Homework for Math 3210 §1, Spring 2019

A. Treibergs, Instructor

April 16, 2019

Our text is by Joseph L. Taylor, *Foundations of Analysis,* American Mathematical Soceity, Providence (2012). Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 19, whichever comes first.

Your written work reflects your professionalism. Make answers complete and self contained. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 not later than 3:00 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 – A5 on Friday, January 11.

Supplementary notes by Anne Roberts, "Basic Logic Concepts," 2005 are available

`http://www.math.utah.edu/%7Earoberts/M3210-1d.pdf`

A1. Truth table. Construct a truth table for the following statement.

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q.$$

A2. Equivalent Statements. Verify using truth tables that

$$P \wedge [\sim (Q \wedge R)]$$

is equivalent to

$$(P \wedge [\sim Q]) \vee (P \wedge [\sim R]).$$

A3. Quantified Statements. Determine the truth value of each statement assuming that x, y, z are real numbers. Prove your answers.

$$(\exists x)(\forall y)(\exists z)(x + y = z);$$
$$(\exists x)(\forall y)(\forall z)(x + y = z);$$
$$(\forall x)(\forall y)(\exists z)[(z > y) \Rightarrow (z > x + y)];$$
$$(\forall x)(\exists y)(\forall z)[(z > y) \Rightarrow (z > x + y)].$$

A4. Negate and Interpret. Write formally, with quantifiers in the right order. Negate the sentence and interpret the negation as an English sentence.

"Everybody doesn't like something but nobody doesn't like Sara Lee."

A5. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

```
7[2, 5]
```

1.1.2 If A, B and C are sets, prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

1.1.5 What is the intersection of all closed intervals containing the open interval $(0, 1)$? Justify your answer.

Please hand in problems B1 – B2 on Friday, Jan. 18.

B1. Image and intersection. Let A and B be sets and $f : A \to B$ be a function. Show that f is one-to-one if and only if for all subsets $E, F \subset A$,

$$f(E \cap F) = f(E) \cap f(F).$$

B2. Functions and Induction. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
7[10, 12]
15[8, 9, 12, 14, 16]
```

Please hand in problem C1 on Friday, Jan. 25.

C1. Fields. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
20[3, 5, 7]
```

Please hand in problem D1 on Friday, Feb. 1.

D1. Order and Defects in the Rationals. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
20[8, 10, 12]
25[2, 4]
```

Please hand in problem E1 – E2 on Friday, Feb. 8.

E1. Postponed to next week.

E2. Completeness of the Reals. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
25[7, 8, 9]
```

Please hand in problem F1 – F3 on Friday, Feb. 15.

F1. Sup of Sum. Let f and g be functions defined on a set containing A as a subset. Show

$$\sup_A (f+g) \le \sup_A f + \sup_A g.$$

F2. Triangle Inequality Prove that if a, b, x, y are real numbers that satisfy the inequalities

$$|x-a| < 1, \qquad |y-b| < 2, \qquad |a-b| > 7$$

then $|x-y| > 4$.

F3. Limits. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
31[2, 8a, 9]
38[5]
42[5, 8, 9, 10]
```

Please hand in problems G1 on Friday, Feb. 22.

G1. Exercises from Taylor's *Foundations of Analysis.*

```
45[1, 8, 11, 12]
50[3, 4, 5, 9]
```

Please hand in problems H1 on Friday, Mar. 1.

H1. Exercises from Taylor's *Foundations of Analysis.*

```
54[1, 2, 3(Hint: bisection), 7, 8]
58[9]
```

Please hand in problems I1 on Friday, Mar. 8.

I1. Exercises from Taylor's *Foundations of Analysis.*

```
54[9, 10, 11, 12]
58[7]
64[3, 8, 11, 12]
```

Please hand in problems J1 on Friday, Mar. 22.

J1. Exercises from Taylor's *Foundations of Analysis.*

```
68[2, 4, 8, 9]
73[1, 3, 4, 7, 8]
```

Please hand in problems K1 on Friday, Mar. 29.

K1. Exercises from Taylor's *Foundations of Analysis.*

```
77[5, 7, 8, 12]
84[2, 4, 11]
88[1, 9, 11]
```

Please hand in problems L1 on Friday, Apr. 5.

L1. Exercises from Taylor's *Foundations of Analysis.*

```
88[7, 10]
92[3, 5, 7, 8]
```

Please hand in problems M1 on Friday, Apr. 12 .

M1. Problems from Taylor's *Foundations of Analysis.*

```
 98[1, 2, 4, 5, any two from 6-15]
107[2, 5, 8, 11]
113[7, 9, 10, 13]
```

Please hand in problems N1 on Friday, Apr. 19 .

N1. Problems from Taylor's *Foundations of Analysis.*

```
119[4, 5, 8, 9]
127[1, 9, 13]
133[4, 5, 6, 9, 13]
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The FINAL EXAM is Mon., April 29, 10:30 am - 12:30 pm.in the usual room AEB 360.