Math 3080 § 1. **Spray Data: 2^3 Factorial with** Name: Example
Treibergs **Multiple Replicates Per Cell ANOVA** April 7, 2010

Data File Used in this Analysis:

```
# Math 3081 - 1                    Spray Data                April 17,2010
# Treibergs
#
# From Devore "Probability and Statistics for Engineering and the Sciences,
# 5th ed.," (Duxbury 1999)
#
# A  2^3  experiment with 2 reps per cell to study robot spray efficiency on
# an assembly line.
# Factors are
#   Spray Volume
#   Belt Speed
#   Brand used in chemical
# Response
#   Uniformity rating for coating
#
"Run"  "Spray Volume"  "Belt Speed"  "Brand"  "Replication 1"  "Replication 2"
1  -1  -1  -1  40  36
2  +1  -1  -1  25  28
3  -1  +1  -1  30  32
4  +1  +1  -1  50  48
5  -1  -1  +1  45  43
6  +1  -1  +1  25  30
7  -1  +1  +1  30  29
8  +1  +1  +1  52  49
```

R Session:

```
R version 2.10.1 (2009-12-14)
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ISBN 3-900051-07-0

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[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]
```

```
[Workspace restored from /Users/andrejstreibergs/.RData]

> tt <- read.table("M3081DataSpray.txt",header=TRUE); tt

  Run Spray.Volume Belt.Speed Brand Replication.1 Replication.2
1   1          -1         -1    -1            40            36
2   2           1         -1    -1            25            28
3   3          -1          1    -1            30            32
4   4           1          1    -1            50            48
5   5          -1         -1     1            45            43
6   6           1         -1     1            25            30
7   7          -1          1     1            30            29
8   8           1          1     1            52            49
> attach(tt)
>#===============REPLICATES COME AS TWO COLUMNS. SET FACTORS ACCORDINGLY===
> A <- factor(rep(Spray.Volume,times=2));A
 [1] -1 1  -1 1  -1 1  -1 1  -1 1  -1 1  -1 1  -1 1
Levels: -1 1
> B <- factor(rep(Belt.Speed,times=2));B
 [1] -1 -1 1  1  -1 -1 1  1  -1 -1 1  1  -1 -1 1  1
Levels: -1 1
> C <- factor(rep(Brand,times=2));C
 [1] -1 -1 -1 -1 1  1  1  1  -1 -1 -1 -1 1  1  1  1
Levels: -1 1
> Y<-c(Replication.1,Replication.2)
> tt2 <- data.frame(A,B,C,Y);tt2
    A  B  C  Y
1  -1 -1 -1 40
2   1 -1 -1 25
3  -1  1 -1 30
4   1  1 -1 50
5  -1 -1  1 45
6   1 -1  1 25
7  -1  1  1 30
8   1  1  1 52
9  -1 -1 -1 36
10  1 -1 -1 28
11 -1  1 -1 32
12  1  1 -1 48
13 -1 -1  1 43
14  1 -1  1 30
15 -1  1  1 29
16  1  1  1 49

>#=================PLOT DESIGN, INTERACTIONS=======================


> layout(matrix(1:3,ncol=3))
> plot.design(tt2)
> interaction.plot(A,B,Y)
> interaction.plot(A,C,Y)
```

```
>#======================RUN ANOVA. NOTE DOUBLE * NOTATION=========
> f1 <- aov(Y~.*.*.,tt2)
> anova(f1)
Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq  F value    Pr(>F)
A          1   30.25   30.25   6.7222 0.0319774 *
B          1  144.00  144.00  32.0000 0.0004776 ***
C          1   12.25   12.25   2.7222 0.1375685
A:B        1 1122.25 1122.25 249.3889 2.584e-07 ***
A:C        1    1.00    1.00   0.2222 0.6499402
B:C        1   12.25   12.25   2.7222 0.1375685
A:B:C      1   16.00   16.00   3.5556 0.0960716 .
Residuals  8   36.00    4.50
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
> ?aov
starting httpd help server ... done

>#==================================MEANS=========================
> model.tables(f1,"means",se=TRUE)
Tables of means
Grand mean

37

 A
A
   -1     1
35.62 38.38

 B
B
-1  1
34 40

 C
C
   -1     1
36.12 37.88
```

```
 A:B
    B
A    -1    1
  -1 41.00 30.25
  1  27.00 49.75

 A:C
    C
A    -1    1
  -1 34.50 36.75
  1  37.75 39.00

 B:C
    C
B    -1    1
  -1 32.25 35.75
  1  40.00 40.00

 A:B:C
, , C = -1

    B
A    -1   1
  -1 38.0 31.0
  1  26.5 49.0

, , C = 1

    B
A    -1   1
  -1 44.0 29.5
  1  27.5 50.5


Standard errors for differences of means
           A     B     C   A:B   A:C   B:C A:B:C
       1.061 1.061 1.061 1.500 1.500 1.500 2.121
replic.     8     8     8     4     4     4     2
```

```
>#====================================EFFECTS==========================
> model.tables(f1,"effects",se=TRUE)
Tables of effects

 A
A
    -1      1
-1.375  1.375

 B
B
-1  1
-3  3

 C
C
    -1      1
-0.875  0.875

 A:B
    B
A    -1     1
  -1  8.375 -8.375
  1  -8.375  8.375

 A:C
    C
A    -1    1
  -1 -0.25  0.25
  1   0.25 -0.25

 B:C
    C
B    -1     1
  -1 -0.875  0.875
  1   0.875 -0.875

 A:B:C
, , C = -1

    B
A    -1 1
  -1 -1  1
  1   1 -1

, , C = 1

    B
A    -1 1
  -1  1 -1
  1  -1  1
```

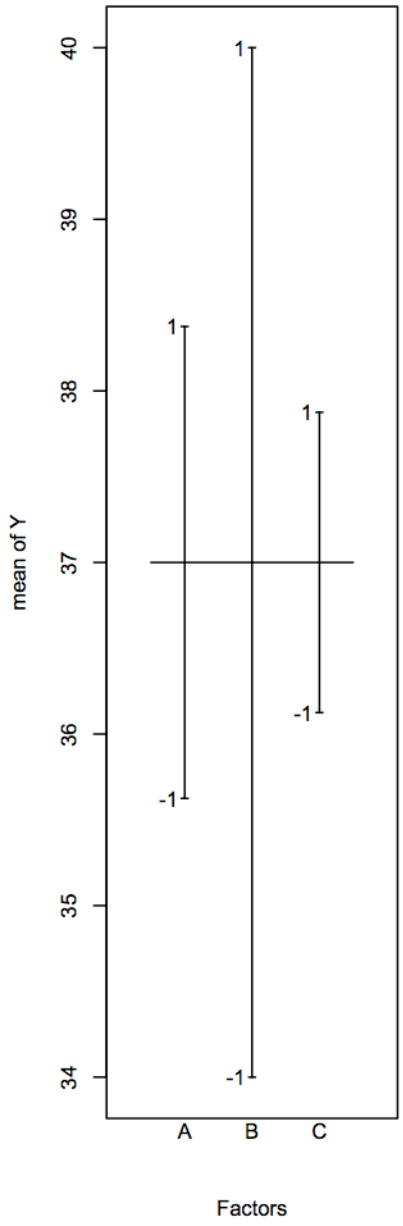
```
Standard errors of effects
            A     B     C   A:B   A:C   B:C A:B:C
        0.750 0.750 0.750 1.061 1.061 1.061 1.500
replic.     8     8     8     4     4     4     2

>#============================PLOT USUAL DIAGNOSTICS====================
> layout(matrix(1:4,ncol=2))
> plot(Y~B)
> plot(rstandard(f1)~fitted(f1),xlab="Predicted Values",ylab="Standard. resid.",
  ylim=max(abs(rstandard(f1)))*c(-1,1));abline(h=c(0,-2,2),lty=c(2,3,3))
> plot(fitted(f1)~Y,ylab="Predicted Values Y hat");abline(0,1)
> qqnorm(rstandard(f1),ylab="Standard. resid.",
  ylim=max(abs(rstandard(f1)))*c(-1,1));abline(h=c(0,-2,2),lty=c(2,3,3))
> abline(0,1)

>#====================2^3 EXPERIMENT. DO ANOVA "BY HAND"===============

>#======================CONTRAST VECTORS, FACTOR NAMES IN STD. ORDER====
> LA <-Spray.Volume;LA
[1] -1  1 -1  1 -1  1 -1  1
> LB <-Belt.Speed; LC <- Brand; LAB <- LA*LB; LAC <- LA*LC; LBC <- LABC<-LA*LB*LC
> L <- rep(1,times=8)

>#==========NAMES VECTOR FOR ANOVA IN STD. ORDER============================
> c1<- c("a","b","ab","c","ac","bc","abc","E","T")

>#=======================TABLE OF CONTRAST COEFFICIENTS FOR 2^3 EXPERIMENT===

> matrix(c(L,LA,LB,LAB,LC,LAC,LBC,LABC),ncol=8,dimnames=list(c("1",c1[1:7]),c("1",c1[1:7])))
    1  a  b ab  c ac bc abc
1   1 -1 -1  1 -1  1  1  -1
a   1  1 -1 -1 -1 -1  1   1
b   1 -1  1 -1 -1  1 -1   1
ab  1  1  1  1 -1 -1 -1  -1
c   1 -1 -1  1  1 -1 -1   1
ac  1  1 -1 -1  1  1 -1  -1
bc  1 -1  1 -1  1 -1  1  -1
abc 1  1  1  1  1  1  1   1

>#=======================CELL SUMS, SST====================================
> YB <- xtabs(Y~A+B+C)
> YV <-as.vector(YB)
> YV
[1]  76  53  62  98  88  55  59 101
> mubar <- mean(YV)
> SST <- sum(Y*Y)-mubar^2*16;SST
>#=======================IN 2^3 EXPERIMENT, SS ARE SQUARES OF CONTRASTS=====
> SSA <- sum(LA*YV)^2/16
> SSB <- sum(LB*YV)^2/16
> SSAB <- sum(LAB*YV)^2/16
> SSC <- sum(LC*YV)^2/16
```

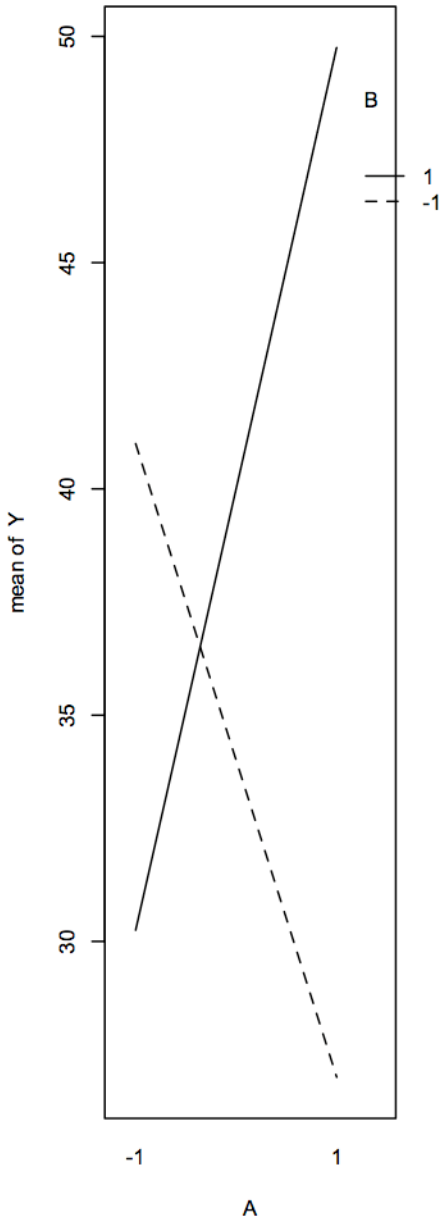
```
> SSAC <- sum(LAC*YV)^2/16
> SSBC <- sum(LBC*YV)^2/16
> SSABC <- sum(LABC*YV)^2/16
>#=========================DF VECTOR=======================================
> c2<-c(1,1,1,1,1,1,1,8 , 15)
>#====================SSE FROM SQUARES IDENTITY. SS COLUMN================
> SSE <- SST-SSA-SSB-SSAB-SSC-SSAC-SSBC-SSABC
> c3 <- c(SSA,SSB,SSAB,SSC,SSAC,SSBC,SSABC,SSE,SST)
>#======================MS COLUMN=========================================
> MSE <- SSE/8
> c4 <- c(SSA,SSB,SSAB,SSC,SSAC,SSBC,SSABC,MSE,-1)
>#==================F COLUMN FOR FIXED EFFECTS============================
> c5 <- c(SSA/MSE,SSB/MSE,SSAB/MSE,SSC/MSE,SSAC/MSE,SSBC/MSE,SSABC/MSE,-1,-1)
>#=================P-VALUES. USE CUM. F WITH 1 AND 8 DF====================
> w<-function(z){pf(z,1,8,lower.tail=FALSE)}
> c6 <- c(w(SSA/MSE),w(SSB/MSE),w(SSAB/MSE),w(SSC/MSE),w(SSAC/MSE),w(SSBC/MSE), w(SSABC/MSE),-1,-1)
>#=============ANOVA TABLE BY HAND=========================================
> matrix(c(c2,c3,c4,c5,c6),ncol=5,dimnames=list(c1,c("DF","SS","MS","F","Pr(>F)")))

    DF      SS      MS           F        Pr(>F)
a    1   30.25   30.25   6.7222222  3.197737e-02
b    1  144.00  144.00  32.0000000  4.776141e-04
ab   1 1122.25 1122.25 249.3888889  2.584228e-07
c    1   12.25   12.25   2.7222222  1.375685e-01
ac   1    1.00    1.00   0.2222222  6.499402e-01
bc   1   12.25   12.25   2.7222222  1.375685e-01
abc  1   16.00   16.00   3.5555556  9.607159e-02
E    8   36.00    4.50  -1.0000000 -1.000000e+00
T   15 1374.00   -1.00  -1.0000000 -1.000000e+00

>#============================COMPARE TO=========================================
> anova(f1)
Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq  F value    Pr(>F)
A          1   30.25   30.25   6.7222 0.0319774 *
B          1  144.00  144.00  32.0000 0.0004776 ***
C          1   12.25   12.25   2.7222 0.1375685
A:B        1 1122.25 1122.25 249.3889 2.584e-07 ***
A:C        1    1.00    1.00   0.2222 0.6499402
B:C        1   12.25   12.25   2.7222 0.1375685
A:B:C      1   16.00   16.00   3.5556 0.0960716 .
Residuals  8   36.00    4.50
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
>
>#====================================EFFECTS FROM AOV============
> model.tables(f1,"effects",se=TRUE)
Tables of effects

 A
```

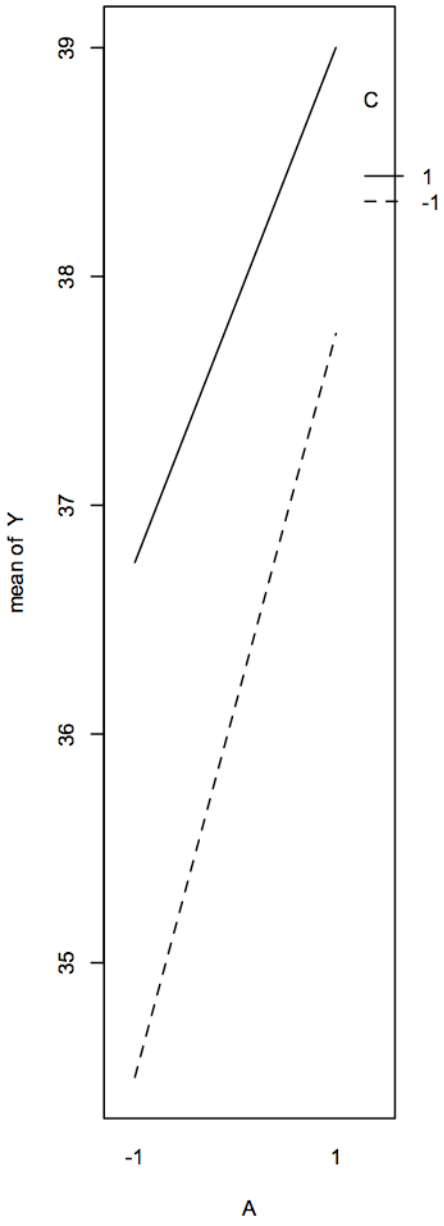
```
A
    -1      1
-1.375  1.375

 B
B
-1  1
-3  3

 C
C
    -1      1
-0.875  0.875

 A:B
    B
A    -1     1
  -1  8.375 -8.375
  1  -8.375  8.375

 A:C
    C
A    -1    1
  -1 -0.25  0.25
  1   0.25 -0.25

 B:C
    C
B    -1     1
  -1 -0.875  0.875
  1   0.875 -0.875

 A:B:C
, , C = -1

    B
A    -1 1
  -1 -1  1
  1   1 -1

, , C = 1

    B
A    -1 1
  -1  1 -1
  1  -1  1


Standard errors of effects
            A     B     C   A:B   A:C   B:C A:B:C
        0.750 0.750 0.750 1.061 1.061 1.061 1.500
replic.     8     8     8     4     4     4     2
```

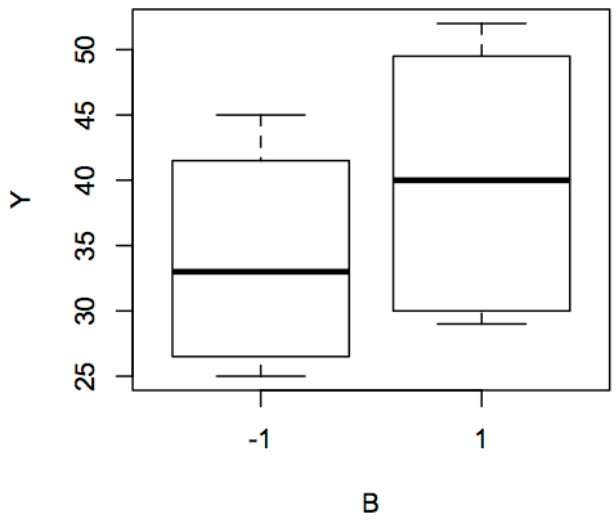
```
>#=============EFFECTS BY HAND=========================================
> es <- function(z){sum(z*YV)/16}
> c7<- c(es(L),es(LA),es(LB),es(LAB),es(LC),es(LAC),es(LBC),es(LABC))

>#=============PICK OFF EFFECTS COMPUTED BY AOV=======================
> model.tables(f1,"effects")$tables
$A
A
    -1      1
-1.375  1.375

$B
B
-1  1
-3  3

$C
C
    -1      1
-0.875  0.875

$`A:B`
    B
A    -1     1
  -1  8.375 -8.375
  1  -8.375  8.375

$`A:C`
    C
A    -1    1
  -1 -0.25  0.25
  1   0.25 -0.25

$`B:C`
    C
B    -1     1
  -1 -0.875  0.875
  1   0.875 -0.875

$`A:B:C`
, , C = -1
    B
A    -1 1
  -1 -1  1
  1   1 -1

, , C = 1
    B
A    -1 1
  -1  1 -1
  1  -1  1
```

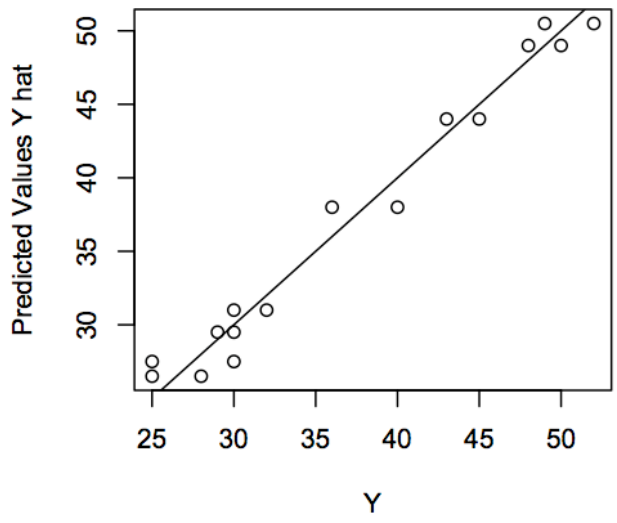
```
> f <- function(z){model.tables(f1,"effects")$tables[[z]][[2]]}

>#==THE 2ND NO. IN Zth EFFECTS TABLE IS POS. FOR A,B,C,ABC BUT NEG. FOR AB,AC,BC========
>#==...$table LISTS EFFECTS IN ANOTHER ORDER: A, B, C, AB AC, BC, ABC===================

> c8<-c(mubar,f(1),f(2),-f(4),f(3),-f(5),-f(6),f(7))
>

>#================PLOT CONTRASTS BY HAND VS. EFFECTS BY AOV============================
> matrix(c(c7,c8),ncol=2,dimnames=list(c("1",c1[1:7]),c("Eff. by hand","Eff.by aov")))
    Eff. by hand Eff.by aov
1         37.000     37.000
a          1.375      1.375
b          3.000      3.000
ab         8.375      8.375
c          0.875      0.875
ac        -0.250     -0.250
bc        -0.875     -0.875
abc        1.000      1.000
```

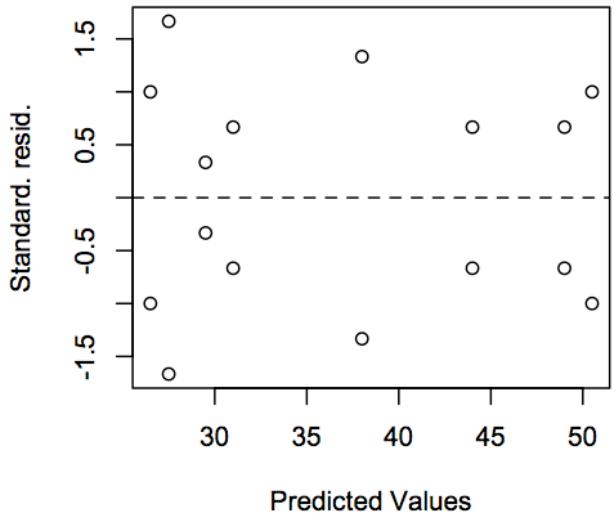

mean of Y
40
39
38
37
36
35
34
1
1
1
-1
-1
-1
A
B
C
Factors

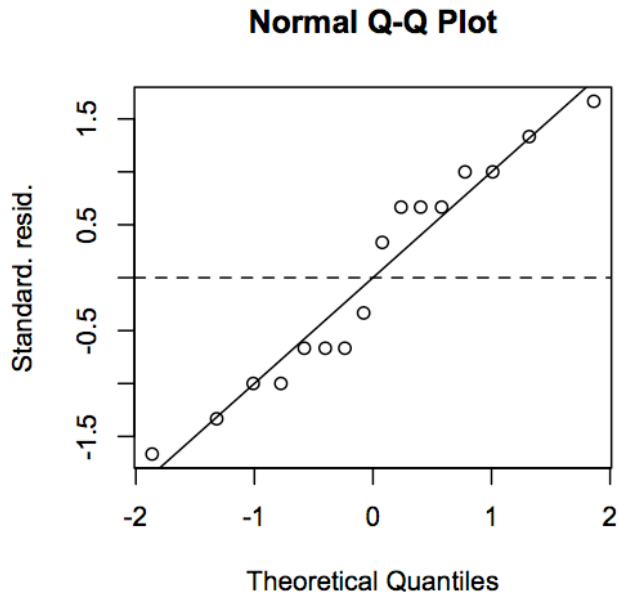

mean of Y
50
45
40
35
30
B
1
-1
-1
1
A


mean of Y
39
38
37
36
35
C
1
-1
-1
1
A


Y
25
30
35
40
45
50
-1
1
B


Predicted Values Y hat
30
35
40
45
50
25
30
35
40
45
50
Y


Standard. resid.
-1.5
-0.5
0.5
1.5
30
35
40
45
50
Predicted Values


Normal Q-Q Plot
Standard. resid.
-1.5
-0.5
0.5
1.5
-2
-1
0
1
2
Theoretical Quantiles