Math 5470 **Applied Dynamical Systems** **Spring 1999**

M W F 9:40-10:30 pm

NS 201

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Final Exam **Due: 12-noon, Tuesday, May 4, 1999**

NAME..

Provide an explanation for each of your answers.

1. What is the dimension of the phase space for a system of two particles, which is described by the following equations:

$$\ddot{\mathbf{r}}_1 + 0.1\dot{\mathbf{r}}_1 = (\mathbf{r}_2 - \mathbf{r}_1)(\mathbf{r}_1 - \mathbf{r}_2)^2 t$$
$$\ddot{\mathbf{r}}_2 + 0.01\dot{\mathbf{r}}_1 = (\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{r}_2 - \mathbf{r}_1)^2 t$$

($\mathbf{r}_1$ and $\mathbf{r}_2$ are 3-dimensional vectors that chatacterize the locations of the particles)?

2. Consider the system

$$\dot{x} = x(1 - \mu x + x^2)$$

($x(t)$ is a scalar function, μ is a real parameter). How many bifurcation points does it have?
Plot its bifurcation diagram for $-\infty < \mu < +\infty$, and indicate the stability of various branches of fixed points.

3. Find the index of the unit circle $\{(x, y) : x^2 + y^2 = 1\}$ with respect to the vector field defined by the following system

$$\dot{x} = x^2 + y^2, \quad \dot{y} = x^2 - y^2.$$

4. Consider the system

$$\dot{x} = y(1 - x), \quad \dot{y} = x(1 - y).$$

Does it have a periodic orbit?

5. Can a fixed point be stable but not attracting?
   Can a fixed point be attracting but not stable?

6. The system

$$\begin{aligned}\dot{x} &= 1 - \mu x + x^2 y,\\ \dot{y} &= (\mu - 1)x - x^2 y\end{aligned}$$

   is known to have a Hopf bifurcation at some critical value of the parameter μ. What is this critical value?

7. Consider an oscillator with "small" dissipation:

$$\ddot{x} + \epsilon\dot{x} + x = 0$$

(ϵ is a small parameter, $0 \leq \epsilon \ll 1$). This second order differential equation can be written as a system of two first order equations:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x - \epsilon y \end{aligned}$$

Take the positive x-axis as the Poincare section line and find the corrse- ponding Poincare map.

8. Give an example of a system which simultaneously possesses the following two properties
(1) it is dissipative, i.e. any volume in phase space contracts under the flow,
(2) "almost" any trajectoty goes to infinity as time $t \to \infty$.

9. Is it true that all trajectoties in the Lorenz system do not go to infinity as $t \to \infty$?
What does it mean that the Lorenz system exhibits sensitive dependence on initial conditions?

10. Consider the iterated map $x_{n+1} = 2 + \mu x_n$
(μ is a parameter, $-\infty < \mu < +\infty$).
1). Does it exhibit sensitive dependence on initial conditions?
2). Is the dynamics of this map chaotic?

11. What is the Lorenz map? How did Lorenz decide that the attractor in his system is not just a “vert long” stable periodic orbit?

12. Consider Feigenbaum's renormalization equation

$$g(x) = \alpha g^2(x/\alpha).$$

Verify that

$$\text{the function } g(x) = \frac{x}{1+cx} \text{ together with the constant } \alpha = 2 \qquad (1)$$

satisfies this equation (c is an arbitrary constant). Explain why the solution (1) does not describe the sequence of period doubling bifurcations in the logistic map.

13. Devide the closed interval $[0, 1]$ into four quarters. Delete the open second quarter from the left. This produces S_1. Repeat this construction indefinitely; i.e. generate S_{n+1} from S_n by deleting the second quarter of each of the intervals in S_n.
    1). Sketch the sets $S_1, \ldots, S_4$.
    2). Compute the box dimension of the limitting set S_∞.
    3). Is S_∞ self-similar?