MATH 3210 - SUMMER 2008 - ASSIGNMENT #2

Induction and $\sum$ notation

(1) Expand the following expressions i.e. write the first 3 terms and last three terms (with dots in between) in each sum:

(a) $\displaystyle\sum_{i=0}^{n} i$

(b) $\displaystyle\sum_{i=0}^{n}(n-i)$

(c) $\displaystyle\sum_{i=0}^{n} 2^i$

(d) $\displaystyle\sum_{k=1}^{n+1} 2^{k-1}$

(e) $\displaystyle\sum_{i=0}^{n} \frac{3}{4}2^i$

(f) $\displaystyle\sum_{j=-2}^{n-2} 3 \cdot 2^j$

(2) Write the following expressions in $\sum$ form:

(a) $1+3+5+\cdots+13$

(b) $6+9+12+\cdots+24$

(c) $1+3+9+27+81$

(3) Find and prove formulas for the following expressions:

(a) $\displaystyle\sum_{i=0}^{n} i - \sum_{j=1}^{n+1} j =$

(b) $\displaystyle\sum_{i=0}^{2n}(-3i^2-2i) - \sum_{j=n}^{2n}(-3j^2-2j)$

(c) $\displaystyle\sum_{k=1}^{n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k+1}$

(d) $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)}$ (hint: use the previous item).

(4) Prove the following statements using induction (can you prove them in another way?)

(a) If $0 < a < b$ then for all $n \in \mathbb{N}$: $a^n < b^n$

(b) Prove that $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$

(c) Prove that $\sum_{k=1}^{n} (7 + 3(k-1)) = \frac{3}{2}n^2 + \frac{11}{2}n$

(d) Prove that for any real a, q and for any integer n: $\sum_{i=0}^{n} aq^i = \frac{a - aq^{n+1}}{1-q}$

(5) (a) Using the formula for $\binom{n}{k}$ prove that: $\binom{n}{k} = \binom{n}{n-k}$

(b) Prove the same thing but only using the definition of $\binom{n}{k}$ (i.e. that it is the number of k-element subsets of the set $\{1, \dots, n\}$).

(6) (a) Using the formula for $\binom{n}{k}$ prove that: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

(b) Prove the same thing but only using the definition of $\binom{n}{k}$ (i.e. that it is the number of k-element subsets of the set $\{1, \dots, n\}$).

(7) Using induction prove the binomial formula (hint: if you get stuck you can use the proof in page 12 of the text but you must explain every step).