which="B",ordered=TRUE)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = Y ~ A * B)

$B
           diff       lwr      upr     p adj
20-30  60.16667 -3.013705 123.3470 0.0648873
10-30 129.16667 65.986295 192.3470 0.0000314
10-20  69.00000  5.819629 132.1804 0.0294625

>#======================RUN PURE ADDITIVE MODEL FOR COMPARISON PURPOSES=============
>#============================YOU HAVE ALREADY TESTED FOR INTERACTION================
>#================USING THESE SMALLER HSD INTERVALS IS THEREFORE INVALID=============

> f2<- aov(Y~A+B);anova(f2)
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
A          2 106912   53456  9.1382 0.0004238 ***
B          2 150390   75195 12.8545 3.252e-05 ***
Residuals 49 286637    5850
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
> TukeyHSD(f2,which="A",ordered=TRUE)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = Y ~ A + B)

$A
           diff       lwr      upr     p adj
20-10  80.11111  18.49286 141.7294 0.0078651
30-10 104.05556  42.43730 165.6738 0.0004759
30-20  23.94444 -37.67381  85.5627 0.6183722

>#==========SHAPIRO-WILK TEST TO SEE IF STANDARDIZED RESIDUALS ARE NORMAL============

> shapiro.test(rstandard(f1))

Shapiro-Wilk normality test

data:  rstandard(f1)
W = 0.9847, p-value = 0.7196
```

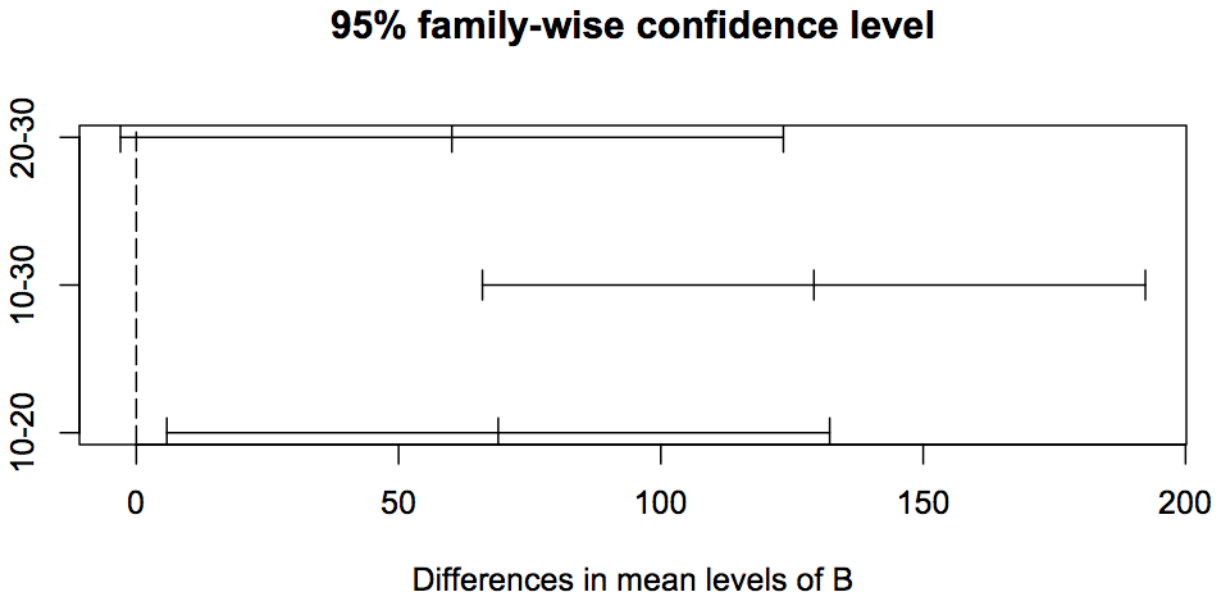
```
>#=========================EFFECTS AND MEANS=======================================
> model.tables(f1,"effects",se=TRUE)
Tables of effects

 A
A
    10     20     30
-61.39  18.72  42.67

 B
B
    10     20     30
 66.06  -2.94 -63.11

 A:B
    B
A    10      20      30
  10   9.056 -22.944  13.889
  20 -16.722   9.611   7.111
  30   7.667  13.333 -21.000

Standard errors of effects
            A     B   A:B
        18.43 18.43 31.93
replic.    18    18     6
> model.tables(f1,"means",se=TRUE)
Tables of means
Grand mean

758.4444

 A
A
   10    20    30
697.1 777.2 801.1

 B
B
   10    20    30
824.5 755.5 695.3

 A:B
    B
A    10    20    30
  10 772.2 671.2 647.8
  20 826.5 783.8 721.2
  30 874.8 811.5 717.0

Standard errors for differences of means
            A     B   A:B
        26.07 26.07 45.15
replic.    18    18     6
```

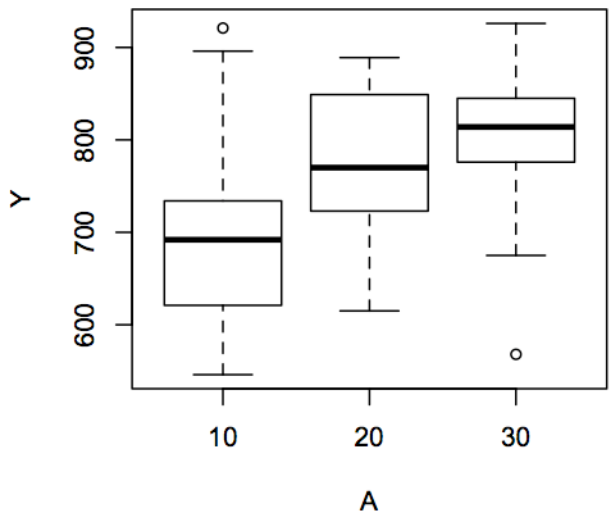
```
>#============================PLOT HSD FOR BOTH FACTORS, DIAGNOSTICS==================
> layout(matrix(1:2,ncol=1))
> plot(TukeyHSD(f1,which="A",ordered=TRUE));abline(v=0,lty=5)
> plot(TukeyHSD(f1,which="B",ordered=TRUE));abline(v=0,lty=5)
> layout(matrix(1:4,ncol=2))
> plot(Y~A)
> plot(rstandard(f1)~fitted(f1),ylab="Standard. Resid.",xlab="Predicted Values",
  ylim=max(abs(rstandard(f1)))*c(-1,1));abline(h=c(0,-2,2),lty=c(2,3,3))
> plot(fitted(f1)~Y,ylab="Y hat");abline(0,1)
> qqnorm(rstandard(f1),ylab="Standard. Resid.",
  ylim=max(abs(rstandard(f1)))*c(-1,1));abline(h=c(0,-2,2),lty=c(2,3,3))
> abline(0,1)

>
```

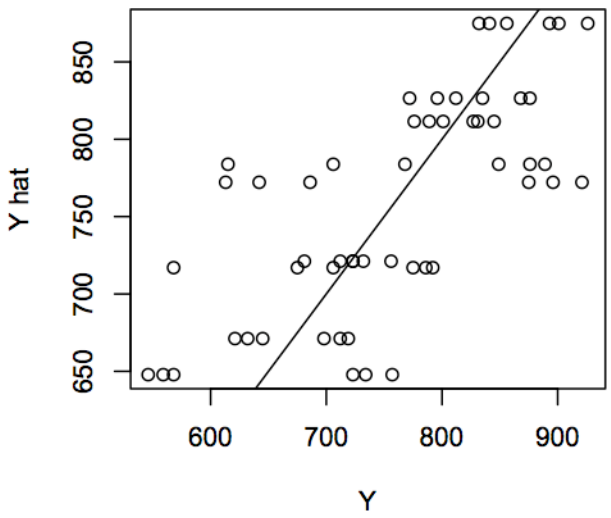

mean of Y
820
800
780
760
740
720
700
10
30
20
20
10
30
A
B
Factors

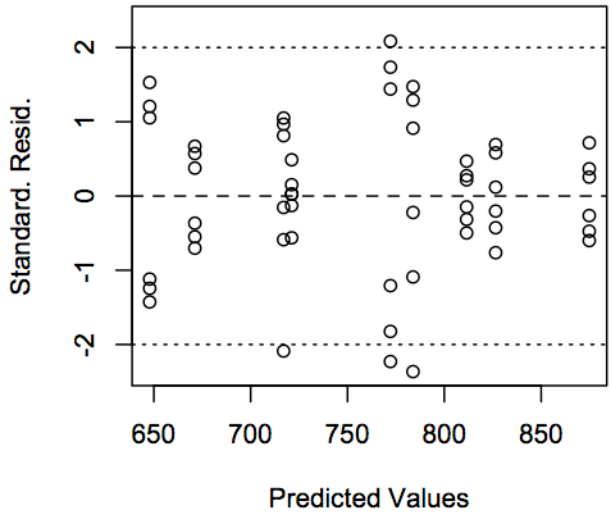

mean of Y
850
800
750
700
650
B
10
20
30
10
20
30
A

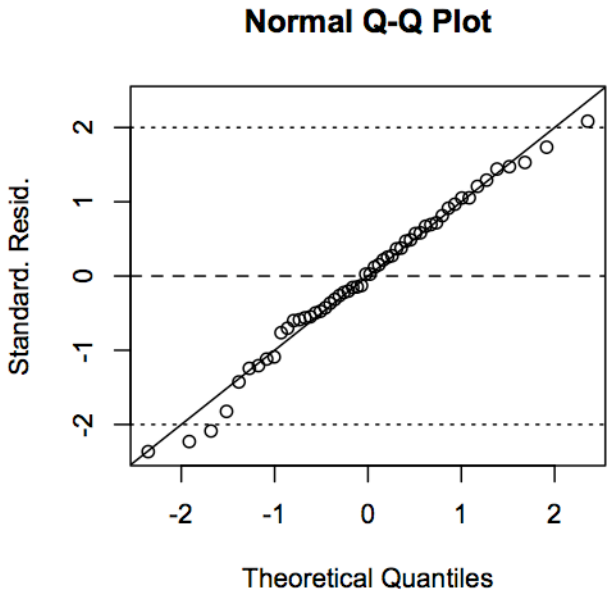

95% family-wise confidence level
20-10
30-10
30-20
0
50
100
150
Differences in mean levels of A


95% family-wise confidence level
20-30
10-30
10-20
0
50
100
150
200
Differences in mean levels of B


Y
600
700
800
900
10
20
30
A


Y hat
650
700
750
800
850
600
700
800
900
Y


Standard. Resid.
-2
-1
0
1
2
650
700
750
800
850
Predicted Values


Normal Q-Q Plot
Standard. Resid.
-2
-1
0
1
2
-2
-1
0
1
2
Theoretical Quantiles