Math 3080 § 1. **Colleges Example: Regression with Quantitative** Name: Example
Treibergs **and Categorical Variables and the Design Matrix** March 22, 2014

This **R**© program explores adding categorical variables to regression. In effect, the regression model subsumes the ANOVA model which has only categorical independent variables. We show that factors as independent variables are automatically handled by the linear models routine and the expected design matrix is generated.

This data was taken from Levine Ramsey & Smidt, *Applied Statistics for Engneers and Scientists,* Prentice Hall, Upper Saddle River, NJ, 2001 about a study about expenditures and descriptive variables about various colleges and universities in the day.

The variables available in the dataset are type of term (semester = 1, other = 0), location (urban = 1, suburban = 2, rural = 3), type of school (public = 0, private = 1), average total SAT score, TOEFL score (less than 550 = 0, at least 550 = 1), room and board expenses (in thousands of dollars), annual total cost (in thousands of dollars), and the average indebtedness at graduation (in thousands of dollars). In today's discussion we shall try to predict the average indebtedness Y from two quantitative variables annual total cost C and room and board expenses RB and two categorical variables, private Pri and location Loc.

0.1 Categorical Variables in Multiple Regression.

Suppose that the response variable Y depends on one quantitative variable X and one categorical variable Z which has J levels. Suppose $J = 3$ and the levels are called A, B and C. Then for each observation $i = 1, \ldots, n$, the regression model takes the form

$$Y_i = \epsilon_i + \begin{cases} \beta_{0A} + \beta_{1A} X_i, & \text{if } Z_i = A; \\ \beta_{0B} + \beta_{1B} X_i, & \text{if } Z_i = B; \\ \beta_{0C} + \beta_{1C} X_i, & \text{if } Z_i = A; \end{cases} \qquad (1)$$

where ϵ_i are independent, identically distributed normal variables with mean zero and variance σ^2 and the $\beta_{0A}, \ldots, \beta_{1C}$ are six constants. Thus at each level of the factor Z, Y is assumed to satisfy a different linear equation whose coefficients depend on the level. To encode this as a pure regression model one introduces $J-1$ new variables in the regression for each qualitative variable. In the case above, we would add $J - 1 = 2$ new variables, call them ZB and ZC (as **R**© calls them). They take the values

$$\begin{aligned} ZB &= 0, & ZC &= 0 & &\text{if } Z = A; \\ ZB &= 1, & ZC &= 0 & &\text{if } Z = B; \\ ZB &= 0, & ZC &= 1 & &\text{if } Z = C. \end{aligned}$$

Then the regression model with quadratic interactions recovers (1). Indeed if we model the response by

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 ZB_i + \beta_3 ZC_i + \beta_4 X_i \cdot ZB_i + \beta_5 X_i \cdot ZC_i + \epsilon_i$$

we get exactly the same model. The coefficients are related by

$$\begin{aligned} \beta_{0A} &= \beta_0; & \beta_{1A} &= \beta_1 \\ \beta_{0B} &= \beta_0 + \beta_2; & \beta_{1B} &= \beta_1 + \beta_4 \\ \beta_{0C} &= \beta_0 + \beta_3; & \beta_{1C} &= \beta_1 + \beta_6. \end{aligned}$$

Notice that $XB_i \cdot XC_i$ is missing as it is always dead zero.

Be sure to declare your qualitative variables as "factors" to get **R**© to handle them correctly in the linear model routine. If the levels of Z were called 1, 2 and 3 instead, then **R**© will interpret Z as a numerical variable and will not supply different coefficients for each level. The levels may not make sense as quantities 1, 2 and 3.

Of course you may choose to try simpler models with the coded qualitative variables. For example if you try the model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 ZB_i + \beta_3 ZC_i + \epsilon_i$$

you are assuming that the slopes are the same for each level of the categorical variable.

0.2 Matrix Formulation of Multiple Regression.

Multiple regression has a particularly elegant formulation using matrix algebra. Students having had Math 2270 or Math 2250 will be familiar with this notation. Devore does not develop it, although most Math 3080 texts will discuss this point of view. When Devore says "a complicated formula involving the x_i's" he is alluding to something relatively easy to deduce involving determinants or matrix inverses.

First assume there are k independent variables $X_1, \ldots, X_k$ and that we are trying to fit the linear model

$$Y_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \cdots + \beta_k X_{i,k} + \epsilon_i$$

This equation may be written in the matrix form

$$y = X\beta + \epsilon.$$

Let the $n \times 1$ matrices (column vectors), the $(k+1) \times 1$ matrix

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}; \qquad \epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{pmatrix}; \qquad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}.$$

and the $(k+1) \times n$ matrix that contains the X_j's as columns is called the *design matrix*

$$X = \begin{pmatrix} 1 & x_{1,1} & x_{1,2} & \cdots & x_{1,k} \\ 1 & x_{2,1} & x_{2,2} & \cdots & x_{2,k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n,1} & x_{n,2} & \cdots & x_{n,k} \end{pmatrix}$$

Let X^T denote the $n \times (k+1)$ dimensional transposed matrix (the transposed matrix swaps the rows and columns ${X^T}_{j,i} = X_{i,j}$.) The predicted value for a particular $\widehat{\beta}$

$$\widehat{y_i} = \widehat{\beta_0} + \widehat{\beta_1}\, x_{i,1} + \cdots + \widehat{\beta_k}\, x_{i,k}$$

may be written in matrix form as

$$\hat{y} = X\hat{\beta}.$$

Thus the sum of squares of deviations for some choice of coefficients may be written

$$Q(\beta) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (y - \hat{y})^T (y - \hat{y}) = \left(y - X\widehat{\beta}\right)^T \left(y - X\widehat{\beta}\right)$$

It is minimized where the gradient with respect to $\widehat{\beta}$ vanishes, when

$$0 = \nabla_{\widehat{\beta}}^T Q(\widehat{\beta}) = -2X^T \left(y - X\widehat{\beta}\right)$$

This is equivalent to the equation

$$(X^T X)\widehat{\beta} = X^T y.$$

Solving the equation gives the expression for the coefficients of the fitted line

$$\widehat{\beta} = (X^T X)^{-1} X^T y.$$

The matrix $(X^T X)$ is invertible, if for example the matrix X itself has rank $k+1$. This would be guaranteed if the columns of X are independent vectors. An accurate numerical inversion of the matrix requires that it not be ill conditioned, which means that the columns should not be close to linearly dependent (the columns of X should not be highly correlated.)

Let us work out these matrices in the case of simple linear regression $k = 1$. The formula boils down to the familiar expressions for $\widehat{\beta_0}$ and $\widehat{\beta_1}$. To see this, observe that

$$X^T y = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{pmatrix} \begin{pmatrix} y_i \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

$$X^T X = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{pmatrix} \begin{pmatrix} 1 & x_i \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & y_n \end{pmatrix} = \begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix}$$

so $\widehat{\beta} = (X^T X)^{-1} X^T y$ equals

$$\begin{aligned} \begin{pmatrix} \widehat{\beta_0} \\ \widehat{\beta_1} \end{pmatrix} &= \begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix} \\ &= \frac{1}{n \sum x_i^2 - \left(\sum x_i\right)\left(\sum y_i\right)} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix} \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix} \\ &= \frac{1}{n S_{xx}} \begin{pmatrix} \left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right) \\ -\left(\sum x_i\right)\left(\sum y_i\right) + n\left(\sum x_i y_i\right) \end{pmatrix} \end{aligned}$$

where $S_{xx} = \sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2$. The second equation simplifies

$$\widehat{\beta_1} = \frac{n\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{nS_{xx}} = \frac{S_{xy}}{S_{xx}}$$

as usual, where $S_{xy} = \sum x_i y_i - \frac{1}{n}\left(\sum x_i\right)\left(\sum y_i\right)$. The first also simplifies

$$\begin{aligned}
\widehat{\beta_0} &= \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{nS_{xx}} \\
&= \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)^2\left(\sum y_i\right) + \left(\sum x_i\right)^2\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{nS_{xx}} \\
&= \frac{\bar{y}\left[\sum x_i^2 - \left(\sum x_i\right)^2\right]}{S_{xx}} + \frac{\bar{x}\left[\left(\sum x_i\right)\left(\sum y_i\right) - \left(\sum x_i y_i\right)\right]}{S_{xx}} \\
&= \bar{y}\frac{S_{xx}}{S_{xx}} - \bar{x}\frac{S_{xy}}{S_{xx}} = \bar{y} - \widehat{\beta_1}\bar{x}
\end{aligned}$$

which is also the usual formula.

There are some immediate consequences. The first is that $\hat{\beta}$ is unbiased since

$$\begin{aligned}
E(\hat{\beta}) &= E\left((X^TX)^{-1}X^Ty\right) = E\left((X^TX)^{-1}X^T(X\beta + \epsilon)\right) \\
&= (X^TX)^{-1}(X^TX)\beta + (X^TX)^{-1}X^TE(\epsilon) = \beta + 0.
\end{aligned}$$

We have only used the fact that the expectation is linear. If A is a constant matrix and y and ϵ are random variables then $E(Ay + \epsilon) = AE(y) + E(\epsilon)$.

The second is that the fitted values are linear combinations of the y's, namely

$$\hat{y} = X\hat{\beta} = X(X^TX)^{-1}X^Ty = Hy$$

where $H = X(X^TX)^{-1}X^T$ is an $n \times n$ matrix called the "hat matrix" (as it maps y to $\hat{y}$.)

Finally, if **R**© is requested to print out the X matrix, it can do so. We show that if categorical variables are introduced, then they are encoded by **R**© as described above.

Data Set Used in this Analysis :

```
# Math 3080-1                 Radon Data             March 22, 2014
# Treibergs
#
# From Levine Ramsey & Smidt, Applied Statistics for Engneers and
# Scientists, Prentice Hall, Upper saddle River, NJ, 2001.
#
# List of Universities and Colleges.
# Variables
#    Name  School,
#    Sem   type of term (semester = 1, other = 0),
#    Loc   location (urban = 1, suburban = 2, rural = 3),
#    Pri   type of school (public = 0, private = 1),
#    SAT   average total SAT score,
#    TOE   TOEFL score (less than 550 = 0, at least 550 = 1),
#    RB    room and board expenses (in thousands of dollars),
#    C     annual total cost (in thousands of dollars),
#    Y     average indebtedness at graduation (in thousands of dollars).
"Name ""Sem" "Loc" "Pri" "SAT" "TOE" "RB" "C" "Y"
"ArizonaStateUniversity"        1  2  0  1080  0  4.3  12.7  12.900
"BallStateUniversity"           1  1  0   985  1  4.0  12.5   8.210
"Cal.StateUniv.-Fresno"         1  1  0   955  0  5.4  13.1   8.761
"ClemsonUniversity"             1  3  0  1130  1  3.9  12.4   9.983
"CollegeofWilliam&Mary"         1  2  0  1295  1  4.5  19.4  13.424
"FloridaInternationalUniv."     1  1  0  1135  0  2.7  10.0   4.139
"FloridaStateUniversity"        1  1  0  1180  1  4.5  11.5  16.500
"GeorgeMasonUniversity"         1  2  0  1055  1  5.0  17.0  13.000
"GeorgiaStateUniversity"        0  1  0  1115  0  7.4  15.4   8.080
"MontclairStateUniversity"      1  2  0  1025  0  5.3  10.2   4.500
"NorthCarolinaStateUniv."       1  1  0  1145  1  4.0  14.3  14.991
"OregonStateUniversity"         0  1  0  1072  1  4.4  15.5  10.500
"PurdueUniversity"              1  1  0  1095  1  4.5  15.2  11.839
"SanDiegoStateUniversity"       1  1  0   945  1  6.2  13.6   6.750
"SlipperyRockUniv.ofPenn."      1  3  0   955  0  3.6  12.9  17.000
"SUNY-Binghamton"               1  2  0  1039  1  4.6  13.4   6.245
"TexasA&MUniversity"            1  1  0  1150  1  3.9  12.7  14.100
"Univ.ofGeorgia"                0  1  0  1180  1  4.0  11.9  10.800
"Univ.ofHawaii-Manoa"           1  1  0  1075  0  4.7  12.6   3.617
"Univ.ofHouston"                1  1  0  1065  1  4.1  12.1   9.404
"Univ.ofMaryland"               1  2  0  1170  1  5.5  15.7  16.637
"Univ.ofMass.-Amherst"          1  2  0  1100  1  4.2  16.4  10.200
"Univ.ofNevada-LasVegas"        1  1  0   980  0  5.5  12.3  10.000
"Univ.ofNewHampshire"           1  3  0  1110  1  4.4  18.6   9.660
"Univ.ofNorthCarolina-C.H."     1  1  0  1225  1  4.5  15.2   9.406
"Univ.ofTexas-Austin"           1  1  0  1215  1  3.9  12.9  10.200
"Univ.ofVermont"                1  1  0  1115  1  5.1  22.4  21.500
"VirginiaCommonwealthUniv."     1  1  0  1005  1  4.3  16.3  14.730
"VirginiaTech"                  1  3  0  1265  1  3.5  14.9  10.327
"WestVirginiaUniversity"        1  1  0  1025  1  4.6  11.7  10.700
"BabsonCollege"                 1  2  1  1165  1  7.6  26.4  18.000
```

"BostonCollege"	1	2	1	1285	1	7.5	26.8	15.855
"BostonUniversity"	1	1	1	1235	1	7.0	27.9	14.463
"BowdoinCollege"	1	2	1	1345	1	6.0	27.8	13.640
"BryantCollege"	0	2	1	1080	1	6.7	20.6	18.000
"BucknellUniversity"	1	3	1	1255	1	5.0	25.4	12.500
"CanisiusCollege"	1	1	1	1143	0	5.9	18.8	14.824
"CarnegieMellonUniversity"	1	1	1	1335	1	6.1	25.6	15.683
"CaseWesternReserveUniv."	1	1	1	1330	1	5.0	22.2	26.031
"ClarkUniversity"	1	1	1	1121	1	4.4	24.4	17.500
"ColbyCollege"	0	1	1	1275	1	5.7	27.9	11.630
"ColgateUniversity"	1	3	1	1300	1	5.9	27.6	9.236
"CollegeofHolyCross"	1	1	1	1275	1	6.7	26.8	12.634
"EmoryUniversity"	1	1	1	1310	1	6.5	26.6	15.309
"FordhamUniversity"	1	1	1	1150	1	7.4	23.4	8.593
"Franklin&MarshallCollege"	1	1	1	1260	1	4.5	26.4	11.500
"GeorgeWashingtonUniversity"	1	1	1	1235	1	6.9	26.7	14.368
"GeorgetownUniversity"	1	1	1	1330	1	7.5	27.5	14.013
"GettysburgCollege"	1	3	1	1200	1	4.8	26.4	11.750
"HarvardUniversity"	1	1	1	1465	1	7.0	28.9	11.650
"IonaCollege"	1	2	1	955	1	7.3	19.8	18.000
"LafayetteCollege"	1	1	1	1185	1	6.3	26.7	11.499
"LaSalleUniversity"	1	1	1	1105	0	6.7	20.8	11.700
"LehighUniversity"	1	1	1	1225	1	6.0	26.8	13.840
"ManhattanCollege"	1	1	1	952	1	7.1	22.0	9.268
"NewYorkUniversity"	1	1	1	1260	1	7.8	28.6	17.318
"NiagaraUniversity"	1	2	1	1065	0	5.4	17.6	11.577
"NortheasternUniversity"	0	1	1	1055	1	8.2	23.4	25.603
"NorthwesternUniversity"	0	2	1	1350	1	6.1	24.2	11.980
"ProvidenceCollege"	1	1	1	1185	1	6.7	22.9	17.500
"RiceUniversity"	1	1	1	1395	1	6.0	18.0	2.323
"RochesterInst.Technology"	0	2	1	1185	0	6.1	21.8	17.500
"SeattleUniversity"	0	1	1	1100	1	5.3	19.5	12.000
"SetonHallUniversity"	1	2	1	1030	1	7.1	20.8	14.900
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"SouthernMethodistUniversity"	1	2	1	1150	1	5.3	21.3	12.113
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"Univ.ofScranton"	0	1	1	1115	0	6.6	21.6	13.500
"VanderbiltUniversity"	1	1	1	1295	1	7.1	27.3	14.500
"VillanovaUniversity"	1	2	1	1242	1	7.0	24.8	17.125
"WakeForestUniversity"	1	2	1	1280	1	5.2	23.7	18.703
"YaleUniversity"	1	1	1	1450	1	6.7	28.9	13.574

R Session:

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R version 2.13.1 (2011-07-08)
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ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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Type 'q()' to quit R.

[R.app GUI 1.41 (5874) i386-apple-darwin9.8.0]

[History restored from /Users/andrejstreibergs/.Rapp.history]

> tt=read.table("M3082DataUniv.txt",header=T)
> tt
                         Name. Sem Loc Pri  SAT TOE  RB    C      Y
1       ArizonaStateUniversity   1   2   0 1080   0 4.3 12.7 12.900
2          BallStateUniversity   1   1   0  985   1 4.0 12.5  8.210
3        Cal.StateUniv.-Fresno   1   1   0  955   0 5.4 13.1  8.761
4            ClemsonUniversity   1   3   0 1130   1 3.9 12.4  9.983
5        CollegeofWilliam&Mary   1   2   0 1295   1 4.5 19.4 13.424
6    FloridaInternationalUniv.   1   1   0 1135   0 2.7 10.0  4.139
7       FloridaStateUniversity   1   1   0 1180   1 4.5 11.5 16.500
8        GeorgeMasonUniversity   1   2   0 1055   1 5.0 17.0 13.000
9       GeorgiaStateUniversity   0   1   0 1115   0 7.4 15.4  8.080
10    MontclairStateUniversity   1   2   0 1025   0 5.3 10.2  4.500
11     NorthCarolinaStateUniv.   1   1   0 1145   1 4.0 14.3 14.991
12       OregonStateUniversity   0   1   0 1072   1 4.4 15.5 10.500
13            PurdueUniversity   1   1   0 1095   1 4.5 15.2 11.839
14     SanDiegoStateUniversity   1   1   0  945   1 6.2 13.6  6.750
15    SlipperyRockUniv.ofPenn.   1   3   0  955   0 3.6 12.9 17.000
16              SUNY-Binghamton   1   2   0 1039   1 4.6 13.4  6.245
17          TexasA&MUniversity   1   1   0 1150   1 3.9 12.7 14.100
18              Univ.ofGeorgia   0   1   0 1180   1 4.0 11.9 10.800
19         Univ.ofHawaii-Manoa   1   1   0 1075   0 4.7 12.6  3.617
20              Univ.ofHouston   1   1   0 1065   1 4.1 12.1  9.404
21             Univ.ofMaryland   1   2   0 1170   1 5.5 15.7 16.637
22        Univ.ofMass.-Amherst   1   2   0 1100   1 4.2 16.4 10.200
23      Univ.ofNevada-LasVegas   1   1   0  980   0 5.5 12.3 10.000
```

24	Univ.ofNewHampshire	1	3	0	1110	1	4.4	18.6	9.660
25	Univ.ofNorthCarolina-C.H.	1	1	0	1225	1	4.5	15.2	9.406
26	Univ.ofTexas-Austin	1	1	0	1215	1	3.9	12.9	10.200
27	Univ.ofVermont	1	1	0	1115	1	5.1	22.4	21.500
28	VirginiaCommonwealthUniv.	1	1	0	1005	1	4.3	16.3	14.730
29	VirginiaTech	1	3	0	1265	1	3.5	14.9	10.327
30	WestVirginiaUniversity	1	1	0	1025	1	4.6	11.7	10.700
31	BabsonCollege	1	2	1	1165	1	7.6	26.4	18.000
32	BostonCollege	1	2	1	1285	1	7.5	26.8	15.855
33	BostonUniversity	1	1	1	1235	1	7.0	27.9	14.463
34	BowdoinCollege	1	2	1	1345	1	6.0	27.8	13.640
35	BryantCollege	0	2	1	1080	1	6.7	20.6	18.000
36	BucknellUniversity	1	3	1	1255	1	5.0	25.4	12.500
37	CanisiusCollege	1	1	1	1143	0	5.9	18.8	14.824
38	CarnegieMellonUniversity	1	1	1	1335	1	6.1	25.6	15.683
39	CaseWesternReserveUniv.	1	1	1	1330	1	5.0	22.2	26.031
40	ClarkUniversity	1	1	1	1121	1	4.4	24.4	17.500
41	ColbyCollege	0	1	1	1275	1	5.7	27.9	11.630
42	ColgateUniversity	1	3	1	1300	1	5.9	27.6	9.236
43	CollegeofHolyCross	1	1	1	1275	1	6.7	26.8	12.634
44	EmoryUniversity	1	1	1	1310	1	6.5	26.6	15.309
45	FordhamUniversity	1	1	1	1150	1	7.4	23.4	8.593
46	Franklin&MarshallCollege	1	1	1	1260	1	4.5	26.4	11.500
47	GeorgeWashingtonUniversity	1	1	1	1235	1	6.9	26.7	14.368
48	GeorgetownUniversity	1	1	1	1330	1	7.5	27.5	14.013
49	GettysburgCollege	1	3	1	1200	1	4.8	26.4	11.750
50	HarvardUniversity	1	1	1	1465	1	7.0	28.9	11.650
51	IonaCollege	1	2	1	955	1	7.3	19.8	18.000
52	LafayetteCollege	1	1	1	1185	1	6.3	26.7	11.499
53	LaSalleUniversity	1	1	1	1105	0	6.7	20.8	11.700
54	LehighUniversity	1	1	1	1225	1	6.0	26.8	13.840
55	ManhattanCollege	1	1	1	952	1	7.1	22.0	9.268
56	NewYorkUniversity	1	1	1	1260	1	7.8	28.6	17.318
57	NiagaraUniversity	1	2	1	1065	0	5.4	17.6	11.577
58	NortheasternUniversity	0	1	1	1055	1	8.2	23.4	25.603
59	NorthwesternUniversity	0	2	1	1350	1	6.1	24.2	11.980
60	ProvidenceCollege	1	1	1	1185	1	6.7	22.9	17.500
61	RiceUniversity	1	1	1	1395	1	6.0	18.0	2.323
62	RochesterInst.Technology	0	2	1	1185	0	6.1	21.8	17.500
63	SeattleUniversity	0	1	1	1100	1	5.3	19.5	12.000
64	SetonHallUniversity	1	2	1	1030	1	7.1	20.8	14.900
65	SienaCollege	1	2	1	1095	0	5.4	17.6	18.248
66	SouthernMethodistUniversity	1	2	1	1150	1	5.3	21.3	12.113
67	St.BonaventureUniversity	1	3	1	1098	1	5.1	17.5	14.000
68	StanfordUniversity	0	2	1	1430	1	7.3	27.8	12.774
69	SyracuseUniversity	1	1	1	1180	1	7.2	24.3	14.500
70	TulaneUniversity	1	1	1	1270	1	6.3	27.5	13.845
71	Univ.ofChicago	0	1	1	1370	1	7.3	28.8	14.073
72	Univ.ofMiami	1	2	1	1145	1	7.1	25.7	16.068
73	Univ.ofNotreDame	1	2	1	1320	1	4.8	23.8	16.572
74	Univ.ofPennsylvania	1	1	1	1355	1	7.5	28.6	17.621
75	Univ.ofPortland	1	1	1	1135	0	4.5	18.9	13.896

```
76            Univ.ofScranton   0   1   1 1115   0 6.6 21.6 13.500
77       VanderbiltUniversity   1   1   1 1295   1 7.1 27.3 14.500
78        VillanovaUniversity   1   2   1 1242   1 7.0 24.8 17.125
79       WakeForestUniversity   1   2   1 1280   1 5.2 23.7 18.703
80             YaleUniversity   1   1   1 1450   1 6.7 28.9 13.574

> attach(tt)
> names(tt)
[1] "Name." "Sem"   "Loc"   "Pri"   "SAT"   "TOE"   "RB"    "C"     "Y"

> ##########  CODE THE CATEGORICAL VARIABLES AS FACTORS #################
> #  IN THIS STUDY, WE ONLY CONSIDER THE VARIABLES  Y, C, RB, Pri AND Loc
> pri=factor(Pri)
> loc=factor(Lri)

> ##########  PLOT EACH PAIR. NOTE THAT THE FACTOR Loc IS PLOTTED AS 1,2,3
> pairs(Y~C+RB+pri+loc)
>
> ##########  RUN THE LINEAR MODEL. #####################################
> #  NOTE THAT THE FACROR IS ENTERED AS A VARIABLE. x=T  REQUESTS X MATRIX
> #  NOTE ALSO THAT THE MODEL IS LOUSY BUT ILLUSTRATES CATEGORICAL VARS.
> #
> m4=lm(Y~C+RB+pri+loc,x=T)
> anova(m4);summary(m4)
Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq F value    Pr(>F)
C          1  256.97 256.972 16.9016 0.0001008 ***
RB         1    1.06   1.057  0.0695 0.7927848
pri        1   10.41  10.411  0.6848 0.4106084
loc        2   35.63  17.813  1.1716 0.3155594
Residuals 74 1125.09  15.204
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Call:
lm(formula = Y ~ C + RB + pri + loc, x = T)

Residuals:
    Min      1Q  Median      3Q     Max
-10.303  -2.299  -0.189   2.034  12.252

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   7.9315     2.7430   2.892  0.00503 **
C             0.2419     0.1464   1.652  0.10272
RB           -0.1371     0.5254  -0.261  0.79491
pri1          1.1629     1.7157   0.678  0.50002
loc2          1.3261     0.9948   1.333  0.18661
loc3         -0.7937     1.5855  -0.501  0.61815
---
```

```
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 3.899 on 74 degrees of freedom
Multiple R-squared: 0.2128,Adjusted R-squared: 0.1596
F-statistic:     4 on 5 and 74 DF,  p-value: 0.002883

> ####  PRINT THE FACTOR vs. X MATRIX  #########################
> # COMPARE FACTORS  pri  AND loc  VS. pri2, loc2 and loc3  VARS.
> cbind(Y,C,RB,pri,loc,m4$x)
        Y    C  RB pri loc (Intercept)    C  RB pri1 loc2 loc3
1  12.900 12.7 4.3   1   2           1 12.7 4.3    0    1    0
2   8.210 12.5 4.0   1   1           1 12.5 4.0    0    0    0
3   8.761 13.1 5.4   1   1           1 13.1 5.4    0    0    0
4   9.983 12.4 3.9   1   3           1 12.4 3.9    0    0    1
5  13.424 19.4 4.5   1   2           1 19.4 4.5    0    1    0
6   4.139 10.0 2.7   1   1           1 10.0 2.7    0    0    0
7  16.500 11.5 4.5   1   1           1 11.5 4.5    0    0    0
8  13.000 17.0 5.0   1   2           1 17.0 5.0    0    1    0
9   8.080 15.4 7.4   1   1           1 15.4 7.4    0    0    0
10  4.500 10.2 5.3   1   2           1 10.2 5.3    0    1    0
11 14.991 14.3 4.0   1   1           1 14.3 4.0    0    0    0
12 10.500 15.5 4.4   1   1           1 15.5 4.4    0    0    0
13 11.839 15.2 4.5   1   1           1 15.2 4.5    0    0    0
14  6.750 13.6 6.2   1   1           1 13.6 6.2    0    0    0
15 17.000 12.9 3.6   1   3           1 12.9 3.6    0    0    1
16  6.245 13.4 4.6   1   2           1 13.4 4.6    0    1    0
17 14.100 12.7 3.9   1   1           1 12.7 3.9    0    0    0
18 10.800 11.9 4.0   1   1           1 11.9 4.0    0    0    0
19  3.617 12.6 4.7   1   1           1 12.6 4.7    0    0    0
20  9.404 12.1 4.1   1   1           1 12.1 4.1    0    0    0
21 16.637 15.7 5.5   1   2           1 15.7 5.5    0    1    0
22 10.200 16.4 4.2   1   2           1 16.4 4.2    0    1    0
23 10.000 12.3 5.5   1   1           1 12.3 5.5    0    0    0
24  9.660 18.6 4.4   1   3           1 18.6 4.4    0    0    1
25  9.406 15.2 4.5   1   1           1 15.2 4.5    0    0    0
26 10.200 12.9 3.9   1   1           1 12.9 3.9    0    0    0
27 21.500 22.4 5.1   1   1           1 22.4 5.1    0    0    0
28 14.730 16.3 4.3   1   1           1 16.3 4.3    0    0    0
29 10.327 14.9 3.5   1   3           1 14.9 3.5    0    0    1
30 10.700 11.7 4.6   1   1           1 11.7 4.6    0    0    0
31 18.000 26.4 7.6   2   2           1 26.4 7.6    1    1    0
32 15.855 26.8 7.5   2   2           1 26.8 7.5    1    1    0
33 14.463 27.9 7.0   2   1           1 27.9 7.0    1    0    0
34 13.640 27.8 6.0   2   2           1 27.8 6.0    1    1    0
35 18.000 20.6 6.7   2   2           1 20.6 6.7    1    1    0
36 12.500 25.4 5.0   2   3           1 25.4 5.0    1    0    1
37 14.824 18.8 5.9   2   1           1 18.8 5.9    1    0    0
38 15.683 25.6 6.1   2   1           1 25.6 6.1    1    0    0
39 26.031 22.2 5.0   2   1           1 22.2 5.0    1    0    0
40 17.500 24.4 4.4   2   1           1 24.4 4.4    1    0    0
41 11.630 27.9 5.7   2   1           1 27.9 5.7    1    0    0
42  9.236 27.6 5.9   2   3           1 27.6 5.9    1    0    1
```

```
43 12.634 26.8 6.7   2   1          1 26.8 6.7    1    0    0
44 15.309 26.6 6.5   2   1          1 26.6 6.5    1    0    0
45  8.593 23.4 7.4   2   1          1 23.4 7.4    1    0    0
46 11.500 26.4 4.5   2   1          1 26.4 4.5    1    0    0
47 14.368 26.7 6.9   2   1          1 26.7 6.9    1    0    0
48 14.013 27.5 7.5   2   1          1 27.5 7.5    1    0    0
49 11.750 26.4 4.8   2   3          1 26.4 4.8    1    0    1
50 11.650 28.9 7.0   2   1          1 28.9 7.0    1    0    0
51 18.000 19.8 7.3   2   2          1 19.8 7.3    1    1    0
52 11.499 26.7 6.3   2   1          1 26.7 6.3    1    0    0
53 11.700 20.8 6.7   2   1          1 20.8 6.7    1    0    0
54 13.840 26.8 6.0   2   1          1 26.8 6.0    1    0    0
55  9.268 22.0 7.1   2   1          1 22.0 7.1    1    0    0
56 17.318 28.6 7.8   2   1          1 28.6 7.8    1    0    0
57 11.577 17.6 5.4   2   2          1 17.6 5.4    1    1    0
58 25.603 23.4 8.2   2   1          1 23.4 8.2    1    0    0
59 11.980 24.2 6.1   2   2          1 24.2 6.1    1    1    0
60 17.500 22.9 6.7   2   1          1 22.9 6.7    1    0    0
61  2.323 18.0 6.0   2   1          1 18.0 6.0    1    0    0
62 17.500 21.8 6.1   2   2          1 21.8 6.1    1    1    0
63 12.000 19.5 5.3   2   1          1 19.5 5.3    1    0    0
64 14.900 20.8 7.1   2   2          1 20.8 7.1    1    1    0
65 18.248 17.6 5.4   2   2          1 17.6 5.4    1    1    0
66 12.113 21.3 5.3   2   2          1 21.3 5.3    1    1    0
67 14.000 17.5 5.1   2   3          1 17.5 5.1    1    0    1
68 12.774 27.8 7.3   2   2          1 27.8 7.3    1    1    0
69 14.500 24.3 7.2   2   1          1 24.3 7.2    1    0    0
70 13.845 27.5 6.3   2   1          1 27.5 6.3    1    0    0
71 14.073 28.8 7.3   2   1          1 28.8 7.3    1    0    0
72 16.068 25.7 7.1   2   2          1 25.7 7.1    1    1    0
73 16.572 23.8 4.8   2   2          1 23.8 4.8    1    1    0
74 17.621 28.6 7.5   2   1          1 28.6 7.5    1    0    0
75 13.896 18.9 4.5   2   1          1 18.9 4.5    1    0    0
76 13.500 21.6 6.6   2   1          1 21.6 6.6    1    0    0
77 14.500 27.3 7.1   2   1          1 27.3 7.1    1    0    0
78 17.125 24.8 7.0   2   2          1 24.8 7.0    1    1    0
79 18.703 23.7 5.2   2   2          1 23.7 5.2    1    1    0
80 13.574 28.9 6.7   2   1          1 28.9 6.7    1    0    0
>
> ######  INCLUDE THE QUADRATIC INTERACTIONS IN REGRESSION #####
> m5=lm(Y~(C+RB+pri+loc)^2,x=T)
> summary(m5);anova(m5)

Call:
lm(formula = Y ~ (C + RB + pri + loc)^2, x = T)

Residuals:
     Min       1Q   Median       3Q      Max
-10.4234  -1.9187  -0.3507   1.7624  12.3995

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)  1.93783   12.31860   0.157   0.8755
C            0.98378    0.88963   1.106   0.2729
RB          -1.28659    2.56648  -0.501   0.6179
pri1        10.01089   16.96392   0.590   0.5572
loc2        -0.01457    6.57071  -0.002   0.9982
loc3        21.16598   13.83064   1.530   0.1308
C:RB         0.01905    0.17663   0.108   0.9145
C:pri1      -0.89795    0.42006  -2.138   0.0363 *
C:loc2      -0.31856    0.32331  -0.985   0.3281
C:loc3      -0.66165    0.52151  -1.269   0.2091
RB:pri1      0.81917    2.17992   0.376   0.7083
RB:loc2      0.81489    1.23335   0.661   0.5111
RB:loc3     -3.17735    3.92929  -0.809   0.4217
pri1:loc2    4.10110    3.69088   1.111   0.2706
pri1:loc3    9.25011    6.37261   1.452   0.1514
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 3.766 on 65 degrees of freedom
Multiple R-squared: 0.3549,Adjusted R-squared: 0.216
F-statistic: 2.554 on 14 and 65 DF,  p-value: 0.005582

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
C          1 256.97 256.972 18.1175 6.815e-05 ***
RB         1   1.06   1.057  0.0745   0.78575
pri        1  10.41  10.411  0.7340   0.39473
loc        2  35.63  17.813  1.2559   0.29165
C:RB       1   0.02   0.019  0.0013   0.97121
C:pri      1 114.53 114.525  8.0744   0.00599 **
C:loc      2  32.82  16.408  1.1568   0.32087
RB:pri     1   0.93   0.935  0.0659   0.79818
RB:loc     2  16.53   8.266  0.5828   0.56123
pri:loc    2  38.33  19.165  1.3512   0.26611
Residuals 65 921.94  14.184
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

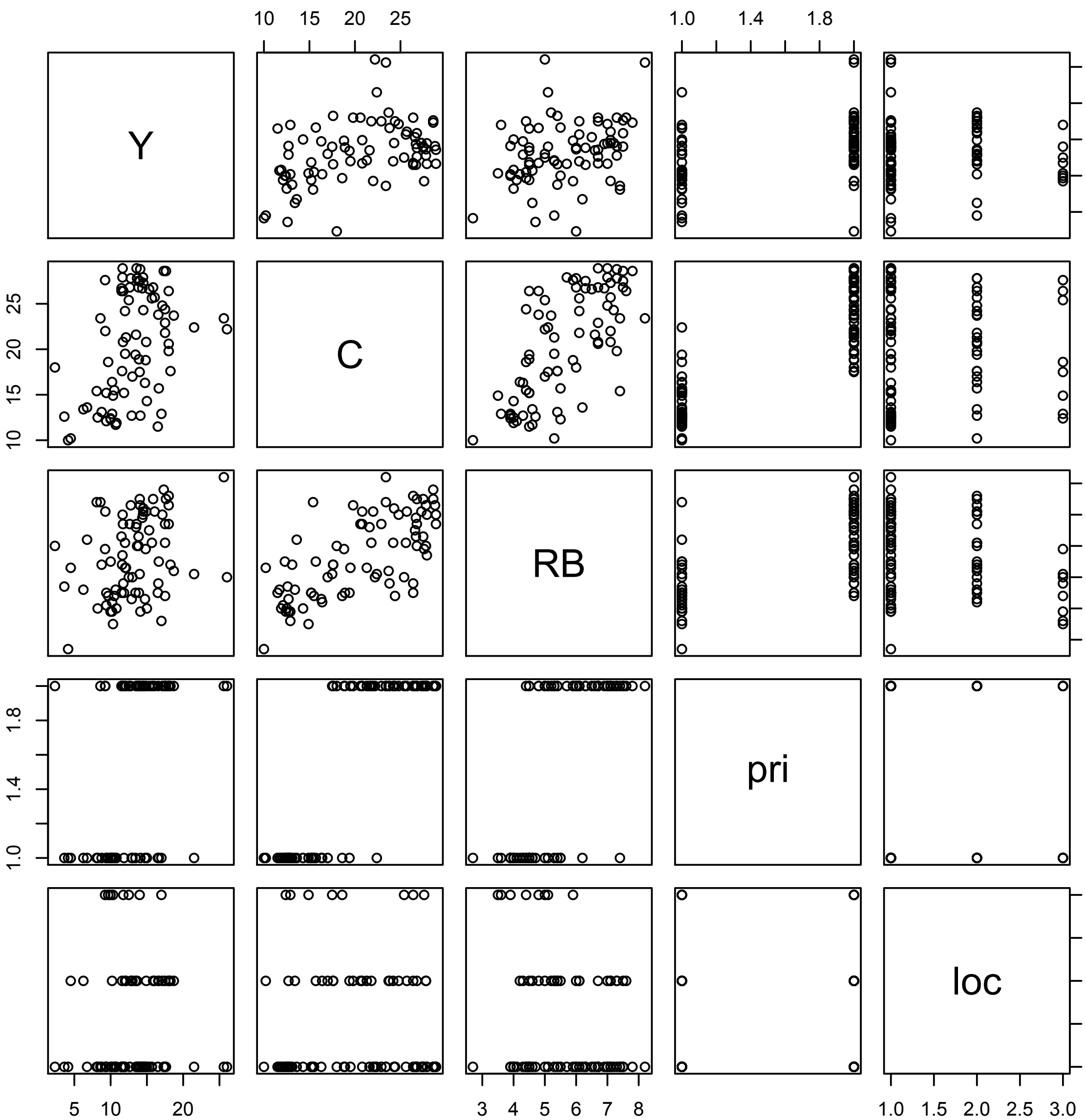

> ###########  PRINT THE DESIGN MATRIX  X   FOR QUADRATIC MODEL  #########
> m5$x
   (Intercept)    C  RB pri1 loc2 loc3   C:RB C:pri1 C:loc2 C:loc3 RB:pri1
1            1 12.7 4.3    0    1    0  54.61    0.0   12.7    0.0     0.0
2            1 12.5 4.0    0    0    0  50.00    0.0    0.0    0.0     0.0
3            1 13.1 5.4    0    0    0  70.74    0.0    0.0    0.0     0.0
4            1 12.4 3.9    0    0    1  48.36    0.0    0.0   12.4     0.0
5            1 19.4 4.5    0    1    0  87.30    0.0   19.4    0.0     0.0
6            1 10.0 2.7    0    0    0  27.00    0.0    0.0    0.0     0.0
7            1 11.5 4.5    0    0    0  51.75    0.0    0.0    0.0     0.0
8            1 17.0 5.0    0    1    0  85.00    0.0   17.0    0.0     0.0
```

9	1	15.4	7.4	0	0	0	113.96	0.0	0.0	0.0	0.0
10	1	10.2	5.3	0	1	0	54.06	0.0	10.2	0.0	0.0
11	1	14.3	4.0	0	0	0	57.20	0.0	0.0	0.0	0.0
12	1	15.5	4.4	0	0	0	68.20	0.0	0.0	0.0	0.0
13	1	15.2	4.5	0	0	0	68.40	0.0	0.0	0.0	0.0
14	1	13.6	6.2	0	0	0	84.32	0.0	0.0	0.0	0.0
15	1	12.9	3.6	0	0	1	46.44	0.0	0.0	12.9	0.0
16	1	13.4	4.6	0	1	0	61.64	0.0	13.4	0.0	0.0
17	1	12.7	3.9	0	0	0	49.53	0.0	0.0	0.0	0.0
18	1	11.9	4.0	0	0	0	47.60	0.0	0.0	0.0	0.0
19	1	12.6	4.7	0	0	0	59.22	0.0	0.0	0.0	0.0
20	1	12.1	4.1	0	0	0	49.61	0.0	0.0	0.0	0.0
21	1	15.7	5.5	0	1	0	86.35	0.0	15.7	0.0	0.0
22	1	16.4	4.2	0	1	0	68.88	0.0	16.4	0.0	0.0
23	1	12.3	5.5	0	0	0	67.65	0.0	0.0	0.0	0.0
24	1	18.6	4.4	0	0	1	81.84	0.0	0.0	18.6	0.0
25	1	15.2	4.5	0	0	0	68.40	0.0	0.0	0.0	0.0
26	1	12.9	3.9	0	0	0	50.31	0.0	0.0	0.0	0.0
27	1	22.4	5.1	0	0	0	114.24	0.0	0.0	0.0	0.0
28	1	16.3	4.3	0	0	0	70.09	0.0	0.0	0.0	0.0
29	1	14.9	3.5	0	0	1	52.15	0.0	0.0	14.9	0.0
30	1	11.7	4.6	0	0	0	53.82	0.0	0.0	0.0	0.0
31	1	26.4	7.6	1	1	0	200.64	26.4	26.4	0.0	7.6
32	1	26.8	7.5	1	1	0	201.00	26.8	26.8	0.0	7.5
33	1	27.9	7.0	1	0	0	195.30	27.9	0.0	0.0	7.0
34	1	27.8	6.0	1	1	0	166.80	27.8	27.8	0.0	6.0
35	1	20.6	6.7	1	1	0	138.02	20.6	20.6	0.0	6.7
36	1	25.4	5.0	1	0	1	127.00	25.4	0.0	25.4	5.0
37	1	18.8	5.9	1	0	0	110.92	18.8	0.0	0.0	5.9
38	1	25.6	6.1	1	0	0	156.16	25.6	0.0	0.0	6.1
39	1	22.2	5.0	1	0	0	111.00	22.2	0.0	0.0	5.0
40	1	24.4	4.4	1	0	0	107.36	24.4	0.0	0.0	4.4
41	1	27.9	5.7	1	0	0	159.03	27.9	0.0	0.0	5.7
42	1	27.6	5.9	1	0	1	162.84	27.6	0.0	27.6	5.9
43	1	26.8	6.7	1	0	0	179.56	26.8	0.0	0.0	6.7
44	1	26.6	6.5	1	0	0	172.90	26.6	0.0	0.0	6.5
45	1	23.4	7.4	1	0	0	173.16	23.4	0.0	0.0	7.4
46	1	26.4	4.5	1	0	0	118.80	26.4	0.0	0.0	4.5
47	1	26.7	6.9	1	0	0	184.23	26.7	0.0	0.0	6.9
48	1	27.5	7.5	1	0	0	206.25	27.5	0.0	0.0	7.5
49	1	26.4	4.8	1	0	1	126.72	26.4	0.0	26.4	4.8
50	1	28.9	7.0	1	0	0	202.30	28.9	0.0	0.0	7.0
51	1	19.8	7.3	1	1	0	144.54	19.8	19.8	0.0	7.3
52	1	26.7	6.3	1	0	0	168.21	26.7	0.0	0.0	6.3
53	1	20.8	6.7	1	0	0	139.36	20.8	0.0	0.0	6.7
54	1	26.8	6.0	1	0	0	160.80	26.8	0.0	0.0	6.0
55	1	22.0	7.1	1	0	0	156.20	22.0	0.0	0.0	7.1
56	1	28.6	7.8	1	0	0	223.08	28.6	0.0	0.0	7.8
57	1	17.6	5.4	1	1	0	95.04	17.6	17.6	0.0	5.4
58	1	23.4	8.2	1	0	0	191.88	23.4	0.0	0.0	8.2
59	1	24.2	6.1	1	1	0	147.62	24.2	24.2	0.0	6.1
60	1	22.9	6.7	1	0	0	153.43	22.9	0.0	0.0	6.7

61	1	18.0	6.0	1	0	0	108.00	18.0	0.0	0.0	6.0
62	1	21.8	6.1	1	1	0	132.98	21.8	21.8	0.0	6.1
63	1	19.5	5.3	1	0	0	103.35	19.5	0.0	0.0	5.3
64	1	20.8	7.1	1	1	0	147.68	20.8	20.8	0.0	7.1
65	1	17.6	5.4	1	1	0	95.04	17.6	17.6	0.0	5.4
66	1	21.3	5.3	1	1	0	112.89	21.3	21.3	0.0	5.3
67	1	17.5	5.1	1	0	1	89.25	17.5	0.0	17.5	5.1
68	1	27.8	7.3	1	1	0	202.94	27.8	27.8	0.0	7.3
69	1	24.3	7.2	1	0	0	174.96	24.3	0.0	0.0	7.2
70	1	27.5	6.3	1	0	0	173.25	27.5	0.0	0.0	6.3
71	1	28.8	7.3	1	0	0	210.24	28.8	0.0	0.0	7.3
72	1	25.7	7.1	1	1	0	182.47	25.7	25.7	0.0	7.1
73	1	23.8	4.8	1	1	0	114.24	23.8	23.8	0.0	4.8
74	1	28.6	7.5	1	0	0	214.50	28.6	0.0	0.0	7.5
75	1	18.9	4.5	1	0	0	85.05	18.9	0.0	0.0	4.5
76	1	21.6	6.6	1	0	0	142.56	21.6	0.0	0.0	6.6
77	1	27.3	7.1	1	0	0	193.83	27.3	0.0	0.0	7.1
78	1	24.8	7.0	1	1	0	173.60	24.8	24.8	0.0	7.0
79	1	23.7	5.2	1	1	0	123.24	23.7	23.7	0.0	5.2
80	1	28.9	6.7	1	0	0	193.63	28.9	0.0	0.0	6.7

	RB:loc2	RB:loc3	pri1:loc2	pri1:loc3
1	4.3	0.0	0	0
2	0.0	0.0	0	0
3	0.0	0.0	0	0
4	0.0	3.9	0	0
5	4.5	0.0	0	0
6	0.0	0.0	0	0
7	0.0	0.0	0	0
8	5.0	0.0	0	0
9	0.0	0.0	0	0
10	5.3	0.0	0	0
11	0.0	0.0	0	0
12	0.0	0.0	0	0
13	0.0	0.0	0	0
14	0.0	0.0	0	0
15	0.0	3.6	0	0
16	4.6	0.0	0	0
17	0.0	0.0	0	0
18	0.0	0.0	0	0
19	0.0	0.0	0	0
20	0.0	0.0	0	0
21	5.5	0.0	0	0
22	4.2	0.0	0	0
23	0.0	0.0	0	0
24	0.0	4.4	0	0
25	0.0	0.0	0	0
26	0.0	0.0	0	0
27	0.0	0.0	0	0
28	0.0	0.0	0	0
29	0.0	3.5	0	0
30	0.0	0.0	0	0
31	7.6	0.0	1	0

32	7.5	0.0	1	0
33	0.0	0.0	0	0
34	6.0	0.0	1	0
35	6.7	0.0	1	0
36	0.0	5.0	0	1
37	0.0	0.0	0	0
38	0.0	0.0	0	0
39	0.0	0.0	0	0
40	0.0	0.0	0	0
41	0.0	0.0	0	0
42	0.0	5.9	0	1
43	0.0	0.0	0	0
44	0.0	0.0	0	0
45	0.0	0.0	0	0
46	0.0	0.0	0	0
47	0.0	0.0	0	0
48	0.0	0.0	0	0
49	0.0	4.8	0	1
50	0.0	0.0	0	0
51	7.3	0.0	1	0
52	0.0	0.0	0	0
53	0.0	0.0	0	0
54	0.0	0.0	0	0
55	0.0	0.0	0	0
56	0.0	0.0	0	0
57	5.4	0.0	1	0
58	0.0	0.0	0	0
59	6.1	0.0	1	0
60	0.0	0.0	0	0
61	0.0	0.0	0	0
62	6.1	0.0	1	0
63	0.0	0.0	0	0
64	7.1	0.0	1	0
65	5.4	0.0	1	0
66	5.3	0.0	1	0
67	0.0	5.1	0	1
68	7.3	0.0	1	0
69	0.0	0.0	0	0
70	0.0	0.0	0	0
71	0.0	0.0	0	0
72	7.1	0.0	1	0
73	4.8	0.0	1	0
74	0.0	0.0	0	0
75	0.0	0.0	0	0
76	0.0	0.0	0	0
77	0.0	0.0	0	0
78	7.0	0.0	1	0
79	5.2	0.0	1	0
80	0.0	0.0	0	0



Y
C
RB
pri
loc