Math 3210 § 2. **Solutions of Homework Problems** Name: *SAMPLE*
Treibergs September 15, 2008

These problems were due on September 8, 2009. They are taken from An Introduction to Analysis by Wm. R. Wade, Prentice Hall 2004.

[6.] Let X and Y be sets and $f : X \to Y$. Prove that the following are equivalent.
a.) f is 1–1 on X.
b.) $f(A \setminus B) = f(A) \setminus f(B)$ for all subsets A and B of X.
c.) $f^{-1}(f(E)) = E$ for all subsets E of X.
d.) $f(A \cap B) = f(A) \cap f(B)$ for all subsets A and B of X.

[7.] Let $E_\alpha \subset X$ be subsets for all $\alpha \in A$. Then

$$\left(\bigcup_{\alpha\in A} E_\alpha\right)^c = \bigcap_{\alpha\in A} E_\alpha^c, \tag{1}$$

$$\left(\bigcap_{\alpha\in A} E_\alpha\right)^c = \bigcup_{\alpha\in A} E_\alpha^c. \tag{2}$$

[8.] Let X, Y be sets and $f : X \to Y$ be a function. Then
(iii.) If $E_\alpha \subset X$ are subsets for $\alpha \in A$, then

$$f^{-1}\left(\bigcup_{\alpha\in A} E_\alpha\right) = \bigcup_{\alpha\in A} f^{-1}(E_\alpha), \tag{3}$$

$$f^{-1}\left(\bigcap_{\alpha\in A} E_\alpha\right) \subset \bigcap_{\alpha\in A} f^{-1}(E_\alpha). \tag{4}$$

(iv.) If $B, C \subset Y$ then

$$f^{-1}(C \setminus B) = f^{-1}(C) \setminus f^{-1}(B). \tag{5}$$

(v.) If $S \subset f(X)$ then $f\left(f^{-1}(S)\right) = S$. If $E \subset X$ then $f^{-1}(f(E)) \supset E$.

[9.] Suppose X, Y, Z are sets and $f : X \to Y$ and $g : Y \to Z$ are functions. Let $g \circ f : X \to Z$ be defined by $g \circ f(x) = g(f(x))$ for all $x \in X$.
(i.) If both f and g are one-to-one then $g \circ f$ is one-to-one.
(ii.) If both f and g are onto, then $g \circ f$ is onto.

These problems were taken from Introduction to Analysis by A. Mattuck, Prentice Hall 1999.

[B.1.1] A tail of a sequence $\{a_n\}_{n\in\mathbb{N}}$ is the sequence after removing the first $N-1$ terms of $\{a_n\}$. Write the given sentence with quantifiers in the right order; Interchange the order of the first two quantifiers if possible and interpret.

Every tail of $\{a_n\}_{n\in\mathbb{N}}$ has a maximal element.

[B.1.2.] Render in English a statement equivalent of the negated sentence. Then write the given sentence with quantifiers and negations in the right order. Use the rules for negating quantifiers to rewrite the original sentence.

$\mathcal{S} :=$ "The sequence $\{a_n\}_{n\in\mathbb{N}}$ has no minimum."

[B.1.3.] Write the given sentence with quantifiers in the right order. Use the rules for negating quantifiers to negate the sentence. Then verify that the given is a counterexample to the sentence.

f is bounded on the interval $I = (0, 1]$. $\quad f(x) = \dfrac{1}{x}$ is not bounded above on I.