MATH 3210 - SUMMER 2008 - PRACTICE FINAL

You have two and a half hours to complete this test. Show all your work. There are a total of 105 points. The maximum grade is 100.

question	**grade**	**out of**
1		20
2a		15
2b		10
2c		10
3		15
4		10
5a		5
5b		5
5c		5
5d		10
total		105

Student Number: ______________________

(1) (20 pts) State and prove the monotone convergence theorem. If you use other theorems in your proof you must state them in full but don't prove them.

(2) (35 pts)

(a) (15 pts) Prove by Cauchy's definition that $\lim_{x \to 2} \frac{x-3}{x+1} = -\frac{1}{3}$

(b) (10 pts) State the sequential characterization of $\lim_{x \to a} f(x) = L$ for a, L finite (this is Heine's definition).

(c) (10 pts) Consider $D(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \notin \mathbb{Q} \end{cases}$ Prove that for any $a \in \mathbb{R}$ the limit $\lim_{x \to a} D(x)$ doesn't exist.

(d) (15 pts) Compute the limit $\lim_{x\to 1} \frac{\int_1^{x^2} e^{\cos(t)}dt}{x-1}$. You must explain every step, and quote the theorems that you are using.

(e) (10 pts) Prove: Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable functions. Suppose $f(a) = g(a)$ and for all $x > a$: $f'(x) \leq g'(x)$ then for all $x > a$: $f(x) \leq g(x)$.

You must explain every step and quote the theorems that you're using.

(f) (25 pts) For each of the following statements, determine if it is true or false. If the statement is false find a counter example. If it is true, prove it. You are allowed and encouraged to appeal to the theorems proven in class and in your homework as long as you quote them in full.

(i) (5 pts) True/False:

The equation $e^{-x} - e^{x} = \pi$ has a real solution.

(ii) (5 pts) True/False:

Every integrable function on $[a, b]$ is continuous on $[a, b]$

(iii) (5 pts) True/False:

Every differentiable function on $[a, b]$ is continuous on $[a, b]$

(iv) (10 pts) True/False:

Suppose $f(x)$ is a function which satisfies:

$$|f(y) - f(x)| \leq K|y - x|^2$$

for some constant $K > 0$ and for all $x, y \in \mathbb{R}$.

Then $f'(x) = 0$ for all $x \in \mathbb{R}$.