| | | |
|---|---|---|
| Math 3080 § 1. | **Peanut Example:** | Name: Example |
| Treibergs | **$2^5$ Half Replication Factorial Experiment** | Feb. 8, 2014 |

We describe a $2^5$ experimental design with one half replicate per cell. It comes from a paper by Kligo, "An application of fractional experimental factorial designs" *Quality Engineering,*1, (1989) 45-54 who studied a process to extract oil from peanuts. It is quoted from Levine, Ramsey and Smidt, *Applied Statistics for Engineers and Scientists,*Prentice Hall, 2001.

The defining contrast is $ABCDE$. The testing combinations are the words that have an even number of symbols in common with $ABCDE$. The paired ones are the odds which are confounded (in data set order)

```
Evens  Odds
   1   ABCDE
  AE     BED
  BE     ACD
  AB     CDE
  CE     ABD
  AC     BDE
  BE     ACD
ABCE       D
  DE     ABC
  AD     BCE
  BD     ACE
ABDE       C
  CD     ABE
ACDE       B
BCDE       A
ABCD       E
```

The estimators for effects may be determined as contrasts. For example, the experimental combination is included if the combination word. For example, to determine "+1" signs in the contrast, see which treatment combination includes the word, for example $A$ is included if $A \in W$ or the power of $A$ in $W$ is odd:

$$\begin{aligned} A \in & \langle 1, AE, BE, AB, CE, AC, BE, ABCE, DE, AD, BD, ABDE, CD, ACDE, BCDE, ABCD \rangle \\ &= \langle 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1 \rangle \end{aligned}$$

so

$$\widehat{(\alpha)_H} = \frac{1}{16} \begin{pmatrix} -(1) + (AE) - (BE) + (AB) - (CE) + (AC) - (BE) + (ABCE) \\ -(DE) + (AD) - (BD) + (ABDE) - (CD) + (ACDE) - (BCDE) + (ABCD) \end{pmatrix}$$

Similarly, the estimator for a two letter effect is included if there is an even number of letters in common

$$\begin{aligned} AB \in & \langle 1, AE, BE, AB, CE, AC, BE, ABCE, DE, AD, BD, ABDE, CD, ACDE, BCDE, ABCD \rangle \\ &= \langle 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1 \rangle \end{aligned}$$

so

$$\widehat{(\alpha\beta)_H} = \frac{1}{16} \begin{pmatrix} (1) - (AE) - (BE) + (AB) + (CE) - (AC) - (BE) + (ABCE) \\ +(DE) - (AD) - (BD) + (ABDE) + (CD) - (ACDE) - (BCDE) + (ABCD) \end{pmatrix}$$

**Data Set Used in this Analysis :**

---

```
# Math 3080 - 1         Peanut Data           Feb. 9, 2014
# Treibergs
#
# The paper by Kligo, "An application of fractional experimental factorial
# designs" Quality Engineering 1, (1989) 45-54 studied a process to extract
# oil from peanuts. Quoted from Levine, Ramsey and Smidt, Applied
# Statistics for Engineers and Scientists, Prentice Hall, 2001.
#
# The response is the amount of oil (solubility) that could dissolve in the
# carbon dioxide.
#
# Factors    (Levels H=high, L=low)
# A = Carbon dioxide pressure
# B = Carbon dioxide temperature
# C = Peanut moisture
# D = Carbon dioxide flow rate
# E = Peanut particle size
#
"A" "B" "C" "D" "E" "Solubility"
L L L L L  29.2
H L L L H  23.0
L H L L H  37.0
H H L L L 139.7
L L H L H  23.3
H L H L L  38.3
L H H L L  42.6
H H H L H 141.4
L L L H H  22.4
H L L H L  37.2
L H L H L  31.3
H H L H H  48.6
L L H H L  22.9
H L H H H  36.2
L H H H H  33.6
H H H H L 172.6
```

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**R Session:**

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```
R version 2.13.1 (2011-07-08)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

R is a collaborative project with many contributors.
```
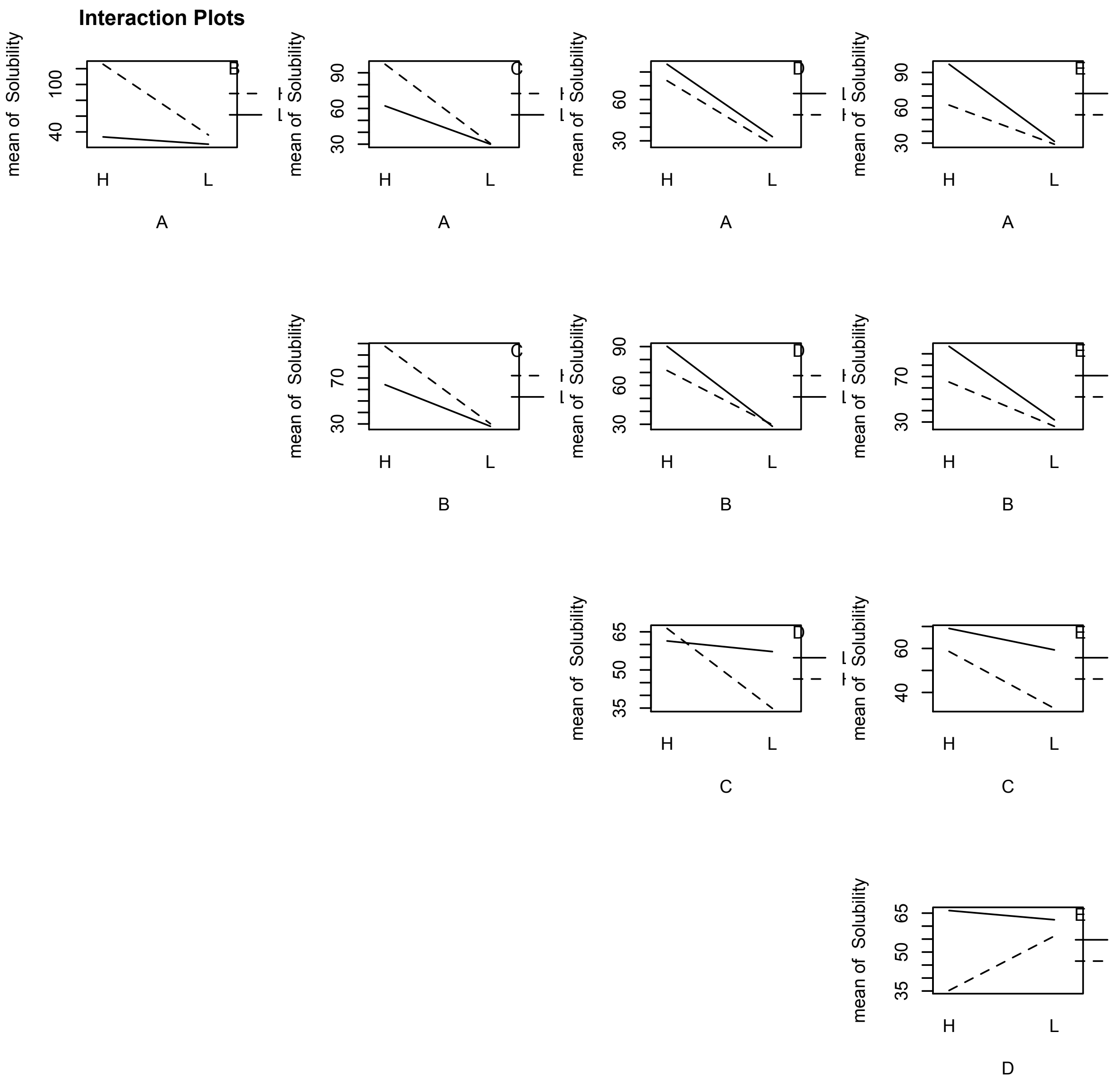
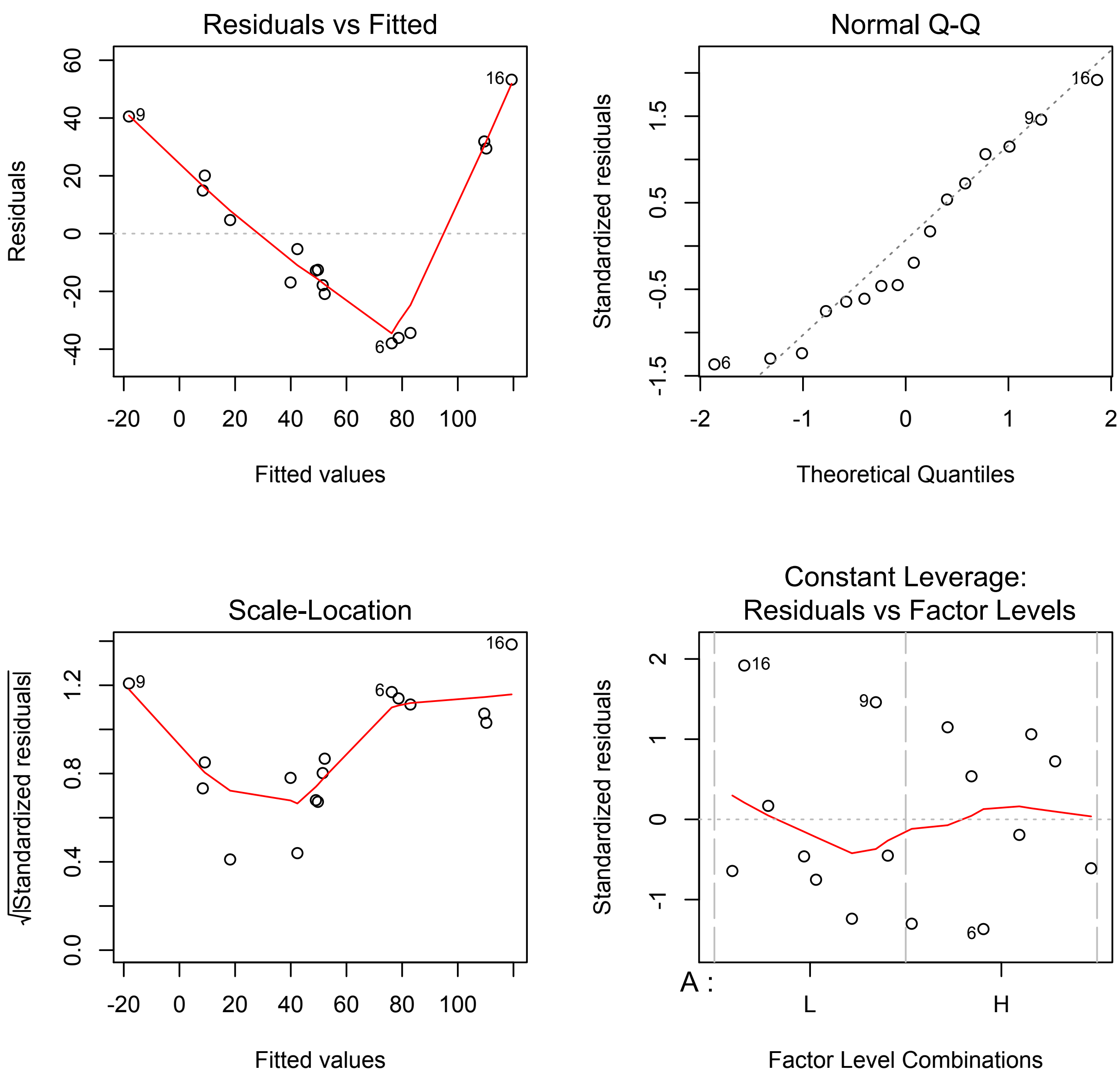
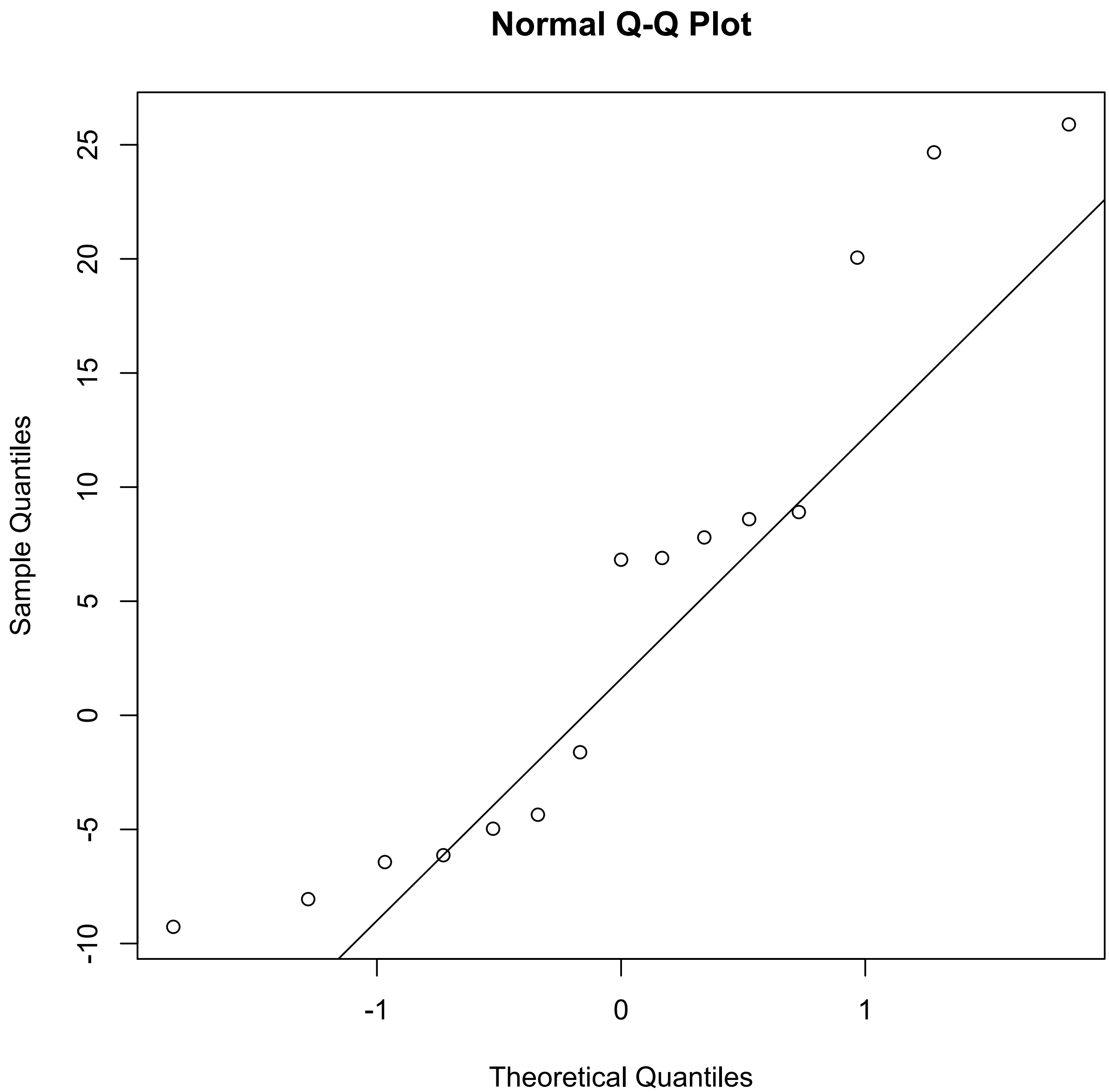

```
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.41 (5874) i386-apple-darwin9.8.0]

[History restored from /Users/andrejstreibergs/.Rapp.history]

> tt=read.table("M3082DataPeanut.txt",header=T)
> attach(tt)
> tt
   A B C D E Solubility
1  L L L L L       29.2
2  H L L L H       23.0
3  L H L L H       37.0
4  H H L L L      139.7
5  L L H L H       23.3
6  H L H L L       38.3
7  L H H L L       42.6
8  H H H L H      141.4
9  L L L H H       22.4
10 H L L H L       37.2
11 L H L H L       31.3
12 H H L H H       48.6
13 L L H H L       22.9
14 H L H H H       36.2
15 L H H H H       33.6
16 H H H H L      172.6
> A=factor(A); B=factor(B); C=factor(C); D=factor(D); E=factor(E)

> ######## ANALYSIS OF VARIANCE #################################
> a1=aov(Solubility~A+B+C+D+E); summary(a1)
            Df  Sum Sq Mean Sq F value  Pr(>F)
A            1  9736.8  9736.8  7.8954 0.01848 *
B            1 10727.8 10727.8  8.6990 0.01455 *
C            1  1269.1  1269.1  1.0291 0.33428
D            1   303.6   303.6  0.2462 0.63048
E            1  1374.6  1374.6  1.1146 0.31592
Residuals   10 12332.2  1233.2
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> ########### INTERACTION PLOTS ###################################
> layout(matrix(c(1,11,12,13,2,5,14,15,3,6,8,16,4,7,9,10),ncol=4))
> interaction.plot(A,B,Solubility,main="Interaction Plots")
> interaction.plot(A,C,Solubility)
> interaction.plot(A,D,Solubility)
> interaction.plot(A,E,Solubility)
> interaction.plot(B,C,Solubility)
```

```
> interaction.plot(B,D,Solubility)
> interaction.plot(B,E,Solubility)
> interaction.plot(C,D,Solubility)
> interaction.plot(C,E,Solubility)
> interaction.plot(D,E,Solubility)
>
> ############# TABLES OF MEANS AND EFFECTS ###################
> model.tables(a1,"means")
Tables of means
Grand mean
54.95625

A
    H     L
79.62 30.29

B
    H     L
80.85 29.06

C
    H     L
63.86 46.05

D
    H     L
50.60 59.31

E
    H     L
45.69 64.23

> model.tables(a1)
Tables of effects
A    H       L
 24.669 -24.669

B    H       L
 25.894 -25.894

C    H      L
 8.906 -8.906

D   H      L
-4.356  4.356

E   H      L
-9.269  9.269

> ################# DIAGNOSTIC PLOTS ##############################
> plot(a1)
```

```
> ############ EFFECTS AS CONTRASTS ###############################
> # GRAND MEAN
> e1=sum(Solubility*c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1))/16; e1
[1] 54.95625
> eA=sum(Solubility*c(-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1))/16; eA
[1] 24.66875
> eB=sum(Solubility*c(-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1))/16; eB
[1] 25.89375
> eAB=sum(Solubility*c(1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1))/16; eAB
[1] 20.05625
> eC=sum(Solubility*c(-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1,1,1))/16; eC
[1] 8.90625
> eAC=sum(Solubility*c(1,-1,1,-1,-1,1,-1,1,   1,-1,1,-1,-1,1,-1,1))/16; eAC
[1] 8.59375
> eBC=sum(Solubility*c(1,1,-1,-1,-1,-1,1,1,   1,1,-1,-1,-1,-1,1,1))/16; eBC
[1] 7.79375
> eD=sum(Solubility*c(-1,-1,-1,-1,-1,-1,-1,-1,   1,1,1,1,1,1,1,1))/16; eD
[1] -4.35625
> eAD=sum(Solubility*c(1,-1,1,-1,1,-1,1,-1,   -1,1,-1,1,-1,1,-1,1))/16; eAD
[1] -1.61875
> eBD=sum(Solubility*c(1,1,-1,-1,1,1,-1,-1,   -1,-1,1,1,-1,-1,1,1))/16; eBD
[1] -4.96875
> eCD=sum(Solubility*c(1,1,1,1,-1,-1,-1,-1,   -1,-1,-1,-1,1,1,1,1))/16; eCD
[1] 6.81875
> eE=sum(Solubility*c(-1,1,1,-1,1,-1,-1,1,  1,-1,-1,1,-1,1,1,-1))/16; eE
[1] -9.26875
> eAE=sum(Solubility*c(1,1,-1,-1,-1,-1,1,1,  -1,-1,1,1,1,1,-1,-1))/16; eAE
[1] -8.05625
> eBE=sum(Solubility*c(1,-1,1,-1,-1,1,-1,1,  -1,1,-1,1,1,-1,1,-1))/16; eBE
[1] -6.43125
> eCE=sum(Solubility*c(1,-1,-1,1,1,-1,-1,1,  -1,1,1,-1,1,1,1,-1))/16; eCE
[1] 6.89375
> eDE=sum(Solubility*c(1,-1,-1,1,-1,1,1,-1,  1,-1,-1,1,-1,1,1,-1))/16; eDE
[1] -6.13125
> ef=c(eA,eB,eAB,eC,eAC,eBC,eD,eAD,eBD,eCD,eE,eAE,eBE,eCE,eDE)
> ############### PLOT QQPLOT OF EFFECTS. OUTLIERS ARE SIGNIFICANT ######

> layout(1)
> qqnorm(ef);qqline(ef)
```


Interaction Plots
mean of Solubility
H
L
A
B
C
D
E
40
100
30
60
90
70
35
50
65


Residuals vs Fitted
Residuals
Fitted values
Normal Q-Q
Standardized residuals
Theoretical Quantiles
Scale-Location
√|Standardized residuals|
Fitted values
Constant Leverage:
Residuals vs Factor Levels
Standardized residuals
A :
L
H
Factor Level Combinations
16
9
6


Normal Q-Q Plot
25
20
15
10
5
0
-5
-10
Sample Quantiles
-1
0
1
Theoretical Quantiles