MATH 3210 - SUMMER 2008 - ASSIGNMENT #4

Limits of Sequences

(1) Using the definition of convergence of a sequence, prove the following: (don't forget the three steps of proof...)

1) $\lim_{n\to\infty} \frac{500}{n} = 0$

2) $\lim_{n\to\infty} \frac{2n-15}{5n+1} = \frac{2}{5}$

3) $\lim_{n\to\infty} \frac{3n^2+2n}{n^2-n+15} = 3$

4) $\lim_{n\to\infty} \sqrt{2n+5} - \sqrt{2n} = 0$

5) $\lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n^2} = 0$

6) $\lim_{n\to\infty} \frac{1}{2^n} = 0$

7) $\lim_{n\to\infty} \frac{n+\sin(\frac{\pi}{2}n)}{2n+5} = \frac{1}{2}$

8) $\lim_{n\to\infty} \frac{2^n}{n!} = 0$

9) $\lim_{n\to\infty} \frac{2n+5}{n+1} \neq 1$

10) $\lim_{n\to\infty} \frac{1-2n}{3n-13} \neq \frac{2}{3}$

Hint for #8: First prove by induction that for $n > 5$: $\frac{2^n}{n!} \leq \frac{1}{n}$

(2) Prove that the sequence $a_n = \cos(\frac{\pi}{2}n)$ diverges.

(3) Which of the following conditions is equivalent to the definition of $\lim_{n\to\infty} a_n = L$? If it is equivalent prove it. If the condition is not equivalent find an example of a sequence which satisfies the condition but doesn't converge, or a sequence which converges but doesn't satisfy the condition.

(a) For all $\varepsilon > 0$ there is an $N(\varepsilon) \in \mathbb{N}$ such that for all $n > N(\varepsilon)$: $|a_n - L| < 2\varepsilon$

(b) For all $\varepsilon > 0$ there is an $N(\varepsilon) \in \mathbb{N}$ such that for all $n > N(\varepsilon)$: $a_n - L < \varepsilon$

(c) For all $\varepsilon > 0$ there is an $N(\varepsilon) \in \mathbb{N}$ such that for all $n > N(\varepsilon)$: $|a_n - L| \leq \varepsilon$

(d) There is an ε for which there is an $N(\varepsilon) \in \mathbb{N}$ such that for all $n > N(\varepsilon)$: $|a_n - L| < \varepsilon$

(e) For all $\varepsilon > 0$ there is an $N(\varepsilon) \in \mathbb{N}$ such that for all $n > N(\varepsilon)$: $|a_n| - L < \varepsilon$